FOURTH EDITION

THE NEW APPLIED MATHEMATICS

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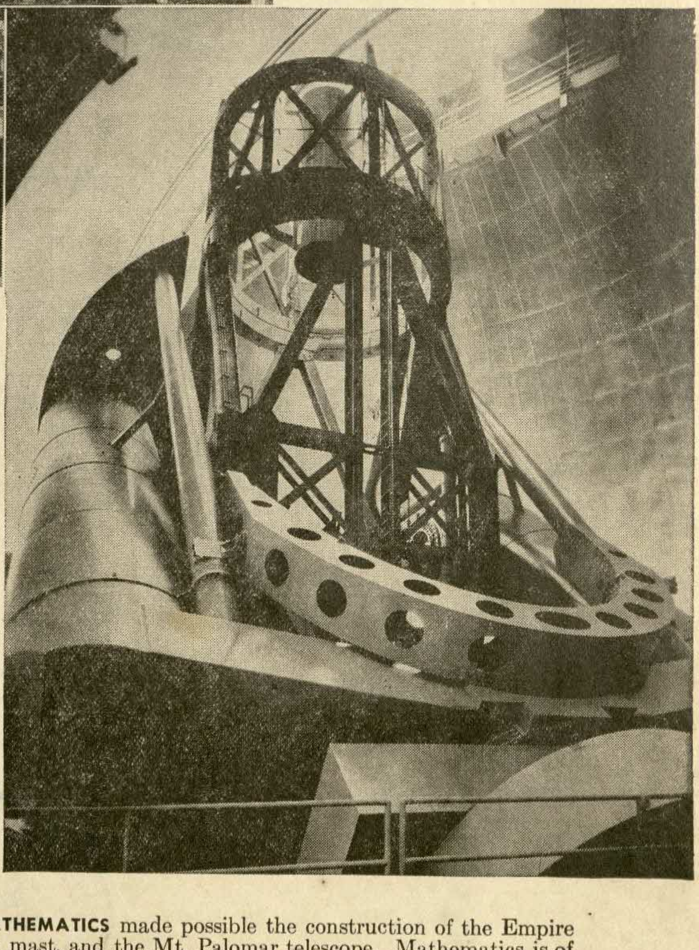
FOURTH EDITION

THE NEW APPLIED MATHEMATICS

The Empire State Building, New York City, boasts 102 stories and a total height of 1,472 feet from sidewalk to top of its 222-foot television mast.



The famous Hale Reflector, a 200-inch reflecting telescope of the California Institute of Technology, Pasadena, is the largest in the world. The telescope penetrates 1,000 million light years in the sky.

A KNOWLEDGE AND USE OF MATHEMATICS made possible the construction of the Empire State Building, with its television mast, and the Mt. Palomar telescope. Mathematics is of utmost importance in the present-day world. Photographs courtesy of Ewing Galloway, New York.

FOURTH EDITION

THE NEW APPLIED MATHEMATICS

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PREFACE

We today realize that a mastery of the basic processes of mathematics is most essential. The demands of the Second World War brought to light a national deficiency in the skills of the fundamentals of mathematics and in the ability to apply mathematical knowledge.

Governmental and industrial surveys have revealed an urgent need for courses in mathematics that will develop a higher degree of skill in the fundamentals of arithmetic, in the use of formulas and equations, and in the development of spatial concepts and mensuration. Consequently, schools are now offering courses which place an increased emphasis on basic arithmetic, basic algebra, and basic geometry.

The fourth edition of *The New Applied Mathematics* is designed to furnish students with mathematical abilities, knowledges, and experiences that will meet their everyday mathematical requirements.

The content of the book follows largely the guidance plan recommended in the *Final Report of the Commission on Post-War Plans* of The National Council of Teachers of Mathematics.

In general, the book offers:

a. A maximum course for one year's work in general mathematics. The large amount of material included provides for a selection of topics in accordance with class needs.

b. Optional exercises which provide for individual differences in a class.

c. A vocabulary study at the beginning of each chapter and a *Glossary of Mathematical Terms* in the *Appendix* to enable students to read the content more intelligently.

d. A plan of procedure to improve skill in the fundamentals of arithmetic and percentage. There are class *Inventory Tests* with individual follow-up *Practice Exercises* designed to overcome any weaknesses. For maintenance and improvement of the skills attained, there are *Review Exercises* supplemented by *Achievement Tests*.

e. Many topics which provide knowledges of value to a consumer in the home or in business. In these chapters there is an abundance of exercises which afford application of mathematical processes.

f. Geometry that is intuitive in nature. The students are made familiar with important geometric forms, mensuration, and constructions.

g. Formulas, equations, graphs, and signed numbers which are the most usable topics of algebra.

There is a *Forms for Practice* supplement, the use of which is optional, that provides printed forms to make the business procedure more real. The exercises appeal to the self-activity of the student, increase interest, and produce greater effort.

Such a program appeals to students and at the same time prepares them to meet everyday mathematical situations.

THE AUTHORS

TABLE OF CONTENTS

PARAGRAPH PAGE

INTRODUCTION

1. The beginnings of arithmetic 1
2. The importance of mathematics 2
3. Plan of procedure to improve the fundamentals of arithmetic . . 2

CHAPTER I—ADDITION, MULTIPLICATION, SUBTRACTION, AND DIVISION OF INTEGERS WITH APPLICATIONS

4. Terms used in addition 5
5. Proving addition by adding in the opposite direction 6
6. Finding the distance between places 6
7. Finding perimeters 7
8. Terms used in multiplication 10
9. Proving multiplication by interchanging the multiplier and the multiplicand 11
10. Proving multiplication by dividing the product by the multiplier or the multiplicand 12
11. Finding the area of rectangular airplane wings 12
12. Terms used in subtraction 13
13. Proving subtraction 14
14. Terms used in division 16
15. Proving division by multiplying the quotient and the divisor and adding the remainder to the product 17
16. Proving division by using the quotient as the divisor 17
17. Averages 17

CHAPTER II—ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF COMMON FRACTIONS WITH APPLICATIONS

18. Common fractions 20
19. Changing the form of a fraction 20
20. Addition of ruler fractions 23
21. Addition of non-ruler fractions 25
22. Changing the amount of a recipe 28
23. A special type of multiplication 29

CHAPTER III—ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF DECIMAL FRACTIONS WITH APPLICATIONS

24. Reading and writing decimal fractions 32
25. Changing common fractions to decimal fractions 33
26. Changing decimal fractions to common fractions 33

PARAGRAPH	PAGE
27. Arranging a series of numbers according to size	34
28. Decimal fractions in shop measurements	36
29. Multiplying a number by 10, 100, 1,000, and so on	39
30. Multiplying a number by a fractional part of 1	40
31. Multiplying a number by a fractional part of 100	41
32. Dividing a number by 10, 100, 1,000, and so on	42
33. To get reasonable results by estimating quotients	43
34. Rounding off numbers	44

CHAPTER IV—PERCENTAGE AND PROBLEM SOLVING

35. The meaning of per cent	48
36. The use of per cent	49
37. Estimating reasonable products	50
38. Problem solving	51
39. Percentage problems	53
40. To find a certain per cent of a number	54
41. To find what per cent one number is of another number	55
42. Calculating grades	55
43. To find a number when a certain per cent of it is known	56

CHAPTER V—DENOMINATE NUMBERS; RATIO AND PROPORTION

44. Denominate number tables	62
45. The meaning of "ratio"	68
46. The writing and use of ratios	69
47. Proportion	71
48. Using proportion	72

CHAPTER VI—DIRECT MEASUREMENT OF LINES AND ANGLES

49. The origin of our measurements	81
50. Measurement	82
51. The ruler	83
52. How to measure a line with a ruler	87
53. How to draw a line with a ruler	88
54. Studying the yardstick, tapeline, and steel tape	89
55. The metric system of measurement	89
56. Uses made of the metric system	91
57. Scale drawing	94
58. Angles	99
59. Lettering and reading angles	99
60. Unit of measurement of angles	99
61. Kinds of angles	100
62. Studying the protractor	100
63. How to measure an angle with a protractor	101
64. How to draw an angle with a protractor	102

PARAGRAPH PAGE

CHAPTER VII—GRAPHS

65. Graphs 104
66. The use of graphs in business 108
67. Pictographs 109
68. Bar graphs 111
69. Line graphs 115
70. Circle graphs 117

CHAPTER VIII—INSURANCE

71. Need for insurance 128
72. Terms of insurance 128
73. Kinds of insurance 128
74. Life insurance 129
75. Dividends 130
76. Kinds of life insurance 131
77. Ordinary life insurance 131
78. Limited payment life insurance 131
79. Endowment life insurance 132
80. Life annuities 133
81. Group insurance 133
82. Health, accident, and hospital insurance 136
83. Workmen's compensation insurance 136
84. Social Security 136
85. Fire protection 139
86. Eighty per cent coinsurance clause 139
87. Automobile insurance 142

CHAPTER IX—BANKS, SAVINGS, AND INVESTMENTS

88. Banks 145
89. Commercial banks 145
90. Opening a checking account 145
91. Signature card 146
92. Deposit tickets 146
93. Checkbook 149
94. Checkbook stub 149
95. Personal check 150
96. Writing the check 150
97. Indorsing the check 151
98. Bank statement 152
99. Reconciliation 153
100. Interest 155
101. Simple interest graph 157
102. Saving for a purpose 158
103. Saving money in a savings bank 158
104. Compound interest 159

PARAGRAPH PAGE
105. Compound interest table 161
106. Saving in a building and loan association 163
107. Investments 164
108. Bonds 165
109. Corporation bonds 165
110. United States Government bonds 166
111. Corporations and their stocks 168
112. Investing in life insurance 170

CHAPTER X—PROBLEMS OF THE CONSUMER

113. Planned spending 173
114. Budget 173
115. Making a weekly budget for a pupil 173
116. Making an annual budget for a family 176
117. Cash record 177
118. Making the cash record 178
119. Balancing the cash record 180
120. Deriving correct values in the home 181
121. Thrift in buying food 182
122. Thrift in buying clothing 185
123. Credit and installment buying 187
124. Consumer borrowing 190
125. Renting or owning a home 193
126. Depreciation 196
127. Making change 197

CHAPTER XI—LOCAL, STATE, AND FEDERAL TAXES

128. Why taxes are necessary 204
129. Kinds of taxes 204
130. General property taxes 205
131. Fixing the amount of property tax 206
132. State sales tax 208
133. Federal income tax 209
134. Import duties 213
135. Excise taxes 215
136. Special taxes 215

CHAPTER XII—POSITION AND FORM OF OBJECTS

137. Horizontal, vertical, and slanting lines 218
138. Parallel lines 219
139. Perpendicular lines 220
140. Straight-line designs 221
141. Position of places on a map 221
142. Plane figures 223

PARAGRAPH PAGE
143. Plane-figure designs 227
144. How to draw a circle, using the compasses 227
145. Solid figures 231
146. Observing solid geometric forms 231

CHAPTER XIII—THE SIZE OF PLANE AND SOLID FIGURES

147. The meaning of "area" 236
148. Perimeter of a rectangle 237
149. Base and altitude of plane figures 239
150. The area of a rectangle 239
151. The area of a square 242
152. The lateral area of a rectangular prism 243
153. The area of a parallelogram 244
154. The area of a triangle 245
155. The area of a trapezoid 247
156. The circumference and diameter of a circle 250
157. The area of a circle 254
158. The lateral area of a cylinder 258
159. Base and altitude of solid figures 259
160. The volume of a solid figure 260
161. The volume of a rectangular prism 260
162. The volume of a cylinder 264
163. The volume of a cone and pyramid 265

CHAPTER XIV—CONSTRUCTIONS

164. Kinds of triangles 268
165. The sum of the angles of a triangle 269
166. A practical use of triangles 270
167. How to bisect a line, using the compasses 270
168. How to bisect an angle, using the compasses 271
169. How to erect a perpendicular to a line from a given point in the line 272
170. How to drop a perpendicular to a line from a given point outside the line 272

CHAPTER XV—INDIRECT MEASUREMENT OF LINES

171. Distinguishing between direct and indirect measurement . . . 275
172. Squares and square roots 275
173. Uses of square root 279

CHAPTER XVI—THE FORMULA

174. Writing formulas 283
175. Some common algebraic terms 285

PARAGRAPH PAGE

176. Combining similar terms 286
177. Making algebraic formulas for perimeters and areas 289
178. Evaluating algebraic expressions 291
179. Making formulas 292
180. Finding the value of a formula 294
181. Graphing formulas 297

CHAPTER XVII—THE EQUATION

182. The meaning and purpose of equations 302
183. Terms used in equations 302
184. Writing and solving simple equations 303
185. Solving equations by using more than one of the preceding truths 310
186. Difficult arithmetic problems made easy by use of equations . . 312
187. Finding the value of a symbol in the right member of an equation 314

CHAPTER XVIII—SIGNED NUMBERS

188. The meaning of signed numbers 317
189. Using signed numbers to indicate opposites 318
190. Using signed numbers to indicate changes 319
191. Adding positive and negative numbers with like signs 321
192. Adding positive and negative numbers with unlike signs . . . 323
193. Adding several positive and negative numbers 325
194. Subtraction of signed numbers 326
195. Graphing positive and negative numbers 327

APPENDIX A—REMEDIAL PROGRAM—PRACTICE EXERCISES . . 331

APPENDIX B—GLOSSARY OF MATHEMATICAL TERMS 359

APPENDIX C—BRAIN TEASERS 364

APPENDIX D—USEFUL TABLES 368

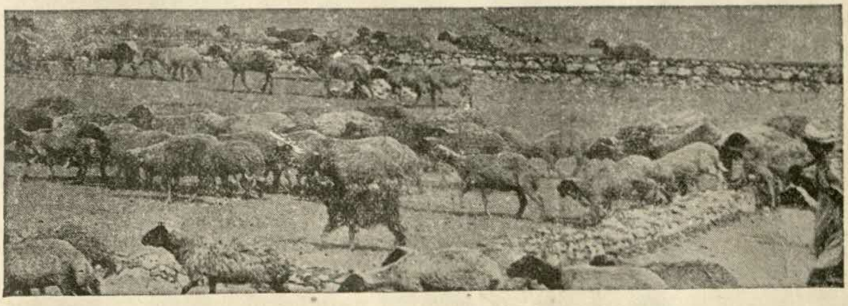
APPENDIX E—ANSWERS TO PRACTICE EXERCISES 371

INDEX 380

INTRODUCTION

1. The beginnings of arithmetic. Arithmetic had its beginning when man first began to count. It served to answer the ever-present question: "How many?"

Our number system has been in the process of development for hundreds of centuries. It is difficult for us to realize that once upon a time there were no names for numbers. Primitive man showed how many sheep he owned by placing as many rocks in a pile, or sticks in a row, as he had sheep.



After the number *one* came into being, primitive man expressed himself as having *one* sheep; or, if he had two or more sheep, he said that he had *many* sheep. Later the word *two* came along, and he expressed himself as having *one, two,* or *many* sheep.

After names for numbers came into existence, it was a long, long time before any symbols were written to represent those names. How well do you think you would succeed in adding a column of numbers expressed in words?

Arithmetic computation did not begin until man began to use symbols for numbers. The simple life of the early days could be lived without it. As tribes and nations developed, calculation became a necessity—so much so that the highly organized life of today calls for figuring at every turn. The time of day, sums of money, and even addresses are today expressed in numbers rather than words.

All of our numbers are combinations of the ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. For example, the number 728 is made up of the digits 7, 2, and 8. These same three digits may be combined to make other whole numbers, such as 782, 278, 287, 827, and 872.

What whole numbers can you form by combining the digits 1, 4, 8, and 5?

Numbers may be whole numbers, called *integers,* or parts of whole numbers, called *fractions.*

2. The importance of mathematics. Huckleberry Finn once said, "I have been to school most all the time and could say the multiplication table up to six times seven is thirty-five, and I don't reckon I could ever get any further than that if I was to live forever. I don't take no stock in mathematics anyway."

Huck Finn did not live in the age of airplanes, automobiles, radios, television, and atomic bombs. He could never have studied aviation, calculated the flight of a bullet, measured an engine part to a thousandth of an inch, squared a number, solved an equation, or read a blueprint. Huck's lack of mathematical knowledge would have made such problems impossible for him.

The days of Huck Finn are past. New problems call for increased mathematical skill. The world demands speed and accuracy. The businessman of today is no longer satisfied with the speed of yesterday, nor will he tolerate any inaccuracy in his records. He knows that inaccuracy may cost him customers and money.

You and I must make a special effort to eliminate all our errors and to speed our work to meet the demands of the times. The first step is to improve our skills in the *fundamentals of arithmetic.*

3. Plan of procedure to improve the fundamentals of arithmetic. Each of you will need to continue your practice in the fundamentals of arithmetic. Some of you will practice in order that you may maintain the high skill that is already yours. Others of you, who have not yet reached a satisfactory score in one or

MISTAKES IN ARITHMETIC ARE COSTLY IN BUSINESS

more of the computations, will need to practice in order to improve your skill.

In this book you will find four types of exercises to aid you in mastering the fundamentals of arithmetic.

The first five chapters contain *Inventory Tests* designed to show you any weak spots in your computations. The follow-up *Practice Exercises* in the Appendix are there to help you overcome those weaknesses.

You will find a series of *Review Exercises*, which are planned to help you maintain and improve the skills that you have mastered. The *Review Exercises* are supplemented by *Achievement Tests*. A chart at the back of this book may be used to tabulate the results of the *Achievement Tests*. The results of these tests will show you how well you are maintaining and improving your skills in the fundamentals of arithmetic.

You will learn more about the tests and exercises later on. Let us now concern ourselves with the *Inventory Tests* and *Practice Exercises*.

The *Inventory Tests* are for all members of the class. Each of these tests covers one phase of computation. The first test covers addition of integers.

It is possible to make one of three ratings on these tests—an A, B, or C rating. An A rating indicates that the person making it is well prepared on this particular computation. The C rating indicates that the person making it is not well prepared on this particular computation. He needs to do the *Practice Exercises* for this computation, and then take the *Inventory Test* again.

The B rating indicates a lack of top accuracy. Many will wish to improve their accuracy by doing the *Practice Exercises* and then repeating the *Inventory Test.* Others may prefer to go directly to the next *Inventory Test.*

The *Inventory Tests* are all five-minute tests.

The *Practice Exercises* provide remedial work on the type of computation covered by the *Inventory Tests.* The *Practice Exercises* are found in the Appendix, and there is no time limit for them.

CHAPTER I

ADDITION, MULTIPLICATION, SUBTRACTION, AND DIVISION OF INTEGERS WITH APPLICATIONS

VOCABULARY

1. arithmetic	4. integer	7. computation	10. perimeter
2. mathematics	5. digit	8. addend	11. length
3. fundamentals	6. addition	9. sum	12. width

4. Terms used in addition. The numbers to be added are called *addends*. The result obtained by addition is called the *sum*.

28 addend
61 addend
89 sum

Inventory Test 1 — ADDITION OF INTEGERS

Time—5 minutes.

Write your answers on folded paper. Add:

8	7	8	9	9	7	7	5	8	4	
6	9	3	8	5	0	6	7	8	7	(10)

$36+8=$ $\quad 47+6=$ $\quad 59+8=$ $\quad 46+7=$ $\quad 28+9=$ (15)

4	23	623	7214	17214	
9	48	245	1806	6805	
8	16	985	49	328	
3	56	646	385	14826	
5	21	546	2738	497	(20)
9	64	421			

Rating	Correct	Next Step
A	19 or 20	Do Inventory Test 2.
B	18	Do Inventory Test 2 or do Practice Exercises 1a to 1d.
C	17 or less	Do Practice Exercises 1a to 1d; then do Inventory Test 1 again.

The *Practice Exercises* will be found in the Appendix.

PROOF

5. Proving addition by adding in the opposite direction. After adding a column of figures from the bottom up, add them from the top down; or first add from the top down, and then add from the bottom up. The two answers in either case should be the same.

EXERCISES

Add the following columns of figures, and prove the answer in each case:

1. $ 649	**2.** $ 72	**3.** $ 427	**4.** $9167	**5.** $5108
76	870	3690	8605	347
3475	598	685	1084	8677
416	16745	486	5837	67
539	6308	5679	8755	4672

6. Finding the distance between places. The development of the automobile has brought paved highways, and the development of paved highways has caused people to travel more and more in automobiles—some on business trips and others on tours for pleasure. Such trips make it rather necessary for the traveler to be able to read maps and to find distances between places.

EXERCISES

Distances on maps are expressed in miles. Prove the sums obtained in each case.

1. In figure 1, find the distance from *A* to *E*, from *C* to *F*, from *B* to *E*, and from *A* to *F*.

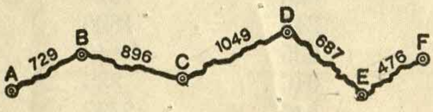


Fig. 1

2. In figure 2, what are the distances from *A* to *F*, from *B* to *H*, from *C* to *I*, and from *A* to *I*?

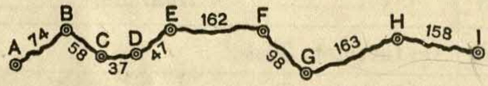
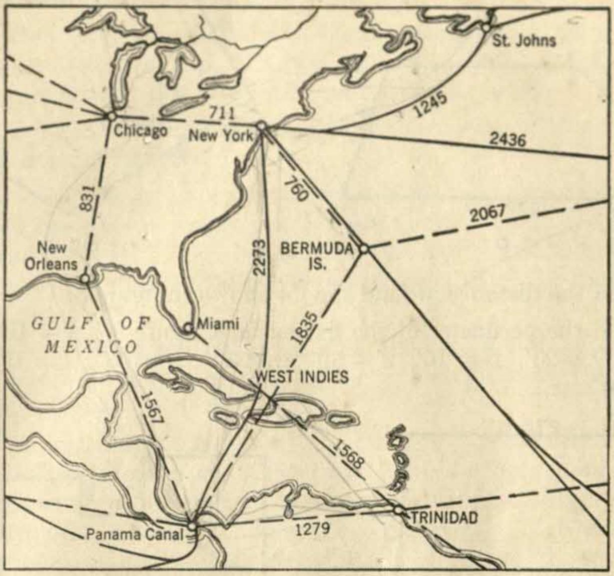


Fig. 2

3. In figure 3 calculate these distances:
(a) From Chicago to St. Johns.
(b) From the Panama Canal to St. Johns via Bermuda.
(c) From Chicago to the Panama Canal via New Orleans.
(d) From Miami to New Orleans via Trinidad.

(e) From Chicago to Trinidad via New Orleans.
(f) From New York to Panama via Chicago.

Fig. 3



7. Finding perimeters. The *perimeter* of an object is the distance around that object. You may find it by adding the lengths of all its sides.

EXERCISES

All distances are in feet. Prove the sums in each case:

1. The foundation for a building is marked as illustrated in figure 4. Find the perimeter of the foundation by adding the lengths of the lines that represent the various sides.

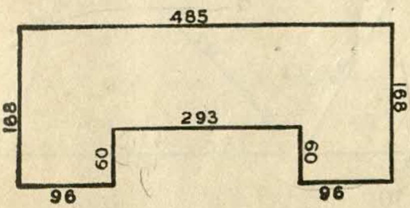


Fig. 4

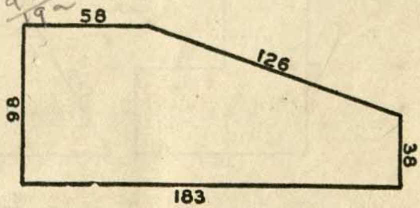


Fig. 5

2. Find the total length of a fence needed to enclose the plot of ground, figure 5.

3. What is the total length of a fence needed to enclose the plot of ground in figure 6 on the next page?

4. Find the perimeter of figure 7 on the next page.

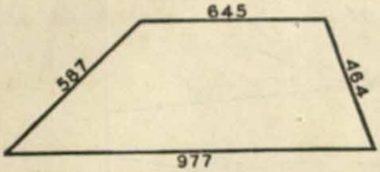


Fig. 6

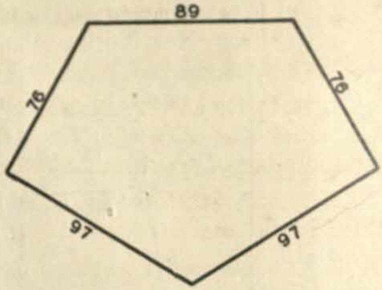


Fig. 7

5. Find the distance around the lot shown in figure 8.

6. Find the perimeter of the flat surface, figure 9. $A = 160'$, $B = 55'$, $C = 19'$, $D = 23'$, $E = 16'$, $F = 39'$, $G = 36'$, $H = 47'$, $I = 19'$. ("160′" means 160 feet.)

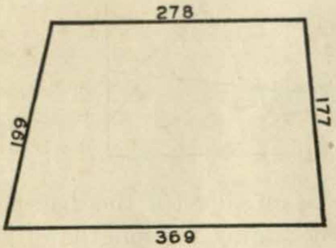


Fig. 8

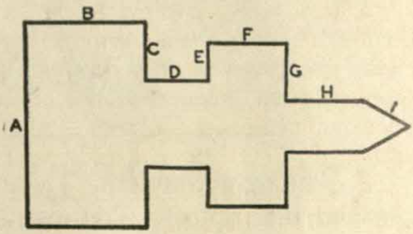


Fig. 9

7. What is the perimeter of each of the lots in figure 10?

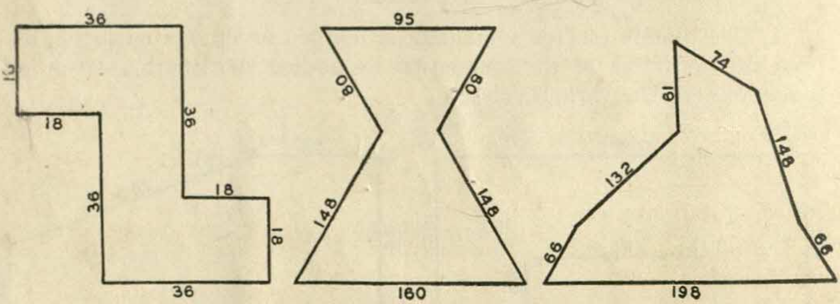


Fig. 10

8. An end door in a railway coach measures 30″ by 80″. How much weatherstripping is needed to seal it?

9. Find the perimeter of the parlor section of a car, if this section measures $27.75' \times 9.6'$.

10. Find the perimeter of a six-sided figure whose sides are 76′, 88′, 73′, 54′, 39′, and 68′.

ADDITIONAL APPLICATIONS—EXERCISES

Prove the results obtained in the following exercises:

1. Ruth Ross works in a factory. During a certain week her production was as follows: 266, 318, 293, 310, 308, and 147 articles. What was the total number of articles that she produced during the week?

2. $4,657, $6,483, $3,675, and $3,946 are invested in a partnership. What is the total investment?

3. The following number of persons were killed in automobile accidents in the United States in a recent year: 1,300 under 5 years of age; 3,000 between 5 and 14 years; 7,500 between 15 and 24 years; 20,800 between 25 and 64 years; and 6,300 aged 65 years and over. Find the total number of fatal accidents in that year.

4. In seven cities of the United States, the number of deaths due to automobile accidents during a certain year was as follows: 128, 123, 41, 583, 101, 304, and 945. Find the total.

5. In a recent survey to learn why customers stopped trading with a certain large department store, these reasons were given: indifference of clerks was given by 114 persons; ignorance of goods, 118 persons; misrepresentation, 87; errors, 87; delays in service, 95; high prices, 208; poor quality of merchandise, 148; substitution of goods, 79; trouble with exchanges, 57. How many customers were questioned in the survey?

6. The weights of five planes are: 3,160 lb., 3,450 lb., 4,520 lb., 5,680 lb., and 4,860 lb. Find the total weights.

7. The graduates of a certain business college have difficulty in a number of activities when they begin a new job. A study of the situation has shown that trouble occurs most often in these activities:

Typing figures	65
Using the telephone	31
Lack of speed in shorthand	30
Spelling	25
English fundamentals	24
Arithmetic	22
	197

Find the total number of difficulties.

8. Find the total miles of foreign air-mail service in one year as reported from these places:

New York to Bermuda	470,787 miles
New York to Montreal	57,610 miles
Miami to San Juan	429,823 miles

9. The daily sales for one week were $105.75, $214.34, $197.67, $268.45, $245.96, $297.58. What was the total for the week? What was the average daily sale for the 6 days?

10. In one year the railroads hauled the following carloads of livestock: cattle and calves, 311,589 carloads; hogs, 218,950; sheep and lambs,

95,828; horses and mules, 11,824; other live animals, 186. Find the total carloads of animals.

11. (OPTIONAL) The land area and population of the different parts of the world are shown in the table. Find the total land area and the total number of people.

Regions	*Square Miles*	*Population*
Africa	11,710,424	157,330,000
Asia	10,347,491	1,154,000,000
Europe	2,092,664	402,800,000
U. S. S. R.	8,176,061	172,000,000
North America	8,664,864	184,260,000
South America	6,937,551	88,680,000
Oceania	3,301,158	10,803,000

MULTIPLICATION

VOCABULARY

1. multiplication 2. multiplicand 3. multiplier 4. product

8. Terms used in multiplication. The number that is to be multiplied is called the *multiplicand.* The number by which the multiplicand is multiplied is called the *multiplier.* The result obtained by multiplication is called the *product.*

72	multiplicand
8	multiplier
576	product

Inventory Test 2 MULTIPLICATION OF INTEGERS

Time—5 minutes.

Write your answers on a folded paper.

Multiply:

8	7	8	9	9	7	7	5	
6	9	3	8	5	0	6	7	(8)
8	4	0	9	5	1	7	0	
8	7	5	4	6	0	8	2	(16)
6	2	4	8	9	9	8	7	
9	0	7	4	6	9	5	7	(24)

608	300	405	807	107	
40	700	708	79	108	(29)

Rating	Correct	Next Step
A	28 or 29	Do Inventory Test 3.
B	27	Do Inventory Test 3 or do Practice Exercises 2a to 2d.
C	26 or less	Do Practice Exercises 2a to 2d; then do Inventory Test 2 again.

Practice Exercises will be found in the Appendix.

9. Proving multiplication by interchanging the multiplier and the multiplicand. To multiply a second time in the same order as the first time is not a good proof, for the same mistake is likely to be made the second time. If the multiplier and the multiplicand are interchanged and the same product is obtained, the work is probably correct.

Examples

1. Multiply 487 by 39; prove.

$$\begin{array}{r} 487 \\ \underline{39} \\ 4383 \\ \underline{1461} \\ 18993 \end{array} \quad \leftarrow \textit{Proof} \rightarrow \quad \begin{array}{r} 39 \\ \underline{487} \\ 273 \\ 312 \\ \underline{156} \\ 18993 \end{array}$$

2. Multiply 597 by 27; prove.

$$\begin{array}{r} 597 \\ \underline{27} \\ 4179 \\ \underline{1194} \\ 16119 \end{array} \quad \leftarrow \textit{Proof} \rightarrow \quad \begin{array}{r} 27 \\ \underline{597} \\ 189 \\ 243 \\ \underline{135} \\ 16119 \end{array}$$

EXERCISES

Multiply each of the following exercises, and prove each answer by interchanging the multiplicand and the multiplier.

1. 478 × 96 **2.** 779 × 84 **3.** 670 × 357

4. 5797 × 82 **5.** 974 × 59 **6.** 508 × 94

7. 544 × 108 **8.** 959 × 68 **9.** 843 × 480

10. 567 × 576 **11.** 638 × 359 **12.** 8557 × 406

10. Proving multiplication by dividing the product by the multiplier or the multiplicand.

Example

$$\begin{array}{r} 487 \\ \underline{39} \\ 4383 \\ \underline{1461} \\ 18993 \end{array} \qquad \textit{Proofs: } \begin{array}{r} 487 \\ 39\overline{)18993} \\ \underline{156} \\ 339 \\ \underline{312} \\ 273 \\ \underline{273} \end{array} \qquad \textit{or: } \begin{array}{r} 39 \\ 487\overline{)18993} \\ \underline{1461} \\ 4383 \\ \underline{4383} \end{array}$$

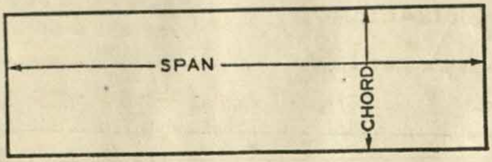
EXERCISES

Multiply in each of the following exercises, and prove each answer by the preceding method:

1. $823 \times 74 =$ **3.** $726 \times 481 =$ **5.** $1582 \times 26 =$

2. $318 \times 58 =$ **4.** $514 \times 188 =$ **6.** $4105 \times 31 =$

11. Finding the area of rectangular airplane wings. Airplane wings are of different shapes. The more nearly rectangular the shape, the easier it is to find the area of the surface.



To find the area of a rectangular wing, multiply the span (length) of the wing by its chord (width). That is:

area = span × chord.

EXERCISES

Find the answers, and prove your products:

1. The area of a rectangular airplane wing whose span is 38 ft. and whose chord is 7 ft. is —— sq. ft.

2. How many square inches are there in a rectangular wing whose span is 459 in. and whose chord is 93 in.?

3. Find the number of square inches in a rectangular wing that has a span of 548 in. and a chord of 76 in.

4. Find the area of a rectangular wing that has a span equal to 487 in. and a chord equal to 86 in.

5. What is the area of a rectangular wing with a span of 508 in. and a chord of 106 in.?

ADDITIONAL APPLICATIONS—EXERCISES

Prove each product:

1. How many gallons of oil are in a tanker containing 62,750 barrels if each barrel holds 42 gallons of oil?

2. Five loads of hay, weighing 2,580 lb., 2,680 lb., 2,740 lb., 2,585 lb., and 2,765 lb., were sold at $17 a ton. How much was received?

3. About 450 bunches of bananas, averaging 145 bananas to the bunch, are contained in the average carload. How many bananas are there in the carload?

4. A firm sold 7,685 washing machines @ $101 each. Find the amount of the total sales.

5. A firm sold 780 automobiles last year @ $3,003 each. What were the total sales?

6. What is the cost of 750 shares of stock at $230 each?

7. An airport runway is 4,800 ft. long and 146 ft. wide. How many square feet are there in the runway?

8. How many square feet has the flight deck of an aircraft carrier if the deck is 728 ft. long and 78 ft. wide?

SUBTRACTION

VOCABULARY

1. subtraction 2. minuend 3. subtrahend 4. remainder

12. Terms used in subtraction. The number from which another number is to be subtracted is called the *minuend*. The number that is to be subtracted from another number is called the *subtrahend*. The result obtained by subtraction is called the *remainder*. The remainder is sometimes called the *difference*.

176	minuend
23	subtrahend
153	remainder

Inventory Test 3 **SUBTRACTION OF INTEGERS**

Time—5 minutes.

Write your answers on a folded paper.
Subtract:

25	44	36	54	23	7	35	
9	9	7	6	5	0	7	(7)

20	40	12	6	31	17	24	
8	7	8	6	2	9	5	(14)

(Inventory Test 3—Continued)

48	73	305	423	916	208	
28	49	74	58	208	23	(20)

500	100	800	700	1,000	
68	27	235	189	407	(25)

12,789	27,845	15,687	47,265	
4,593	7,689	3,908	15,908	(29)

122,044	131,721	300,000	401,010	
63,078	48,608	72,181	64,302	(33)

Rating	Correct	Next Step
A	32 or 33	Do Inventory Test 4.
B	31	Do Inventory Test 4 or do Practice Exercises 3a to 3d.
C	30 or less	Do Practice Exercises 3a to 3d; then do Inventory Test 3 again.

Practice Exercises will be found in the Appendix.

13. Proving subtraction. To prove subtraction, add the subtrahend and the remainder. If their sum is equal to the minuend, the subtraction is correct.

Examples

1. 9,287	*Proof:* 2,988	**2.** $8,296	*Proof:* $3,568
2,988	6,299	3,568	4,728
6,299	9,287	$4,728	$8,296

EXERCISES

Subtract each of the following, and prove each answer:

1. 8,648	**2.** 4,050	**3.** $6,015	**4.** $68,067	**5.** 549,213	**6.** 7,807
2,939	3,987	4,695	8,988	526,785	3,954
7. 6,008	**8.** 2,637	**9.** 574,731	**10.** 36,643	**11.** 8,950	**12.** 7,856
5,163	1,798	93,689	8,557	984	5,687

Solve and prove the following:

1. An electric meter gives the present reading as 7248 K. W. H. (kilowatt hours of electricity); the previous reading was 5879 K. W. H. How much electricity was used?

2. Oil operators recommend that the daily oil allowable for August be 575,500 barrels, an increase of 49,375 barrels over July. What is the amount of oil allowable in July?

3. The Mississippi River including its tributaries is 7,263 mi. long, and the Nile is 4,000 mi. long. How many miles longer is the Mississippi River than the Nile?

4. One year 450,993 miles of air travel service were scheduled to be flown from New York to Montreal. The number of miles actually flown was 429,823. How many scheduled miles were not flown?

5. In five years the number of automobiles in one city increased from 68,350 to 160,000. What was the increase?

6. The total customs collections for one year were $388,773,987. The next year the collections were $346,505,321. Find the decrease.

7. How much more is earned by a motion picture company during a six months' period when $2,769,190 is earned than during the previous six months' period when $1,355,781 was earned?

8. A motor company during one quarter of the year reported earnings of $325,367. During the corresponding quarter of the following year $1,551,978 was earned. What was the increase in earnings?

9. The United States has 227,244 miles of railways and Canada has 42,336 miles. How many more railway miles are there in the United States than in Canada?

10. In a recent year, Canada exported to the United States goods to the value of $451,943 and imported from the United States goods to the value of $744,231,156. How much more goods did Canada receive from the United States than she sent to the United States?

11. The members of a class had $4 to buy food for a picnic. If they bought 48 oranges at 8 for 25¢, how much did they have left for other food?

12. In World War I the monthly average of troops transported by railroad was 475,450 men. In World War II the monthly average was 980,000 men. How many less were transported monthly on an average in World War I?

13. How many more life-insurance policies were in force in 1947 than in 1937, if in 1937 there were 124,157,611 and in 1947 there were 168,139,902 policies in force?

14. How much more postal savings were on deposit in 1947 (when $3,392,773,461 was on deposit) than in 1918 (when there was $148,471,499)?

15. Mr. Jones has $13,745 with which to build a home. The lot cost $2,475. The house cost $14,747. The garage cost $1,785. If the other ex-

penses amounted to $287, how much money must he borrow to complete the home?

16. The population of North America is 192,656,859 and that of South America is 101,399,661. How much greater is the population of North America?

DIVISION

VOCABULARY

1. division 2. dividend 3. divisor 4. quotient

14. Terms used in division. The number that is to be divided is called the *dividend*. The number by which the dividend is divided is called the *divisor*.* The result obtained by division is called the *quotient*.

$$\begin{array}{r} 214 \text{ quotient} \\ \text{divisor } 2\overline{)428} \text{ dividend} \end{array}$$

Inventory Test 4 DIVISION OF INTEGERS

Time—5 minutes.

Express any fraction in the quotient in its lowest terms. Copy exercises 21–25 on paper before starting your test.

Divide:

$8\overline{)48}$	$9\overline{)27}$	$7\overline{)49}$	$9\overline{)63}$	$6\overline{)42}$	
$4\overline{)32}$	$7\overline{)56}$	$8\overline{)72}$	$2\overline{)0}$	$3\overline{)27}$	(10)
$9\overline{)36}$	$7\overline{)28}$	$9\overline{)81}$	$5\overline{)0}$	$8\overline{)64}$	(15)
$7\overline{)357}$	$9\overline{)621}$	$8\overline{)768}$	$6\overline{)3006}$	$9\overline{)5085}$	
$38\overline{)1748}$	$42\overline{)4326}$	$18\overline{)3663}$	$56\overline{)2058}$	$72\overline{)25224}$	(25)

Rating	Correct	Next Step
A	24 or 25	Do Inventory Test 5.
B	23	Do Inventory Test 5 or do Practice Exercises 4a to 4d.
C	22 or less	Do Practice Exercises 4a to 4d; then do Inventory Test 4 again.

Practice Exercises will be found in the Appendix.

*A zero cannot be used as a *divisor*.

15. Proving division by multiplying the quotient and the divisor and adding the remainder to the product.

Example

197		
48)9467	*Proof:*	197
48		48
466		1576
432		788
347		9456
336		+11
11		9467

3

EXERCISES

Divide in each of the following exercises, and prove each answer by the preceding method:

1. 5091 by 39
2. 4003 by 87
3. 7283 by 76
4. 4500 by 18
5. 45,724 by 67
6. 19,336 by 82

16. Proving division by using the quotient as the divisor.

If the quotient is used as the divisor, the quotient of the proof will be the same as the original divisor.

If in the original division the quotient contains a remainder, as in the example, use only the whole number of the quotient as the divisor in the proof, and the remainders will be the same in each division.

Example

352	24
24)8463	352)8463
72	704
126	1423
120	1408
63	15
48	
15	

EXERCISES

Complete each of the following exercises, and prove by the preceding method:

1. 910 ÷ 35
2. 2436 ÷ 42
3. 888 ÷ 37
4. 4514 ÷ 74
5. 1812 ÷ 28
6. 2124 ÷ 18
7. 3719 ÷ 82
8. 1568 ÷ 53
9. 2417 ÷ 64
10. 3267 ÷ 92
11. 10,001 ÷ 34
12. 212,121 ÷ 21

17. Averages. To find the average of numbers, divide their sum by the number of addends.

Ray made the following grades in mathematics one week: 80, 72, 88, 94, and 95. Find his average grade for the week.

The sum (s) of his grades is 429.
The number of grades—addends (n)—is 5.
The average (a) is $85\frac{4}{5}$.

The rule shortened by the use of letters* is: $a = \frac{s}{n}$.

Example

$a = \frac{s}{n}$ $\quad s = 80 + 72 + 88 + 94 + 95 = 429$

$n = 5$

$a = \frac{429}{5}$
$a = 85\frac{4}{5}$

Therefore, the average grade for the week was $85\frac{4}{5}$.

EXERCISES

1. A plane that travels 1,800 mi. between 1 P. M. and 9 P. M. has an average speed of how many miles per hour?

2. What is the average horsepower (H.P.) of six engines having the following H.P. ratings: 320, 290, 375, 400, 338, and 367?

3. Find the average speed of five airplanes whose speeds are: 200 m. p. h., 286 m. p. h., 318 m. p. h., 253 m. p. h., and 319 m. p. h.

4. For three successive years Mr. Carr earned $3,500, $3,800, and $4,600. What was his average salary per year?

5. The attendance at school in a certain class for one week was: Monday, 38; Tuesday, 39; Wednesday, 38; Thursday, 40; Friday, 37. What was the average daily attendance?

6. A man's yearly salary for the past six years has been: $3,200, $3,450, $3,700, $3,950, $4,100, and $4,450. Find his average salary.

7. Find the average of the following temperatures recorded at 7 A. M. for one week: 68°, 62°, 70°, 64°, 79°, 86°, and 82°.

8. The average temperature for a certain city is, month by month, as follows:

January	28°	April	55°	July	78°	October	58°
February	31°	May	65°	August	77°	November	44°
March	43°	June	74°	September	69°	December	32°

Determine the yearly average temperature of the city.

9. (OPTIONAL) A car travels 15 mi. in 1 hour over a rough road, and later travels 60 mi. in 2 hours over a better road. What was the average speed of the car?

*A rule in which letters are used to represent numerical values is called a *formula*.

Solution: The average speed (a) equals the total distance (d) divided by the total time.

$$a=\frac{d}{t} \qquad d = 15 \text{ mi.} + 60 \text{ mi.} = 75 \text{ mi.}$$

$$t = 1 \text{ hr.} + 2 \text{ hr.} = 3 \text{ hr.}$$

$$a=\frac{?}{?}$$

$$a=?$$

Therefore, the average speed was ? m. p. h.

10. (OPTIONAL) A train traveled 480 mi. in 6 hours; then traveled 680 mi. in 8 hours. Find the average speed of the train.

11. (OPTIONAL) Mrs. Case spent $4.00 for 60 bars of soap and $5.00 for 80 bars. What was the average price per bar for the soap?

12. (OPTIONAL) In a class of 30 pupils, the following scores were made on a test: 3 pupils made 100% each, 8 made 90% each, 10 made 80%, 4 made 70%, 2 made 60%, 2 made 40%, and 1 made 0%. What was the average score per pupil? (*Hint:* The total score of the three who made 100% each was 300%.)

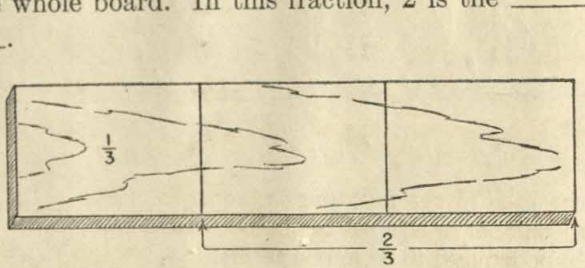
CHAPTER II

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF COMMON FRACTIONS WITH APPLICATIONS

VOCABULARY

1. fraction
2. common fraction
3. proper fraction
4. improper fraction
5. mixed number
6. numerator
7. denominator
8. least common denominator

18. Common fractions. Henry had a board which he divided into three equal parts as shown below. He used one of the three equal parts to make a table top. This part of the board is indicated by the fraction $\frac{1}{3}$. The 1 is the *numerator* and 3 is the *denominator.* The remaining portion, made up of two of the three equal parts, is $\frac{2}{3}$ of the whole board. In this fraction, 2 is the ______ and 3 is the ______.



A fraction that is written with a numerator and a denominator, such as $\frac{5}{9}$, is called a *common fraction.*

If the numerator is less than the denominator, as in $\frac{5}{9}$ or $\frac{2}{3}$, the fraction is called a *proper fraction.*

If the numerator is larger than the denominator, as in $\frac{5}{2}$, the fraction is an *improper fraction.*

A number made up of a whole number and a fraction, such as $4\frac{1}{2}$, is called a *mixed number.*

19. Changing the form of a fraction. In order to succeed with fractions, you must be able to do these four things:

(a) To change a fraction to lower or higher terms:

$$\frac{2}{4}=\frac{1}{2} \qquad \frac{1}{4}=\frac{3}{12}.$$

(b) To change improper fractions to whole or mixed numbers:

$$\tfrac{8}{4} = 2 \qquad \tfrac{5}{4} = 1\tfrac{1}{4}.$$

(c) To change mixed numbers to improper fractions:

$$2\tfrac{1}{2} = \tfrac{5}{2}.$$

(d) To change two or more fractions to fractions having the same denominator:

$$\tfrac{1}{3} \text{ and } \tfrac{1}{2} = \tfrac{2}{6} \text{ and } \tfrac{3}{6}.$$

EXERCISES

1. Change each of the following fractions to its lowest terms by dividing the numerator and the denominator by the largest number that is contained exactly in both of them.

Example

$$\frac{10}{12} = \frac{10 \div 2}{12 \div 2} = \frac{5}{6}$$

If both the numerator and the denominator of a fraction are divided by the same number, the value of the fraction remains the same.

$\frac{12}{33}$	$\frac{9}{27}$	$\frac{8}{16}$	$\frac{10}{22}$	$\frac{18}{26}$	$\frac{24}{60}$	
$\frac{48}{60}$	$\frac{46}{69}$	$\frac{20}{90}$	$\frac{24}{64}$	$\frac{12}{15}$	$\frac{15}{18}$	(12)
$\frac{124}{200}$	$\frac{5}{1000}$	$\frac{40}{640}$	$\frac{70}{2000}$	$\frac{36}{72}$	$\frac{18}{24}$	
$\frac{27}{33}$	$\frac{25}{45}$	$\frac{56}{63}$	$\frac{27}{36}$	$\frac{42}{48}$	$\frac{35}{40}$	(24)

2. Write each of the following improper fractions as a mixed number or as a whole number. Each fraction should be reduced to its lowest terms:

Example

$$\tfrac{18}{4} = 4\tfrac{2}{4} = 4\tfrac{1}{2}$$

$\frac{11}{3}$	$\frac{9}{2}$	$\frac{7}{3}$	$\frac{24}{5}$	$\frac{8}{3}$	$\frac{20}{8}$	$\frac{34}{16}$	$\frac{25}{3}$	(8)
$\frac{34}{2}$	$\frac{25}{4}$	$\frac{10}{8}$	$\frac{32}{5}$	$\frac{9}{4}$	$\frac{17}{6}$	$\frac{6}{5}$	$\frac{8}{5}$	
$\frac{2}{2}$	$\frac{6}{3}$	$\frac{33}{15}$	$\frac{7}{5}$	$\frac{11}{6}$	$\frac{21}{4}$	$\frac{48}{3}$	$\frac{35}{7}$	(24)
$\frac{9}{8}$	$\frac{17}{4}$	$\frac{44}{10}$	$\frac{16}{5}$	$\frac{43}{6}$	$\frac{15}{10}$	$\frac{60}{8}$	$\frac{27}{5}$	
$\frac{7}{3}$	$\frac{42}{6}$	$\frac{18}{8}$	$\frac{12}{9}$	$\frac{35}{10}$	$\frac{16}{3}$	$\frac{54}{6}$	$\frac{40}{12}$	(40)

3. Write each of the following mixed numbers as improper fractions:

Example

$$3\tfrac{2}{5} = \tfrac{17}{5}$$

$8\dfrac{3}{5} = \dfrac{43}{5}$ $\quad 3\dfrac{1}{16} =$ $\quad 4\dfrac{3}{4} =$ $\quad 4\dfrac{1}{5} =$ $\quad 7\dfrac{2}{9} =$ $\quad 3 = \dfrac{\quad}{4}$

$4\frac{3}{4}=\frac{\ }{4}$ $\quad 5\frac{1}{3}=$ $\quad 7\frac{2}{3}=$ $\quad 8\frac{1}{6}=$ $\quad 7\frac{1}{4}=$ $\quad 12=\frac{\ }{2}$ (12)

$12\frac{1}{2}=\frac{\ }{2}$ $\quad 2\frac{3}{16}=$ $\quad 2\frac{5}{6}=$ $\quad 1\frac{3}{4}=$ $\quad 6\frac{3}{4}=$ $\quad 9\frac{2}{3}=$

$2\frac{7}{8}=\frac{\ }{8}$ $\quad 9\frac{1}{3}=$ $\quad 10\frac{3}{5}=$ $\quad 3\frac{1}{7}=$ $\quad 8\frac{2}{3}=$ $\quad 16\frac{2}{3}=$ (24)

Example

$$\frac{1}{3}=\frac{1\times 4}{3\times 4}=\frac{4}{12}$$
$$\frac{1}{4}=\frac{1\times 3}{4\times 3}=\frac{3}{12}$$
$$\frac{5}{6}=\frac{5\times 2}{6\times 2}=\frac{10}{12}$$

When two or more fractions are changed to the least *like* denominators possible, they are changed to the *least common denominator*.

If both the numerator and the denominator of a fraction are multiplied by the same number, the value of the fraction remains the same.

4. Change the following groups of fractions to least common denominators.

1. $\frac{1}{2}=?/6$ $\frac{2}{3}=?/6$	2. $\frac{3}{4}=?/12$ $\frac{5}{6}=?/12$	3. $\frac{1}{4}=?/20$ $\frac{3}{5}=?/20$	4. $\frac{2}{5}=?/15$ $\frac{2}{3}=?/15$	5. $\frac{3}{8}=?/24$ $\frac{5}{6}=?/24$
6. $\frac{1}{2}=$ $\frac{4}{5}=$	7. $\frac{7}{12}=$ $\frac{2}{3}=$	8. $\frac{3}{10}=$ $\frac{4}{15}=$	9. $\frac{1}{6}=$ $\frac{3}{8}=$	10. $\frac{3}{16}=$ $\frac{2}{3}=$
11. $\frac{4}{5}$ $\frac{1}{4}$	12. $\frac{5}{12}$ $\frac{5}{8}$	13. $\frac{11}{12}$ $\frac{3}{5}$	14. $\frac{7}{8}$ $\frac{5}{6}$	15. $\frac{3}{7}$ $\frac{1}{2}$
16. $\frac{3}{4}$ $\frac{2}{3}$	17. $\frac{5}{6}$ $\frac{7}{9}$	18. $\frac{3}{4}$ $\frac{7}{8}$	19. $\frac{1}{2}$ $\frac{5}{12}$	20. $\frac{5}{12}$ $\frac{1}{3}$
21. $\frac{3}{4}$ $\frac{3}{8}$	22. $\frac{11}{12}$ $\frac{1}{4}$	23. $\frac{2}{5}$ $\frac{4}{5}$	24. $\frac{2}{5}$ $\frac{3}{8}$	25. $\frac{5}{6}$ $\frac{7}{10}$
26. $\frac{1}{2}$ $\frac{2}{3}$ $\frac{1}{8}$	27. $\frac{1}{4}$ $\frac{1}{3}$ $\frac{3}{8}$	28. $\frac{2}{3}$ $\frac{1}{4}$ $\frac{5}{6}$	29. $\frac{2}{5}$ $\frac{1}{2}$ $\frac{7}{10}$	30. $\frac{2}{3}$ $\frac{3}{4}$ $\frac{7}{8}$

Inventory Test 5 ADDITION OF COMMON FRACTIONS

Time—5 minutes.

Add; then reduce each sum to its lowest terms:

$\frac{5}{8}+\frac{2}{3}$ $\quad$ $\frac{3}{4}+\frac{2}{5}$ $\quad$ $\frac{3}{16}+\frac{7}{8}$ $\quad$ $\frac{5}{12}+\frac{1}{3}+\frac{5}{6}$

$12\frac{1}{4}+37\frac{2}{3}$ $\quad$ $27\frac{1}{3}+34\frac{4}{5}$ $\quad$ $18\frac{1}{3}+25\frac{7}{8}$ (7)

$$\begin{array}{cccccc}
\frac{1}{8} & \frac{2}{5} & \frac{1}{2} & \frac{5}{9} & \frac{7}{8} & \frac{3}{4} \\
\underline{\frac{1}{5}} & \underline{\frac{7}{10}} & \underline{\frac{1}{3}} & \underline{\frac{2}{3}} & \underline{\frac{4}{5}} & \underline{\frac{1}{6}}
\end{array} \quad (13)$$

$$\begin{array}{rrrrr}
8\frac{5}{9} & 18\frac{2}{5} & 24\frac{3}{16} & 172\frac{1}{2} & 181\frac{6}{7} \\
\underline{6\frac{1}{3}} & \underline{7\frac{3}{4}} & \underline{18\frac{3}{8}} & \underline{256\frac{2}{3}} & \underline{156\frac{2}{3}}
\end{array} \quad (18)$$

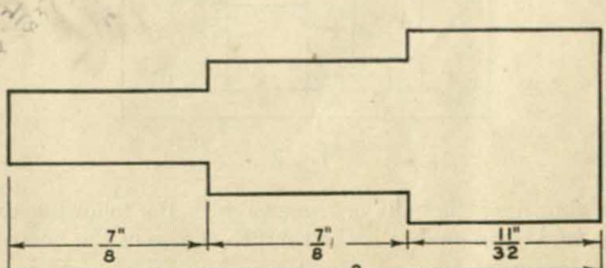
Rating	Correct	Next Step
A	17 or 18	Do Inventory Test 6.
B	15 or 16	Do Inventory Test 6 or Practice Exercises 5a to 5e.
C	14 or less	Do Practice Exercises 5a to 5e; then do Inventory Test 5 again.

See Appendix for *Practice Exercises*.

20. Addition of ruler fractions. The inch on an ordinary ruler is usually marked off into halves, fourths, eighths, and sixteenths. A steel rule, called a *scale*, is further graduated into 32nds and 64ths. It is most essential to a mechanic to be able to make use of these very small divisions.

A mechanic must be able to calculate total lengths of objects shown in shop drawings.

Find the total (over-all) length of the object shown in this shop drawing.



Total length $= \frac{7}{8}'' + \frac{7}{8}'' + \frac{11}{32}'' = ?$

EXERCISES

1. Find the total lengths of these objects:

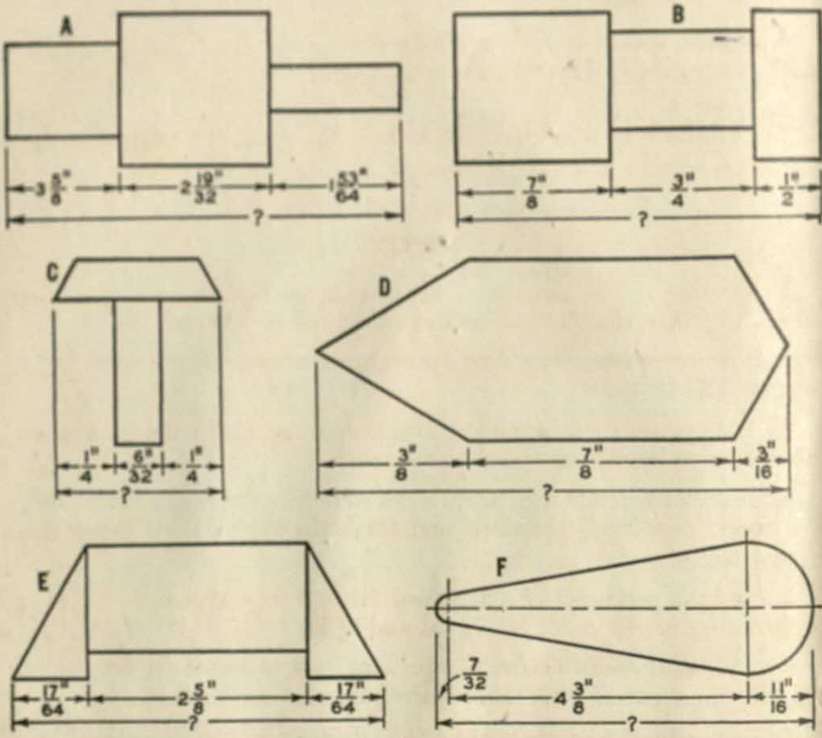


Fig.

2. Find the total length of this lathe chuck, figure 2.

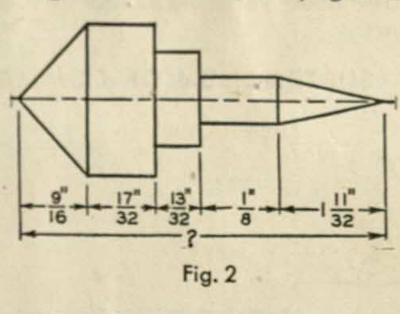


Fig. 2

3. A drawing board is built of three plies of the following thicknesses: $\frac{5}{8}$ in., $\frac{7}{16}$ in., and $\frac{9}{32}$ in. What is the total thickness of the board?

4. A carpenter lays floors in three layers: first the rough sheeting, $\frac{7}{8}$ in. thick; then the furring strips, $\frac{7}{8}$ in. thick; and on top the finished flooring, $\frac{13}{16}$ in. thick. How thick is the floor?

5. When Jim was building a cabinet, he glued together five boards, $8\frac{3}{4}$ in., $6\frac{7}{8}$ in., $7\frac{5}{16}$ in., $5\frac{1}{2}$ in., and $6\frac{3}{8}$ in. wide. What was the combined width of the boards?

6. A panel was built up of five plies $\frac{1}{4}$ in., $\frac{1}{8}$ in., $\frac{3}{16}$ in., $\frac{3}{16}$ in., and $\frac{5}{32}$ in. thick, respectively. How thick was the panel?

21. Addition of non-ruler fractions. Many fractions that are not found on a ruler are in common use.

EXERCISES

1. A farm is divided as follows: $14\frac{3}{4}$ acres in one field, $9\frac{2}{3}$ acres in another, and $17\frac{1}{3}$ acres in the third. Find the total number of acres.

2. How many feet of molding are required to go around a room $14\frac{3}{4}$ ft. long and $23\frac{5}{6}$ ft. wide?

3. Betty traveled $18\frac{7}{10}$ miles by automobile and $124\frac{2}{3}$ miles by airplane. How many miles did she travel on the trip?

4. On an automobile trip, $326\frac{6}{10}$ miles were traveled the first day, $197\frac{3}{10}$ the second day, $388\frac{7}{10}$ the third, and $467\frac{3}{10}$ the fourth. How many miles were traveled?

5. Find the perimeter of a five-sided figure if its sides are the following lengths: $8\frac{7}{12}$ in., $4\frac{2}{3}$ in., $5\frac{5}{6}$ in., $7\frac{3}{4}$ in., and $6\frac{1}{3}$ in.

6. A salesgirl made five sales from one bolt of linen: $4\frac{3}{4}$ yd., $6\frac{1}{2}$ yd., $18\frac{1}{4}$ yd., $9\frac{1}{6}$ yd., and $7\frac{3}{4}$ yd. How many yards did she sell?

7. Five bolts of gingham contain $68\frac{2}{3}$ yd., $47\frac{3}{4}$ yd., $55\frac{5}{6}$ yd., $49\frac{1}{2}$ yd., and $58\frac{11}{12}$ yd. What is the total yardage?

8. Mr. North bought material for four cases that took $1\frac{1}{3}$ yd., $1\frac{5}{8}$ yd., $1\frac{5}{6}$ yd., and $1\frac{1}{2}$ yd. How many yards did he buy for all the cases?

Inventory Test 6 SUBTRACTION OF COMMON FRACTIONS

Time—5 minutes.

$\frac{1}{2}$	$\frac{5}{6}$	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{9}{10}$	$\frac{11}{12}$	
$\underline{\frac{1}{3}}$	$\underline{\frac{2}{3}}$	$\underline{\frac{1}{4}}$	$\underline{\frac{5}{12}}$	$\underline{\frac{4}{5}}$	$\underline{\frac{3}{4}}$	(6)

$\frac{7}{12} - \frac{1}{3}$ $\qquad$ $\frac{5}{6} - \frac{3}{4}$ $\qquad$ $\frac{7}{8} - \frac{3}{16}$ $\qquad$ $\frac{4}{5} - \frac{1}{4}$ (10)

$9\frac{2}{3}$	$5\frac{1}{2}$	$7\frac{5}{6}$	$8\frac{15}{16}$	$9\frac{3}{5}$	
$\underline{3\frac{1}{2}}$	$\underline{1\frac{3}{4}}$	$\underline{3}$	$\underline{7\frac{5}{8}}$	$\underline{4}$	(15)

$26\frac{1}{3}$	48	$72\frac{5}{8}$	43	$16\frac{1}{5}$	
$\underline{12\frac{2}{3}}$	$\underline{24\frac{3}{4}}$	$\underline{16\frac{1}{2}}$	$\underline{10\frac{3}{8}}$	$\underline{9\frac{2}{3}}$	(20)

Rating	Correct	Next Step
A	19 or 20	Do Inventory Test 7.
B	18	Do Inventory Test 7 or do Practice Exercises 6a to 6d.
C	17 or less	Do Practice Exercises 6a to 6d; then do Inventory Test 6 again.

Practice Exercises will be found in the Appendix.

EXERCISES

A. Ruler fractions.

1. A board was reduced from $3\frac{1}{4}$ in. in thickness to $2\frac{5}{8}$ in. by being run through a planer. How much was taken off the board?

2. One steel strip is $\frac{5}{8}$ in. thick and another is $\frac{17}{32}$ in. thick. What is the difference in thickness?

3. If a plank $8\frac{3}{16}$ in. thick must be cut down to $5\frac{3}{4}$ in., how thick is the piece cut off by the saw?

4. How much must a $1\frac{1}{16}$-in. board be planed to make it the required thickness of $\frac{29}{32}$ in.?

5. A rough casting weighed $72\frac{1}{16}$ lb. After being finished in a lathe, it weighed $68\frac{1}{4}$ lb. How much was the loss in finishing?

6. David knows the total bottom and top lengths of this box that he is making. He wishes to know how much longer the top is than the bottom of the box (figure 3). Can you tell him?

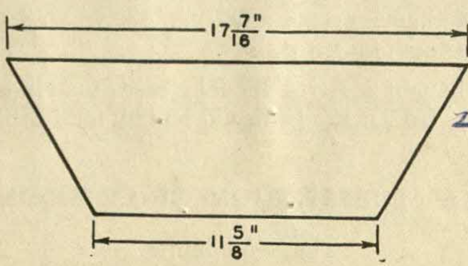


Fig. 3

7. A taper is reduced gradually from $2\frac{19}{32}$ in. at one end to $1\frac{7}{16}$ in. at the other. How much is the reduction?

8. How many feet of baseboard will be required to go around a room $12\frac{1}{4}$ ft. wide and $14\frac{1}{6}$ ft. long, allowing $3\frac{1}{2}$ ft. for a door?

9. Two circular holes are cut in a steel plate. The distance from the outside of one hole to the outside of the other is $5\frac{1}{4}$ in. Their diameters are $1\frac{3}{16}$ in. and $1\frac{3}{4}$ in. What is the length of the metal between the holes? (Make a sketch of the metal plate.)

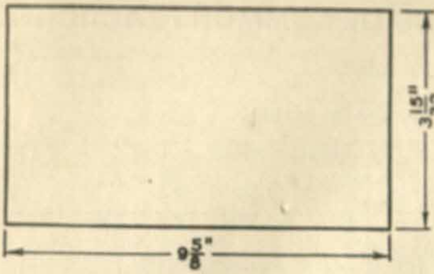


Fig. 4

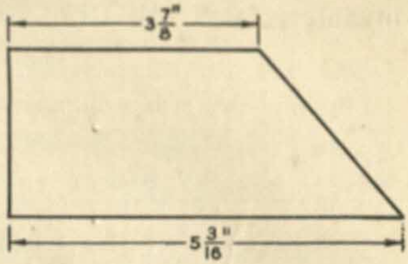


Fig. 5

10. The length of this box (figure 4) is how much more than the width?

11. Find the difference between the length of the top and the bottom of the scale drawing of a hog house, figure 5.

EXERCISES

B. Non-ruler fractions.

1. Grace bought $4\frac{1}{3}$ yd. of ribbon to make a pillow top. She found that she needed $5\frac{3}{8}$ yd. How many more yards did she need to buy?

2. Cora bought $6\frac{1}{3}$ yd. of lace and used $4\frac{1}{2}$ yd. of it on a dress. How many yards were left?

3. Clara is $4\frac{5}{8}$ ft. tall and her mother is $5\frac{1}{2}$ ft. tall. Find their difference in height.

4. If, from a shipment of $21\frac{1}{2}$ tons of coal, $19\frac{3}{4}$ tons are delivered, how many tons remain?

5. Henry had $106\frac{1}{3}$ acres of land and sold $78\frac{4}{5}$ acres. How many acres had he left?

6. From a farm containing 640 acres, $375\frac{1}{2}$ acres are sold. How many acres remain unsold?

7. Joe weighs $86\frac{5}{8}$ lb. and Bob weighs $92\frac{3}{16}$ lb. How much more does Bob weigh than Joe?

8. A sack that holds $1\frac{1}{2}$ bu. contains $\frac{7}{12}$ of a bushel. How many more bushels are needed to fill the sack?

9. A ship travels $\frac{1}{5}$ of its journey in rain, $\frac{2}{3}$ in sunshine, and the remainder in fog. What part of its journey did it travel in fog?

10. A load of coal was driven on the scales to be weighed. The truck weighed $3750\frac{1}{2}$ lb.; the shovel, $3\frac{3}{4}$ lb.; the chute, $27\frac{1}{4}$ lb. What is the weight of the coal, if the gross weight is $8527\frac{1}{4}$ lb.?

11. From a pile of $34\frac{1}{2}$ tons of coal, a dealer sold $6\frac{1}{2}$ tons to one customer, $8\frac{3}{4}$ tons to another, and $6\frac{7}{8}$ tons to the third customer. How many tons of coal were left?

Inventory Test 7 MULTIPLICATION OF COMMON FRACTIONS

Time—5 minutes.

Multiply; then reduce all products to lowest terms:

$\frac{3}{4} \times \frac{2}{5}$	$\frac{7}{8} \times \frac{6}{7}$	$\frac{4}{5} \times \frac{15}{4}$	$\frac{1}{3} \times \frac{1}{3}$	(4)
$8 \times \frac{3}{4}$	$\frac{4}{15} \times 25$	$\frac{3}{8} \times 72$	$2\frac{1}{2} \times 6$	
$3\frac{1}{7} \times 14$	$3\frac{3}{4} \times 1\frac{3}{5}$	$6 \times 3\frac{1}{3}$	$\frac{5}{6} \times \frac{9}{10}$	(12)
$8\frac{1}{3} \times 2\frac{2}{5}$	$3\frac{1}{5} \times 10$	$\frac{2}{3} \times \frac{3}{7} \times \frac{7}{8}$	$\frac{4}{5} \times \frac{7}{8} \times \frac{3}{4}$	
$7 \times \frac{1}{3}$	$12\frac{1}{2} \times \frac{1}{15}$	$180 \times \frac{5}{6}$	$20 \times \frac{1}{3}$	(20)

Rating	Correct	Next Step
A	19 or 20	Do Inventory Test 8.
B	17 or 18	Do Inventory Test 8 or do Practice Exercises 7a to 7c.
C	16 or less	Do Practice Exercises 7a to 7c; then do Inventory Test 7.

22. Changing the amount of a recipe. It is often necessary to make either more or less of a given food than the recipe provides for.

EXERCISES

1. Ingredients for a spice cake:

$\frac{1}{3}$ cup shortening	3 teaspoons baking powder
$1\frac{1}{3}$ cups brown sugar	$\frac{1}{2}$ teaspoon salt
1 egg	1 teaspoon cinnamon
$\frac{2}{3}$ cup milk	$\frac{1}{4}$ teaspoon nutmeg
$1\frac{2}{3}$ cups flour	$\frac{1}{2}$ teaspoon cloves

Find the amount of ingredients necessary to make two spice cakes; to make three spice cakes; to make half a spice cake; to make one-third of a spice cake.

2. To make banana frozen ice cream (six servings):

$\frac{2}{3}$ cup sliced bananas	$\frac{1}{2}$ cup boiling water
$\frac{3}{4}$ cup sugar	2 cups cream

2 teaspoons gelatin

Find the amounts of the ingredients necessary to serve 3 people; to serve 4; to serve 9; to serve 15.

3. Crystallized walnuts:

4 cups walnut meats	$\frac{3}{4}$ cup water
$1\frac{1}{2}$ cups sugar	$\frac{1}{2}$ teaspoon cream of tartar

1 tablespoon honey or syrup

Mix sugar, water, honey, and cream of tartar and boil to 242° F, or a medium soft ball. Add nuts and stir.

Find the amount of ingredients to make:

(a) Double the recipe.
(b) One-half the recipe.
(c) One-fourth the recipe.
(d) One and one-half times the recipe.

ADDITIONAL APPLICATIONS—EXERCISES

1. Clara uses $\frac{1}{3}$ yd. of velvet to cover a jewel box. How many yards of velvet will she need to cover 8 such boxes?

2. How much ribbon must Sally buy to make 6 hair ornaments if each ornament contains $\frac{3}{8}$ yd. of ribbon?

3. If 12 strips of $\frac{5}{8}$-in. steel are stacked on top of each other, how high a stack will they make?

4. How high a stack will 18 bricks, each $2\frac{1}{2}$ in. thick, make?

5. A purchase of $26\frac{1}{2}$ yd. of fencing is how many feet of fencing?

6. What is the weight of 120 boxes if each weighs $23\frac{3}{4}$ oz.?

7. Find the weight of 36 boxes of hose weighing $6\frac{1}{2}$ oz. each.

8. A house valued at \$4,600 is insured for $\frac{4}{5}$ of its value. For what sum of money is it insured?

9. Sound travels at the rate of 1,087 ft. per second in air. How far away is a train whose whistle is heard $3\frac{3}{10}$ seconds after the smoke is seen?

10. A garden plot $48\frac{1}{2}$ ft. by $72\frac{1}{2}$ ft. contains how many square feet?

23. A special type of multiplication. In multiplying a mixed number (a) by an integer or (b) by another mixed number, it is sometimes better to arrange the work in a vertical column.

Example 1

$$\begin{array}{r} 72 \\ \underline{8\frac{2}{3}} \\ 48 \\ \underline{576} \\ 624 \end{array}$$

Example 2

$$\begin{array}{r} 76\frac{1}{3} \\ \underline{84\frac{1}{2}} \\ 28\frac{1}{6} \\ 38 \\ 304 \\ \underline{608} \\ 6450\frac{1}{6} \end{array}$$

In example 1:

$$\begin{aligned} 72 \times \tfrac{2}{3} &= 48 \\ 72 \times 8 &= \underline{576} \\ 72 \times 8\tfrac{2}{3} &= 624 \text{ product.} \end{aligned}$$

In example 2:

$$\begin{aligned} \tfrac{1}{2} \times \tfrac{1}{3} &= \tfrac{1}{6} \\ 84 \times \tfrac{1}{3} &= 28 \\ 76 \times \tfrac{1}{2} &= 38 \\ 76 \times 4 &= 304 \\ 76 \times 8 &= \underline{608} \\ 76\tfrac{1}{3} \times 84\tfrac{1}{2} &= 6450\tfrac{1}{6} \text{ product.} \end{aligned}$$

EXERCISES

Multiply vertically:

1. $27\frac{1}{4}$ $28\frac{1}{3}$	2. $35\frac{1}{3}$ $18\frac{2}{7}$	3. $64\frac{3}{5}$ 75	4. $205\frac{1}{3}$ $78\frac{2}{5}$	5. $72\frac{2}{3}$ $84\frac{5}{6}$
6. $87\frac{1}{2}$ $44\frac{1}{3}$	7. $45\frac{3}{4}$ $48\frac{3}{5}$	8. $21\frac{5}{6}$ $18\frac{3}{7}$	9. $104\frac{3}{8}$ $48\frac{1}{2}$	10. $72\frac{4}{5}$ $65\frac{5}{6}$
11. 69 $84\frac{2}{3}$	12. $93\frac{3}{4}$ 88	13. $81\frac{1}{3}$ $27\frac{4}{9}$	14. $56\frac{1}{2}$ $78\frac{1}{4}$	15. $52\frac{1}{6}$ $36\frac{1}{4}$

Inventory Test 8 DIVISION OF COMMON FRACTIONS

Time—5 minutes.

Divide; then reduce all fractions in quotients to lowest terms:

$\frac{4}{5} \div \frac{2}{3}$	$\frac{7}{8} \div \frac{1}{2}$	$\frac{1}{2} \div \frac{1}{4}$	$\frac{1}{3} \div \frac{1}{6}$	(4)
$\frac{7}{9} \div \frac{2}{3}$	$\frac{3}{4} \div \frac{1}{3}$	$\frac{4}{7} \div \frac{2}{3}$	$\frac{1}{2} \div \frac{2}{3}$	
$9 \div \frac{3}{4}$	$8 \div \frac{1}{2}$	$6 \div \frac{2}{3}$	$12 \div \frac{4}{5}$	(12)
$\frac{7}{8} \div 2$	$\frac{4}{5} \div 8$	$\frac{2}{3} \div 12$	$\frac{8}{9} \div 16$	
$2\frac{1}{2} \div 5$	$3\frac{1}{3} \div 2$	$3\frac{3}{4} \div 2\frac{1}{2}$	$4\frac{4}{5} \div 2\frac{2}{3}$	(20)

Rating	Correct	Next Step
A	19 or 20	Do Inventory Test 9.
B	17 or 18	Do Inventory Test 9 or do Practice Exercises 8a to 8c.
C	16 or less	Do Practice Exercises 8a to 8c; then do Inventory Test 8 again.

EXERCISES

1. How long will it take Sam and Joe to walk to camp, a distance of 10 mi., if they walk at the rate of $3\frac{1}{3}$ m. p. h.?

2. Find the number of shelf boards $2\frac{1}{3}$ ft. long that can be cut from a 14-ft. board.

3. At $8\frac{1}{3}$¢ per pound, how many pounds of sugar can be sold for $1?

4. A certain job requires 45 hours to complete. If a man works at it $2\frac{1}{2}$ hours per day, how many days will it take him to complete the job?

5. How many strips of steel each $\frac{5}{16}$ in. thick will it take to make a pile $7\frac{1}{2}$ in. high?

6. Clyde wishes to rip a board $4\frac{7}{8}$ in. wide down the middle. How far from the edge must he lay out the line where he is to saw?

7. Mrs. Flagg bought a chicken weighing $4\frac{1}{4}$ lb. for \$2.55. What was the cost per pound?

8. Find the average speed of a train that travels 125 mi. in $3\frac{1}{8}$ hours.

9. How many clown costumes can be made from a bolt of cloth containing $38\frac{1}{2}$ yd., if each costume requires $3\frac{2}{3}$ yd.?

10. A scarf is to be made $\frac{1}{4}$ as wide as it is long. What is the required length if the width is to be 9 in.?

11. A remnant of cloth $\frac{7}{8}$ yd. long sold for \$3.50. Another piece of like material $\frac{5}{8}$ yd. long sold for \$3. Which was the better price per yard and how much better?

12. How many sacks containing $2\frac{1}{4}$ bushels each can be filled from a bin of oats containing 270 bushels?

13. How many strips of brass each $\frac{5}{16}$ in. thick will it take to build up a piece for a motor to a thickness of $1\frac{9}{16}$ in.?

14. The leg of a stool tapers from $2\frac{1}{4}$ in. across one end to $1\frac{1}{2}$ in. at the other end. If the length of the leg is 8 in., how much is the reduction per inch?

15. (OPTIONAL) Mary is to cut crosswise, from 32-in. cloth, 6 yd. of banding 4 in. wide. How much cloth must she buy, and how much waste will there be?

16. (OPTIONAL) A dress measures $2\frac{1}{3}$ yd. around the lower edge. An 8-in. ruffle is to be made one-fourth fuller. How much cloth 30 in. wide must be allowed for it?

CHAPTER III

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF DECIMAL FRACTIONS WITH APPLICATIONS

VOCABULARY

1. decimal fraction 2. measurement 3. estimate

24. Reading and writing decimal fractions. Decimal fractions, commonly called *decimals,* are being used more and more in the place of common fractions in industry.

A decimal fraction is a special kind of fraction whose denominator is 10, or 100, or 1,000, and so on. The decimal point is used to show whether the denominator is 10, 100, 1,000, and so on. For example, $\frac{3}{10}$ is written in the form of a decimal as .3; $\frac{42}{100}$ is written .42; $\frac{27}{1000}$ is written .027; 485/10,000 is written .0485.

EXERCISES

Write these common fractions as decimal fractions:

(a) $\frac{9}{10}$ (b) $\frac{3}{100}$ (c) $\frac{7}{1000}$ (d) 1/10,000 (e) $\frac{47}{1000}$ (f) $\frac{51}{100}$
(g) 21/10,000 (h) 17/100,000 (i) 33/1,000,000 (j) $\frac{73}{10}$ (k) $\frac{467}{1000}$

In reading a number that is a combination of a whole number and a decimal, use the word "*and*" to indicate the decimal point. For example, 324.678 is read "three hundred twenty-four and six hundred seventy-eight thousandths." (*Note.* Use the word *and* at no other place in reading decimals.)

Read these decimal fractions aloud:

(a) .6	(b) .5	(c) .64	(d) .26	(e) 1.24
(f) 4.36	(g) .301	(h) 2.282	(i) 5.1767	(j) .365
(k) 3.9	(l) 2.509	(m) 1.1786	(n) 8.1345	(o) 2.27486

Write the following numbers as decimal fractions:

(a) 18 hundredths	(b) twelve and 4 tenths
(c) 27 thousandths	(d) 5 and 3 tenths
(e) six thousandths	(f) 12 ten-thousandths
(g) two and 3 thousandths	(h) 9 hundred-thousandths
(i) one and 7 ten-thousandths	(j) fifty and sixty-six thousandths

In business, we usually read large numbers by repeating only the digits in their order, reading the left digit first. For instance, the number 605.342 may be read—"*Six-O-five-point-three-four-two,*" or "*Six hundred five, point, three forty-two.*"

Read each of the following numbers as if you were at work in a business office:

467.035	862.051	90.062	89.730	57,460.82	47.3609	765.3601
563.892	490.83	10.009	240.451	9080.6	2910.87	2105.43

25. Changing common fractions to decimal fractions. Since in a common fraction the line between the numerator and the denominator indicates division, we can change common fractions to decimal fractions by placing a decimal point after the numerator and annexing as many zeros as needed and then dividing the numerator by the denominator. Our answer must have as many decimal places as there are decimal places in the dividend.

Example

$$\tfrac{5}{7} = 7\overline{)5.00}\quad .71\tfrac{3}{7}$$

EXERCISES

Express as decimal fractions:

(a) $\frac{6}{7}$	(b) $\frac{4}{9}$	(c) $\frac{8}{25}$	(d) $\frac{3}{50}$	(e) $\frac{3}{20}$	(f) $\frac{5}{16}$
(g) $\frac{7}{12}$	(h) $\frac{8}{15}$	(i) $\frac{5}{6}$	(j) $\frac{1}{200}$	(k) $\frac{11}{25}$	(l) $\frac{1}{100}$

26. Changing decimal fractions to common fractions. To write a decimal fraction as a common fraction, leave out the decimal point and write the number as the numerator of the fraction; then write the appropriate denominator under the number and reduce the common fraction to its lowest terms.

Examples

(a) $.65 = \frac{65}{100} = \frac{13}{20}$

(b) $.005 = \frac{5}{1,000} = \frac{1}{200}$

EXERCISES

Express as common fractions in lowest terms:

(a) .85	(b) .72	(c) .7	(d) .8	(e) .006	(f) .004
(g) .18	(h) .015	(i) .62	(j) .45	(k) .125	(l) .625
(m) .0012	(n) 1.5	(o) 3.45	(p) 2.125	(q) .0008	(r) .00125

27. Arranging a series of numbers according to size. In a series of numbers, it is not always easy to decide which is the highest or the lowest in value. If the numbers are common fractions, it is usually best to change them to decimal fractions.

Which is the largest fraction in the series $\frac{2}{5}$, $\frac{3}{8}$, $\frac{4}{9}$, $\frac{3}{7}$? Change each fraction to a decimal. Then, by looking at the first decimal place (tenths), we can easily select $\frac{3}{8}$ (.37$\frac{1}{2}$) to be the smallest number. For the remaining three numbers, we must look to the first two decimal places (hundredths). Arranged in order from largest to smallest, the series is now $\frac{4}{9}$, $\frac{3}{7}$, $\frac{2}{5}$, $\frac{3}{8}$.

Examples

$\frac{2}{5} = .40$
$\frac{3}{8} = .37\frac{1}{2}$
$\frac{4}{9} = .44\frac{4}{9}$
$\frac{3}{7} = .42\frac{6}{7}$

Within some groups of numbers, it is necessary to change the numbers to thousandths or even a lower denomination before determining the order of size.

EXERCISES

1. Recite aloud the numbers in each series in the order of their size.

Largest first:				Smallest first:			
(a)	.20	.03	.16	(g)	.05	.45	.005
(b)	.60	.59	.71	(h)	.06	.66	6.6
(c)	.3	.04	.18	(i)	1	.18	1.8
(d)	.125	.15	.2	(j)	.33	3	.03
(e)	.608	.6	.06	(k)	.8	.88	.08
(f)	.167	.30	.28	(l)	.375	3.75	.07

2. Write the numbers in each series in the order of their size.

Largest first:				Smallest first:			
(a)	$.42\frac{1}{2}$	.42	$.82\frac{1}{8}$	(i)	$\frac{1}{16}$	$\frac{1}{12}$	$\frac{1}{15}$
(b)	.23	$.2\frac{1}{2}$	$.2\frac{3}{4}$	(j)	$\frac{3}{8}$	$\frac{3}{7}$	$\frac{2}{5}$
(c)	.9	.89	$.9\frac{1}{4}$	(k)	$1\frac{1}{4}$	$1\frac{1}{3}$	$1\frac{1}{8}$
(d)	$\frac{3}{4}$	.77	$.07\frac{1}{2}$	(l)	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{8}$
(e)	$\frac{7}{8}$	$\frac{5}{6}$	.85	(m)	2.0	$2\frac{1}{2}$	2.35
(f)	$\frac{5}{8}$	$\frac{3}{5}$	$.6\frac{1}{2}$	(n)	.895	.807	.8594
(g)	$\frac{1}{4}$	$\frac{2}{9}$	$\frac{2}{7}$	(o)	.678	2	.7
(h)	$\frac{3}{16}$	$\frac{1}{8}$	$\frac{1}{7}$	(p)	3.6	$3\frac{5}{8}$	$3.60\frac{1}{2}$

Inventory Test 9 ADDITION AND SUBTRACTION OF DECIMAL FRACTIONS

Working time—5 minutes.

Add:

1. 7.3	2. 5.85	3. 18.61	4. 15.03
1.8	.72	1.896	22.88
3.0	2.	$16\frac{1}{4}$	30.21

5. $1.2 + 3.45 + 8 + 3.4$

6. $32.6 + 7.14 + .8 + 14$

Subtract:

7. 2.8	8. .63	9. 7.9	10. $47\frac{1}{2}$
1.2	.08	.82	2.64

11. $256.34 - 27.1$

12. $78 - 2.71$

Rating	Correct	Next Step
A	11 or 12	Do Inventory Test 10.
B	9 or 10	Either do Inventory Test 10 or do Practice Exercises 9a to 9d.
C	8 or less	Do Practice Exercises 9a to 9d; then do Inventory Test 9 again.

EXERCISES

1. The receipts from a football game were $1,456.40. The expenses were $268.55. Find the profit.

2. A broker received $1,128 to invest in bonds. If his commission is $2.82, how much remains to invest?

3. One quart dry measure contains 67.200625 cu. in., and one quart liquid measure contains 57.75 cu. in. How many more cubic inches are there in a dry quart than in a liquid quart?

4. How deep a cut must be made in a circular steel rod whose diameter is $2\frac{1}{8}$ in. to bring the diameter to 1.275 in.?

5. Jack had $$7\frac{1}{2}$ and paid a debt of $3.86. How much did he have left?

6. A carpenter agrees to build a cabinet for $10.50. The lumber cost him $3.20 and the hardware $1.28. How much was his profit?

7. Scott and Company had a bank balance on April 1 of $1,812.23. During April, deposits of $68.14, $25.87, $75, and $197.55, and with-

drawals of $225, $98.46, $328.78, and $263.79 were made. What is the balance on April 30?

8. If a contractor is awarded a construction job on a bid of $28,452.75, what is his net profit if he estimates the cost at $25,594.49?

9. Joe's balance in the bank was $64.79 before depositing $26.43, $38.79, $17.65, and $29.48. Find his balance after he made the deposits.

10. A merchant's balance in the bank on May 31 was $697.75. During June he deposited $37.44, $86.79, and $97.68 and withdrew $219.49, $59.95, and $197.38. What was his balance on June 30?

11. Find the total cost of a 300-ft. cement walk if the foreman was paid $38.75, laborers were paid $75.80, cement cost $62, and gravel cost $37.79.

12. A grocer's sales for Monday amounted to $267.63; Tuesday, $198.56; Wednesday, $367.98; Thursday, $406.45; Friday, $419.59; and Saturday, $438.36. What were the total sales for the week?

13. Shipments of 1,845.9 lb., 2,384.72 lb., 1,694.2 lb., 2,897.8 lb., and 2,638.64 lb. of coal are placed in a bin. Find the total number of pounds in the bin.

14. In the following table, find the total sales for (a) each clerk, and (b) each day of the week.

Clerk	*Monday*	*Tuesday*	*Wednesday*	*Thursday*	*Friday*	*Saturday*
A	$76.18	$77.77	$68.95	$74.89	$56.42	$79.38
B	$50.27	$38.66	$45.23	$46.37	$52.76	$64.86
C	$38.29	$37.45	$36.48	$59.26	$39.07	$42.18
D	$75.75	$68.58	$49.75	$58.24	$66.38	$72.75

15. A pipe has an outside diameter of 1.315 in. and an inside diameter of 1.04 in. What is the thickness of the pipe?

28. Decimal fractions in shop measurements. Many measurements in the shop require such a high degree of accuracy that a special tool that measures to the thousandth of an inch is used. Consequently, measurements expressed as decimals frequently appear on drawings.

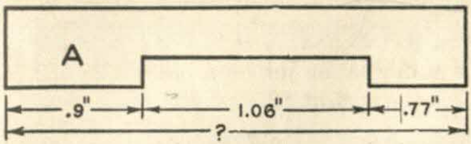
Examples

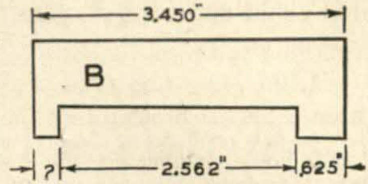
A. In figure A, the total length of the object is:

$.9'' + 1.06'' + .77'' = 2.73''$

B. In figure B, the missing dimension is:

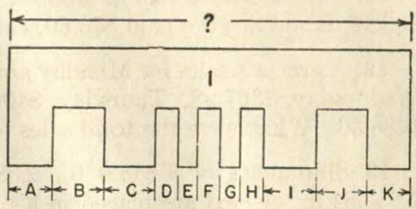
$3.450'' - 2.562'' - .625'' = .263''$

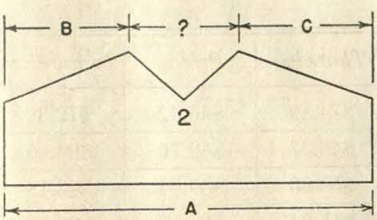




EXERCISES

$A = .37$ $D = .25$ $G = .21$
$B = .48$ $E = .21$ $H = .25$
$C = .62$ $F = .25$ $I = .62$
$J = .48$ $K = .37$

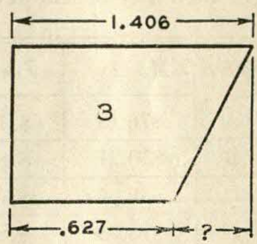




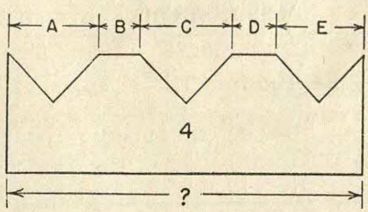
$A = 45.69$

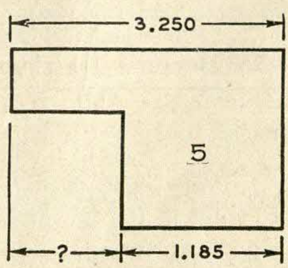
$B = 16.93$

$C = 17.14$



$A = .974$
$B = .389$
$C = .997$
$D = .389$
$E = .974$





1. Find the missing dimensions in diagrams 1, 2, 3, 4, and 5 (all dimensions are in inches).

2. Find the missing dimension of the motor support in figure 1.

3. Find the total length (figure 2): $A = 1.234''$, $B = .887''$, $C = .491''$, $D = 1.196''$, $E = 1.234''$.

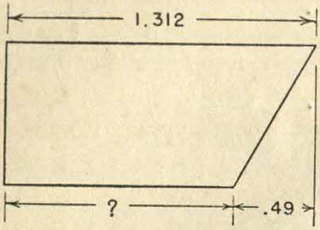


Fig. 1

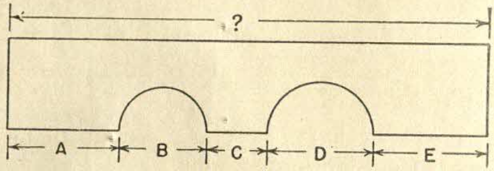


Fig. 2

Inventory Test 10 — MULTIPLICATION OF DECIMAL FRACTIONS

Time—5 minutes.

Multiply:

1. .3 × .8	2. 5.8 × .2	3. .08 × 7	4. 524 × .006	5. 6.7 × .08
6. 2.7 × 3	7. 3.6 × .4	8. 100 × .08	9. 7.92 × 10	
10. 7.4 × 2.8	11. 4.07 × .03	12. .32 × .14	13. 4.29 × .36	14. 5.63 × 6.07

Rating	Correct	Next Step
A	13 or 14	Do Inventory Test 11.
B	11 or 12	Do Inventory Test 11, or do Practice Exercises 10a and 10b.
C	10 or less	Do Practice Exercises 10a and 10b; then do Inventory Test 10 again.

EXERCISES

1. Forty-eight sheets of metal are stacked one on top of the other. If each sheet is .087 in. thick, what is the total height of the sheets?

2. At \$1.84 per foot, what is the cost of 180 ft. of steel tubing?

3. If a $\frac{5}{8}$-in. rivet weighs .175 lb., what is the weight of 1,000 such rivets? Of 750 rivets? Of 8,000 rivets?

4. If a plane that has a wing area of 386 sq. ft. can carry 20.8 lb. per square foot, how many pounds can the plane carry?

5. A certain metal sheet with an area of 18.7 sq. ft. weighs .196 lb. per square foot. What is its total weight?

6. A steamer traveling 16.8 m. p. h. will go how far in 24 hours?

7. An aircraft traveling 228.6 m. p. h. will go how far in 2.8 hours?

8. The airline fare between New York and Los Angeles is \$181.53 for adults. How much passenger fare is received by the airline for a trip from New York to Los Angeles on which 18 adults are carried?

9. What is the weight of 10 aircraft engines whose average weight is 318.76 lb. each?

10. If the flying cost of a certain plane is \$.482 per mile, what will be the cost of a 780-mile flight?

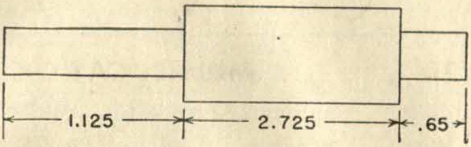


Fig. 3

11. Find the total length of figure 3.

29. Multiplying a number by 10, 100, 1,000, and so on. To multiply a number by 10, 100, 1,000, and so on, move the decimal point as many places to the *right* as there are zeros in the multiplier. Annex zeros if necessary.

Examples

$846. \times 10 = 8{,}460.$
$846. \times 100 = 84{,}600.$
$8.46 \times 10 = 84.6$

EXERCISES— ORAL

Multiply each of the following numbers by 10, 100, and 1,000:

1. 690	**8.** 33.3	**15.** 1298	**22.** 3.24	**29.** 7.2
2. 78	**9.** 200	**16.** 1.8	**23.** .526	**30.** 181
3. 4.78	**10.** 6.7	**17.** .35	**24.** 10	**31.** 666
4. 2344	**11.** 4.5	**18.** .07	**25.** .003	**32.** 3.45
5. 211	**12.** .8	**19.** 21.5	**26.** .6	**33.** .056
6. 21,800	**13.** 27.85	**20.** .4	**27.** 1	**34.** .1
7. 2.4	**14.** .139	**21.** 28	**28.** .5	**35.** 20

36. Mary Thomas makes bank deposits of \$10 per month for 124 months. What is her total deposit in that period?

37. What is the total cost of 100 articles @ \$4.06 each?

38. At the rate of \$10 per month, how much does a telephone cost per year?

39. There are 1,760 yd. in a mile. How many yards are there in 100 mi.?

40. A firm sold 7,685 washing machines @ \$100 each. Find the amount of the total sales.

41. A cubic foot of water weighs 62.5 lb. What do 100 cu. ft. weigh?

42. A cubic foot of space holds 7.48 gal. of water. How many gallons will 1,000 cu. ft. hold?

43. An agent makes 5¢ commission on each Christmas card that he sells. What is his total commission on 1,000 cards?

30. Multiplying a number by a fractional part of 1. (See Appendix D for equivalent fractional parts of one.)

Examples

Multiply 128 by $.12\frac{1}{2}$.

$$\overset{16}{\cancel{128}} \times \frac{1}{\cancel{8}} = 16$$

Multiply 2.52 by $.66\frac{2}{3}$.

$$\overset{.84}{\cancel{2.52}} \times \frac{2}{\cancel{3}} = 1.68$$

Multiply 324 by $1.33\frac{1}{3}$.

$$\overset{108}{\cancel{324}} \times \frac{4}{\cancel{3}} = 432$$

EXERCISES

1. $528 \times .25$

2. $756 \times .33\frac{1}{3}$

3. $4.59 \times .66\frac{2}{3}$

4. $65.6 \times .37\frac{1}{2}$

5. $.16\frac{2}{3} \times 2.28$

6. $.12\frac{1}{2} \times 368$

7. $59.5 \times .14\frac{2}{7}$

8. $51.2 \times .87\frac{1}{2}$

9. $7.2 \times .83\frac{1}{3}$

10. $.08\frac{1}{3} \times 2.4$

11. $.28\frac{4}{7} \times 264.6$

12. $10.48 \times .62\frac{1}{2}$

13. $1.28 \times .06\frac{1}{4}$

14. $.66\frac{2}{3} \times .528$

15. $43.2 \times .37\frac{1}{2}$

16. $.62\frac{1}{2} \times 926.4$

17. $.87\frac{1}{2} \times 62.56$

18. $.84 \times 1.33\frac{1}{3}$

19. $33.6 \times 1.37\frac{1}{2}$

20. $.968 \times 1.87\frac{1}{2}$

21. $17.37 \times 2.33\frac{1}{3}$

22. $1.66\frac{2}{3} \times 5.46$

23. $2.12\frac{1}{2} \times .72$

24. $1.62\frac{1}{2} \times 8.08$

25. Find the cost of 96 yd. of lace @ $33\frac{1}{3}$¢ per yard.

26. How much does a farmer's wife receive if she sells 64 doz. eggs at 50¢ per dozen?

27. Seventy bushels of potatoes at \$1.20 per bushel cost how much?

28. Brokerage at 25¢ per share amounts to how much on 528 shares?

29. At $16\frac{2}{3}$¢ per bulb, find the cost of 2 doz. electric light bulbs.

30. Find the total cost of 42 yd. of cloth at 25¢ per yard.

31. A gross (144) of marbles is sold at 10¢ per marble. What is the total amount of the sale?

32. If 742 pupils of a certain school give 25¢ each to the Red Cross, how much is contributed?

33. At a bargain sale, 524 aprons were sold at 75¢ each; how much was collected from the sale?

34. If eggs are sold at a profit of 10¢ per dozen, what is the profit on a case (30 doz.)?

31. Multiplying a number by a fractional part of 100. To multiply a number by a fractional part of 100, *first* multiply by 100 and then multiply by the fractional part of 100. ($.16\frac{2}{3} = \frac{1}{6}$ of 1; $16\frac{2}{3} = \frac{1}{6}$ of 100.)

Examples

Multiply 846 by $16\frac{2}{3}$.

$$\overset{14{,}100}{\cancel{84{,}600}} \times \frac{1}{\cancel{6}} = 14{,}100$$

Multiply 846 by 75.

$$\overset{21{,}150}{\cancel{84{,}600}} \times \frac{3}{\cancel{4}} = 63{,}450$$

Multiply 8.46 by $166\frac{2}{3}$.

$$\overset{282}{\cancel{846}} \times \frac{5}{\cancel{3}} = 1{,}410$$

EXERCISES

1. 360×50
2. 124×75
3. $488 \times 12\frac{1}{2}$
4. $429 \times 33\frac{1}{3}$
5. $5264 \times 12\frac{1}{2}$
6. $564 \times 16\frac{2}{3}$
7. $720 \times 8\frac{1}{3}$
8. $.344 \times 6\frac{1}{4}$
9. $7664.08 \times 37\frac{1}{2}$
10. $26.72 \times 62\frac{1}{2}$
11. $6.747 \times 66\frac{2}{3}$
12. $224 \times 112\frac{1}{2}$
13. $66\frac{2}{3} \times 3.48$
14. $133\frac{1}{3} \times 57.9$
15. $83\frac{1}{3} \times 7.26$
16. $16\frac{2}{3} \times .852$
17. $26.4 \times 8\frac{1}{3}$
18. $32.62 \times 128\frac{4}{7}$
19. $41\frac{2}{3} \times 456$
20. $17.52 \times 37\frac{1}{2}$
21. $14\frac{2}{7} \times 23.1$
22. $.954 \times 116\frac{2}{3}$
23. $18\frac{3}{4} \times 35.2$
24. $112\frac{1}{2} \times 1.256$

25. What is the cost of $16\frac{2}{3}$ yd. of cloth @ 24¢ per yard?

26. One cubic foot of water weighs $62\frac{1}{2}$ lb. What will 800 cu. ft. weigh?

27. If $75 per month was paid on a house for 120 mo., what was the total amount paid?

28. If there are 75 plants to each row, how many plants are there in 198 rows?

29. Mr. Arthur sells 125 shares of stock quoted at $80 per share. What is the total amount of the sale?

30. If 584 motors are sold @ $75 each, find the total amount of the sales.

31. If 1 ton of alfalfa costs $24.60, what is the cost of $33\frac{1}{3}$ tons?

Inventory Test 11 DIVISION OF DECIMAL FRACTIONS

Working time—5 minutes.

Division:

1. $4\overline{).88}$ **2.** $.3\overline{).72}$ **3.** $.02\overline{)4.8}$ **4.** $.05\overline{)62.5}$

5. $.6\overline{)66}$ **6.** $.2\overline{)8}$ **7.** $.004\overline{)12}$ **8.** $10\overline{)7.56}$

9. $\begin{array}{r} 45 \\ 2.3\overline{)1.035} \end{array}$ **10.** $\begin{array}{r} 37 \\ .72\overline{)26.64} \end{array}$ **11.** $\begin{array}{r} 485 \\ 24\overline{)116.40} \end{array}$

12. $\begin{array}{r} 252 \\ 4.8\overline{).12096} \end{array}$ **13.** $\begin{array}{r} 231 \\ .182\overline{)4.2042} \end{array}$ **14.** $\begin{array}{r} 237 \\ .145\overline{)3.4365} \end{array}$

15. $56.74 \div 10$ **16.** $3.1468 \div 100$

17. $3 \div 100$ **18.** $79.2 \div 1000$

19. $.02 \div 10$ **20.** $90 \div 100$

Rating	Correct	Next Step
A	19 or 20	Do Inventory Test 12.
B	17 or 18	Do Inventory Test 12, or do Practice Exercises 11a to 11c.
C	16 or less	Do Practice Exercises 11a to 11c; then do Inventory Test 11 again.

32. Dividing a number by 10, 100, 1,000, and so on. To divide a number by 10, 100, 1,000, and so on, move the decimal point as many places to the *left* as there are zeros in the divisor. (Prefix zeros if necessary.)

Examples

$846 \div 10 = 84.6$
$846 \div 100 = 8.46$
$8.46 \div 100 = .0846$

EXERCISES

A. Oral. Divide each of the following numbers by 10, 100, and 1,000:

1. 2	**8.** 24.5	**15.** 1267	**22.** 3.17	**29.** .18
2. 1.8	**9.** 300	**16.** 52.32	**23.** .8	**30.** 214
3. .7	**10.** 6.75	**17.** 3.724	**24.** 24	**31.** .07
4. 30	**11.** .425	**18.** 471.6	**25.** 500	**32.** 2000
5. .06	**12.** .007	**19.** .6161	**26.** 7.7	**33.** .4
6. 52	**13.** 28.7	**20.** 3000	**27.** 5	**34.** 1
7. 2.4	**14.** 1.11	**21.** 24.65	**28.** .1	**35.** 10

B. Written

36. What wages are paid for the completion of the following articles:

150	articles	@	$4.80 per 100		3,675	articles	@	$6.00 per 1,000
400	"	@	2.50 " 100		2,693	"	@	1.63 " 1,000
975	"	@	.96 " 100		974	"	@	8.75 " 1,000
2,450	"	@	.75 " 100		543	"	@	10.65 " 1,000

37. A $476 debt is paid in full in 100 equal payments. What is the amount of each payment?

38. An 8-in. space is divided into 100 equal parts. How long is each part?

39. At the rate of $1.65 for each 100 lb. of freight, what is the cost of shipping 29,000 lb.?

40. A freight car loaded with 86,400 lb. of wheat is shipped from Kansas City to St. Louis, Mo., at $19\frac{1}{2}$¢ per 100 lb. What is the total freight charge?

41. Green & Co. shipped 30,580 pounds of freight at the rate of $1.96 per hundred pounds. What were the freight charges?

42. At $63\frac{3}{4}$¢ per 100 lb., what are the freight charges on a shipment weighing 784 lb.?

43. A lot shipment is a shipment containing two or more packages sent to the same address. Find the total weight in a lot shipment of 3 packages weighing 186 lb., 179 lb., and 256 lb.

44. At $1.57 per 100 lb., what is the total cost of sending the lot shipment in exercise 43?

45. What will be the cost of 11,000 cu. ft. of gas @ $1.25 per 1,000 cu. ft.?

33. To get reasonable results by estimating quotients. Inaccurate answers are worthless. Many wrong answers are even unreasonable. You can avoid these absurd answers in division of decimals by following a plan of estimating your quotients.

Example 1

Divide 423.4 by 7.3.

7.3 is about 7 (*substituting a near value that is an easy divisor*)

423.4 is about 420 (*substituting a near value that is divisible by 7*)

$420 \div 7 = 60$ *estimated quotient.*

The correct quotient is 58.

> **Example 2**
>
> Divide .04692 by .051.
> Estimating the approximate quotient:
> 51 is about 50
> 46.92 is about 45
> 45 ÷ 50 = .9 *estimated quotient.*
> The correct quotient is .91

In the second example, you will find it easier to change the divisor to an integer by multiplying by 1,000. Be sure to multiply the dividend by 1,000 also. The problem then becomes 46.92 ÷ 51.

EXERCISES

Estimate each quotient and then divide:

1. 73.8 ÷ 1.8 ~~or 738 ÷ 18~~

   18 is about 20
   738 is about __
   __ ÷ __ = ______ *estimated quotient.*
   ______ correct quotient.

2. 139.2 ÷ 48

   48 is about __
   139.2 is about 150
   ______ ÷ ______ = ______ *estimated quotient.*
   ______ correct quotient.

3. 12.11 ÷ 2.3 .2 2 = 6	**6.** 83.7 ÷ 6.2	**9.** .14981 ÷ 7.1
4. 90.82 ÷ 1.9 90 2 - 45	**7.** .925 ÷ .025 900 .5	**10.** .288 ÷ .048
5. 1.344 ÷ .48 /	**8.** .21312 ÷ .32 21000 32	**11.** 325.6 ÷ 8.8

34. Rounding off numbers. The cost of merchandise is usually calculated to the nearest cent. As cents extend through the second decimal place, it is the third decimal place that determines the nearest cent. If the third decimal number is 5, or more than 5, add 1¢ to the cost. If the third decimal number is less than 5, disregard it and any numbers that may follow it.

Thus, when rounded off to the nearest cent, a cost of $1.535 becomes $1.54, while a cost of $1.534 becomes $1.53.

In amounts of money, the nearest hundredth is close enough for most purposes. However, there are times when a number must be written out to the thousandth or even to the millionth. For example, if the actual cost of certain articles is $.2455 each, and if 1,000 such articles are purchased, the total cost is $245.50. If the price per article had been rounded off to the nearest hundredth, which

would be \$.25 each, the cost of 1,000 would be calculated at \$250, which is \$4.50 more than the real cost per 1,000.

"About how many were at the concert today?" The answer, "About 3,000," might be near enough if by actual count 2,864 people were present. To the nearest thousand, 3,000 would be correct, while to the nearest hundred there were 2,900, and to the nearest ten there were 2,860 present. How close to round off a number, or whether to round it off at all, depends upon the use to be made of the number.

In writing a number correct to a required number of decimal places, look at the number in the decimal place that is one place beyond the decimal required. Do as you did in finding the nearest cent. That is, if the number in the first decimal place beyond the one required is 5 or more, add 1 to the last decimal required; if the number is less than 5, disregard it and any decimals to the right of it.

Examples

3.141592 to the nearest hundred-thousandth is 3.14159
" to the nearest ten-thousandth is 3.1416
" to the nearest hundredth is 3.14
" to the nearest tenth is 3.1.

EXERCISES

1. Round off each decimal to the nearest hundredth:

(a) .623	(b) .8764	(c) 1.472	(d) 3.158	(e) .7247
.705	.2315	2.169	.297	.6666
.816	.1798	1.083	1.604	.1666
.341	.1505	4.032	2.309	.3333
.722	.0637	7.192	5.001	.1717

2. Round each decimal off to the nearest thousandth:

(a) .2348	(b) .1086	(c) 3.1451	(d) .7824	(e) .0681
.1726	.2007	7.0207	.4555	.0092
.3549	.3198	6.3131	1.7232	.0796
.1071	.1670	1.4999	5.6781	.0683

3. Find the cost of each of these novelties to the nearest cent:

(a) 2 for 5¢	(b) 6 for 24¢	(c) 8 for 25¢
3 for 10¢	8 for \$1.00	2 for 7¢
4 for 15¢	12 for \$1.25	4 for 27¢
3 for 13¢	12 for 50¢	12 for \$1.15

4. Find the cost of each article to the nearest tenth of a cent:

(a) 3 for 7¢	(b) 7 for 25¢	(c) one dozen for 45¢
4 for 25¢	3 for 50¢	one dozen for 50¢
3 for 8¢	4 for 75¢	one dozen for 40¢

5. Find each quotient to the nearest hundredth:

(a) $6\overline{)7.51}$ (b) $3\overline{)8.2}$ (c) $.4\overline{).59}$ (d) $.5\overline{)7.126}$

(e) $.08\overline{).0678}$ (f) $2\overline{).0678}$ (g) $.09\overline{).0978}$ (h) $7\overline{)6.585}$

EXERCISES

Find the average of each of these groups of numbers:

1. 9.3 in., 3.4 in., 1.7 in.
2. .6 ft., .5 ft., .8 ft., .9 ft.
3. 15.7 gal., 27.5 gal., 18.3 gal.
4. 7.7%, 9.4%, 6.2%, 3.9%.
5. $1.25, $2.72, $2.18.
6. .5, .25, .375.
7. .66, .5, .838.
8. 6.8, 4.6, 3.9.
9. 1.5, 2.2, 3.6.
10. .1, .7, .8.

11. The diameter of a rod was measured by four different employees. Their measurements of the diameter were: 1.68″, 1.67″, 1.69″, and 1.69″. Find the average diameter.

12. Mr. Reed's earnings for 5 weeks were: $85, $78, $81, $87.50, and $82.50. Find his average earnings per week.

13. A motorist traveled 180 mi. and used $10\frac{1}{2}$ gal. of gasoline. How many miles did he average per gallon?

14. During one month, Joan spent 85¢ for lunches the first week, 90¢ the second week, $1.10 the third week, and 95¢ the fourth week. What was the average cost of her lunches per week?

EXERCISES

1. If the cost of 18 hammers is $22.50, what is the cost of 1 hammer?

2. Mr. Cooper drove 484.4 mi. and used 28 gal. of gasoline. What did he average in miles per gallon?

3. An agent bought 36 doz. eggs, for which he paid $12.60. What was the price per dozen?

4. Beulah paid $2.34 for a chicken weighing $4\frac{1}{2}$ lb. Find the price per pound.

5. At 24¢ per gallon, how many gallons of gasoline can be bought for $17.28?

6. A strip of aluminum 32.96 in. long is to be cut into 16 equal parts. How long will each piece be?

7. How many strips of metal each .016 in. thick will it take to build up a piece for a motor to a thickness of 1.12 in.?

8. How many sacks containing 2.5 bu. can be filled from a bin of 1,800 bu. of wheat?

9. A school with an enrollment of 840 pupils made $327.60 on an entertainment. What was the average amount per pupil?

10. A railroad rises 123.795 ft. in 4126.5 ft. Find the average rise per foot.

11. A keg of $\frac{7}{8}$-in. bolts weighs 498 lb. How many bolts are there in the keg, if each bolt weighs .664 lb.?

12. How many pieces of rope each 7.5 in. long can be cut from a coil 175 ft. long? 86

13. The speedometer on Mr. Jackson's car registered 2814.8 mi. when he left on a 6-day business trip. Upon his return the speedometer registered 3272.2 mi. How many miles did he drive, on the average, per day?

14. The speedometer on Mr. Monroe's car registered 3567.3 mi. before he started on a trip into another state. Upon his return it registered 4020.9 mi. He used 28 gal. of gasoline on the trip. What was the average number of miles per gallon of gasoline used?

15. A carload of wood pulp cinders originated at Lock Haven, Pa., and moved to Luke, Md. The shipment weighed 93,400 pounds and the freight charges were $186.80. What was the freight rate per hundred pounds?

CHAPTER IV

PERCENTAGE AND PROBLEM SOLVING

VOCABULARY

1. per cent
2. percentage

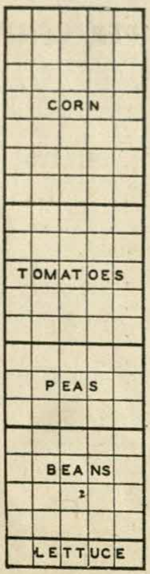
35. The meaning of per cent. A number written with the per cent sign (%) is a special kind of a fraction. It is a fraction expressed as 100ths. The word *per cent* means hundredths, and a number written with the per cent sign may be written:

(a) as a common fraction:

$$3\% = \frac{3}{100}$$

or (b) as a decimal fraction:

$$3\% = .03$$



John Smith laid out a garden of 100 squares (5 squares wide, 20 squares long). He pictured his garden on squared paper as illustrated. John used 5 of the 100 squares for lettuce. That is, he used $\frac{5}{100}$ or 5% of the garden for lettuce. He used 20 of the squares for beans. That is, he used $\frac{20}{100}$ or 20% of the garden for beans. What per cent did he use for peas? for tomatoes? for corn?

What is the total of the 100ths? That is, how many 100ths represent all of the garden? What is the total of the per cents representing the vegetables planted? That is, what per cent represents all of the garden?

One hundred per cent of anything is all of that thing.

36. The use of per cent. Per cent is a convenient and easy way of making comparisons. For example, in September, Edna took an Inventory Test of 50 exercises in the fundamentals of arithmetic. She did 28 exercises correctly. This means that $\frac{28}{50} = \frac{56}{100}$ or 56% of the exercises were correct. After a few weeks' practice, Edna took a similar test but of only 40 exercises (instead of 50) and made a score of 28 again. This was $\frac{28}{40}$ or .70 or 70% of the exercises correct. Thus, Edna could see that a few weeks' practice had raised her mark from 56% to 70%.

In an Inventory Test of 10 exercises in addition and 12 exercises in multiplication, Joe solved 7 correctly in addition and 10 correctly in multiplication. He made a comparison of these results in order to know in which process he needed the more practice. His addition score was $\frac{7}{10}$ or 70%. His multiplication score was $\frac{10}{12}$ ($\frac{5}{6}$) or $83\frac{1}{3}$%. Thus, he saw that while he needed practice in each process, he needed more practice in addition.

Business firms use per cents to make comparisons of costs and of profits, and in budgeting money for various uses. The need for a knowledge of per cent in the business world is great. It is not uncommon for a businessman to advise young people in school to "learn how to use per cent."

Let us try an *Inventory Test* in per cent and see how well prepared we are to use it.

Inventory Test 12 **PER CENT**

Time—5 minutes.

Change these decimal fractions to per cents:

1. .16	2. .07	3. .08	4. .125
5. .025	6. 1.75	7. .1425	8. .005

Change these common fractions and mixed numbers to decimals:

9. $\frac{3}{100}$	10. $\frac{7}{50}$	11. $\frac{4}{5}$	12. $\frac{1}{10}$
13. $\frac{4}{15}$	14. $\frac{2}{7}$	15. $2\frac{3}{4}$	16. $1\frac{1}{6}$

Change these per cents to decimals:

17. 25%	18. 18%	19. 6%	20. 4%
21. 150%	22. 240%	23. $\frac{1}{2}$%	24. .2%

Change these per cents to common fractions in lowest terms:

25. 20%	26. 8%	27. 1%	28. 125%
29. $12\frac{1}{2}$%	30. 1.2%	31. .5%	32. $\frac{1}{4}$%

Rating	Correct	Next Step
A	31 or 32	Do Inventory Test 13.
B	29 or 30	Do Inventory Test 13, or do Practice Exercises 12a to 12f.
C	28 or less	Do Practice Exercises 12a to 12f; then do Inventory Test 12 again.

Inventory Test 13

PER CENT EXERCISES

Time—5 minutes.

Fill in the missing numbers:

1. 50% of $64 = ?$
2. $?\%$ of $24 = 6$
3. 20% of $? = 40$
4. 6% of $150 = ?$
5. $33\frac{1}{3}\%$ of $120 = ?$
6. 3% of $? = 9$
7. 150% of $200 = ?$
8. $?\%$ of $80 = 30$
9. $12 = ?\%$ of 12
10. 1.2% of $400 = ?$
11. $?\%$ of $60 = 10$
12. $8 = ?\%$ of 24
13. $?\%$ of $80 = 8$
14. $8\frac{1}{3}\%$ of $12 = ?$
15. 102% of $40 = ?$
16. $3 = ?\%$ of 15
17. $?\%$ of $50 = 8$
18. $66\frac{2}{3}\%$ of $18 = ?$
19. $1 = ?\%$ of $\frac{1}{2}$, 50
20. $\frac{1}{4} = ?\%$ of 1
21. $5 = ?\%$ of 4
22. 400% of $6 = ?$
23. $37\frac{1}{2}\%$ of $88 = ?$
24. $16\frac{2}{3}\%$ of $42 = ?$

Rating	Correct	Next Step
A	23 to 24	Do Inventory Test 14.
B	21 to 22	Do Inventory Test 14, or do Practice Exercises 13a to 13d.
C	20 or less	Do Practice Exercises 13a to 13d; then do Inventory Test 13 again.

37. Estimating reasonable products. To avoid getting absurd answers, near fractional equivalents of per cents are often used in estimating products in which per cents are a factor.

Example

What is 19% of 724?
19% is close to 20%, or $\frac{1}{5}$.
724 is close to 700.
$\frac{1}{5}$ of $700 = 140$, the estimated product.
The exact product is 137.56.

EXERCISES

Estimate each product; then find the exact product:

1. 26% of \$440
2. 11% of \$690
3. 74% of \$480
4. 17% of \$180
5. 19% of \$250
6. 51% of \$84.50
7. 66% of \$39
8. 13% of \$88
9. 34% of \$9.60
10. 21% of \$55.25
11. 15% of \$7.20
12. 8% of \$120
13. 24% of \$16.80
14. 62% of \$24.80
15. 87% of \$9.76

38. Problem solving. Up to now you have been working *examples.* You have been told whether to add, subtract, multiply, or divide. You are now ready to solve *problems.* On the following pages, you are to use arithmetic computations in the solution of problems of daily life. You must decide for yourself which of the fundamental processes you need to use. You will need to think each problem through carefully, plan its solution, and be accurate in the use of the fundamental processes.

Steps in problem solving. The suggestions found in the following eight steps will help you become skillful at problem solving:

Step 1. *Read* the problem carefully and as many times as necessary to understand its meaning. Look in the dictionary for any word whose meaning is not clear.

Step 2. *Determine* what is to be found.

Step 3. *Select* which of the given facts will be helpful in finding the result.

Step 4. *Plan* your procedure. Will you add, subtract, multiply, or divide?

Step 5. *Estimate* the result in round numbers.

Step 6. *Solve* the problem, keeping the following suggestions in mind:

(a) Put your work in systematic order.
(b) Draw a sketch if it will help in the solution.
(c) Write all numbers legibly, making large decimal points.
(d) Label your result and any other important numbers.
(e) Prove any sizable fundamental processes.

Step 7. *Compare* the result obtained with your estimated result.

Step 8. *Check,* if possible, working the problem backwards. That is, from the result you have found, work back to one of the given facts.

Each of the steps is important and should be used. However, in most problems the solution can be shortened by performing one or more of the steps mentally. *Learn* the eight steps and use them either mentally or in writing.

PROBLEMS

Solve these problems, following the suggestions in problem solving:

1. Mr. Sands used $8\frac{1}{4}$ gal. of gasoline for a trip of 165 mi. How many miles did he average per gallon?

2. Which is the cheaper and how much: to buy 8 oranges at 78¢ per dozen, or to buy 8 oranges at 2 for 15¢?

3. Henry is gathering up scrap metal to sell. He has three pieces weighing $15\frac{1}{2}$ lb., $12\frac{3}{8}$ lb., and $14\frac{1}{8}$ lb. At 5¢ per pound, how much will he receive?

4. Nine tenants decide to rent a garden space for $12.50. The plowing will cost $5.25, and the seeds and plants will cost $4.75. How much will be the average expense for each tenant?

5. The Girl Scouts of a certain troop sold 62 dozen doughnuts at 55¢ per dozen. If their expenses were $11.47, how much was their profit?

6. Gene earned $6.25 per day for 25 days during July. His room and board for the time cost him $48.50. How much of his earnings did he have left?

7. Harold made a bread board and sold it for 65¢. If the lumber cost him 17¢, how much will he receive per hour for the 3 hours of labor that he spent on it?

8. Which is the better buy: (a) to pay $18.75 for a dress, or (b) to buy $3\frac{1}{4}$ yd. of material at $1.24 per yard, 9 buttons at 80¢ per dozen, 8 yd. of braid at $7\frac{1}{2}$¢ per yard, and pay a dressmaker $6.50? How much money would your choice save you?

9. Mr. Parker receives $480 per month, which he budgets as follows: $\frac{1}{3}$ for board and room, $\frac{1}{12}$ for clothing, $\frac{1}{10}$ for life insurance, $\frac{1}{15}$ for church and charity, $\frac{1}{20}$ for recreation, and $\frac{1}{30}$ for personal needs. He saves the remainder. How much does he save?

10. How much did it cost Mr. Adams to go on a 2,400-mi. trip if he got 15 mi. to the gallon of gasoline, which cost 27¢ per gallon?

11. Mr. Palmer used his car 9 days on a business trip. At the beginning of the trip the speedometer read 6748.3 mi., and at the end of the trip it read 8378.2 mi. How many miles did he average per day?

12. The members of a school concert sold $70 worth of tickets. If 200 pupil tickets were sold at 15¢ each, how many adult tickets were sold at 25¢ each?

13. A building valued at $6,400 is insured for $\frac{7}{8}$ of its value at the rate of $3.60 per $1,000. What is the cost of the insurance?

14. Wesley has $20. This is $\frac{5}{6}$ of the money he needs to buy a bicycle. What is the cost of the bicycle?

15. Bill is buying a car. He paid $500 down and is to pay $40 a month for 18 months. For cash he could have bought the car for $1075. How much would he have saved by paying cash?

16. A 950-ft. fence with posts $8\frac{1}{3}$ ft. apart is to be built around a lot. How many posts will be needed?

17. Bert saved $\frac{1}{3}$ of his earnings. He then decided to invest $\frac{3}{4}$ of his savings in Government bonds. What part of his earnings did he invest in the bonds?

18. If caps that cost $17.40 per dozen are sold at $1.75 each, what is the profit on each cap?

19. Twenty-eight music stands costing $18 each were priced to sell for $23.50. After the stands were sold, what was the total profit?

20. If overalls costing $30 a dozen are sold at $3.50 each, what is the profit on each pair sold?

21. If the sun is 93,000,000 miles from the earth and light travels 186,000 miles per second, how many minutes are required for the sun's rays to reach the earth?

22. John can do a certain piece of work in 48 hours, while Carl, his older brother, can do the same work in 32 hours. Which is it the cheaper to hire if John charges $33\frac{1}{3}$¢ per hour and Carl $37\frac{1}{2}$¢ per hour?

23. The windows in a house extend 6′ 6″ above the floor and are 30 in. wide. How many yards of drapery material 36 in. wide are needed for each window? (One strip of material makes one drapery, and the curtains are to extend from the top of the window to the floor. Allow 3 in. to turn under at the top and bottom of each drapery. Two drapes to each window.)

24. A hotel served 108,000 cups of coffee in a year. If $1\frac{1}{2}$ tons of coffee were used, determine the average number of cups obtained from one pound of coffee.

25. (OPTIONAL) A printing press averages 2,500 impressions an hour and prints 64 pages at each impression. At $3 per hour, what will it cost to print 15,000 catalogues containing 384 pages each?

26. (OPTIONAL) A tank full of water has 2 pipes opening from it, one of which will empty one-sixth of it in 1 hour and the other one-fourth of it in 1 hour. If both pipes are open for 1 hour, what part will remain in the tank? What part will be emptied?

39. Percentage problems. Percentage problems are of three different types:

(a) To find a certain per cent of a number.
(b) To find what per cent one number is of another number.
(c) To find a number when a certain per cent of it is known.

40. To find a certain per cent of a number. Jack spelled 40 words, of which 85% were correct. How many of the words were correctly spelled? You need to find a per cent (85%) of a number (40).

Step 1. Read the problem carefully.
Step 2. Find: Number of words correctly spelled.
Step 3. Known:

40 = total number of words.
85% = words correctly spelled.

Step 4. Procedure:

85% of 40 = number correctly spelled.
Factor × factor = product.

Step 5. Estimate:

Use 80% in place of 85%.

80% of 40 = 32.

Step 6. Solution:

85% of 40 = 34.

Hence, 34 words were correctly spelled.

Step 7. Compare results with estimate: 34 words is near to 32.
Step 8. Check:

$\frac{34}{40} = \frac{17}{20} = .85 = 85\%$, per cent of words correct.

EXERCISES

1. John's lesson consisted of 15 problems. He got 80% correct. How many were correct?

2. Mary made a grade of 75% on 24 exercises. How many did she have right?

3. Harold won 20% of the 35 events in an athletic contest. How many events did he win?

4. Rudolph earns $28 per month working after school hours. He saves 15% of his earnings. How much does he save?

5. On a trip of 275 mi., Edward drove 60% of the distance. How many miles did he drive?

6. A radio that cost $42 was sold for 85% of its cost. For how much was it sold?

7. Jane received $8.50 on her birthday. She saved 40% of the money. How much did she save? How much did she spend?

8. Donald bought a wagon for \$6.50. He sold it for 120% of the cost. For how much did he sell the wagon?

9. Last year Sam won 12 events in a contest. This year he won 125% as many as last year. How many events did he win this year?

10. In September, Bonnie read 180 words per minute. By March she read 130% of her September rate. How many words did she read per minute in March?

41. To find what per cent one number is of another number.

In a class of 36 pupils, 33 were present. What per cent were present?

You need to find what per cent one number (33) is of another number (36).

Step 1. Read the problem carefully.
Step 2. Find: Per cent of pupils present.
Step 3. Known:

36 = total pupils in the class.
33 = pupils present.

Step 4. Procedure:

$$\frac{33}{36} = ?\%.$$

Step 5. Estimate:

33 is slightly less than 100% of 36—say, 90%.

Step 6. Solution:

First Method	Second Method
$\frac{33}{36} = \frac{11}{12} = .91\frac{2}{3} = 91\frac{2}{3}\%$.	$?\%$ of $36 = 33$ (Product ÷ known factor = missing factor.) $33 \div 36 = .91\frac{2}{3}$ or $91\frac{2}{3}\%$.

Hence, $91\frac{2}{3}\%$ of the pupils were present.

Step 7. Comparison:

$91\frac{2}{3}\%$ is slightly less than 100%—near 90%.

Step 8. Check:

$91\frac{2}{3}\%$ of $36 = 33$, pupils present.

42. Calculating grades. In a spelling lesson of 10 words, Fay spelled 8 of the words correctly. To find the per cent correctly

spelled, she needs to find what per cent the part spelled correctly (8) is of all the words (10). This is $\frac{8}{10} = .80 = 80\%$. Fay's grade, therefore, is 80%.

Jack solved correctly 13 out of 16 problems. His grade is $\frac{13}{16}$ or $.81\frac{1}{4} = 81\frac{1}{4}\%$.

This method of obtaining a grade can be used in all subjects provided the exercises all have the same value.

EXERCISES

1. In an English language exercise of 32 sentences, Dora had 27 correct. What was her grade?

2. Clara has answered 18 out of 25 history questions correctly. What is her grade?

3. Harvey was given 18 lines of poetry to memorize. He learned all 18 of the lines. What is his grade?

4. Out of 30 points in a science lesson, Gerald had 25 correct. Calculate his grade.

5. Betty had 24 out of 27 points correct in her Spanish lesson. What was her per cent correct?

6. Jerry missed 4 out of 32 problems. What per cent of the problems did he have correct?

7. Sally had 17 exercises to do. She missed 2 of them. What per cent did she have right?

8. Calculate Fern's grade if she has 37 out of 40 points correct.

9. Jean did 4 of her 22 exercises wrong. Find her grade.

10. Of a group of 35 sentences, Jack has 28 correct. What is his grade?

43. To find a number when a certain per cent of it is known.
Mary's class sold 156 tickets for a school play. This was 12% of the tickets sold. How many tickets were sold?

You need to find a number (?) when a certain per cent (12%) is known (156).

Step 1. Think as you read the problem.
Step 2. Find: Total tickets sold.
Step 3. Known:

156 = tickets sold by Mary's class.
12% = tickets sold by Mary's class.

Step 4. Procedure:

12% of tickets = 156
All of tickets = 156 ÷ .12 = ?

Step 5. Estimate:

150 ÷ .10 = 1500.

Step 6. Solution:

First Method	Second Method
12% of tickets = 156	12% of ? = 156
1% of tickets = 156 ÷ 12 = 13	(Product ÷ known factor = missing factor)
100% of tickets = 100 × 13 = 1300	156 ÷ .12 = 1300

Hence, the total number of tickets sold was 1300.

Step 7. Comparison:

1300 is near 1500.

Step 8. Check:

12% of 1300 = 156.

EXERCISES

1. James had 16 of his exercises correct. If this was 80% of his lesson, how many exercises were there in all?

2. If Rose saves 35% of her earnings, how much will she have earned when she has saved $10.50?

3. Harry sold 54 papers, which is 90% of the number he had to sell How many papers did Harry have to sell?

4. A coat is reduced $10.80, which is 12% of its marked price. For how much was the coat marked to sell?

5. Twenty-four pupils, which is 80% of the class, made a score of above 90. How many pupils are in the class?

6. George gets a ride of 45 blocks, which is 90% of the distance to his work. How far does he live from his work?

7. Dorothy, who receives 8% of her sale of cards as a commission, earned a $3.44 commission in December. What did her sales amount to during the month?

8. Paul is saving money to buy a camera. He has saved $18, which is 45% of the cost. What does the camera cost?

9. Twenty per cent of the boys in a class were Boy Scouts. There were 8 Scouts in the class. How many boys were in the class?

10. Andy weighs 125 pounds, which is $62\frac{1}{2}$% of his father's weight. How much does his father weigh?

PERCENTAGE PROBLEMS

Solve:

1. Mr. Jay contributed 9% of a $180 Red Cross fund. How much did he contribute?

2. Ray worked 9 of the 14 weeks of his vacation. What per cent of his vacation did he work?

3. Joel paid $2.52 for a history book. This was 28% of the cost of all his books. How much did his books cost him?

4. An agent sold $1,420 worth of flour at a $1\frac{3}{4}$% commission. How much was his commission?

5. Mr. Powell's rent this past year amounted to $65 per month. His rent was 20% of his income. Find his yearly income.

6. Carl sold his bicycle for $31.20, which was 80% of its cost. What was the cost?

7. A typewriter is listed to sell at $120 less $18. Find the per cent of discount.

8. Miss Peck works in a dress shop. She gets a 15% discount on any dress that she purchases in the shop. A dress marked to sell for $29.75 would cost her how much?

9. John's team won 14 games and lost 11 games. What is the per cent won?

10. A store reduced costs from $150 to $115. Find the per cent of the reduction.

11. If $750 is invested in stock, what profit must be made in order to earn 6%?

12. At 24% commission, what is the amount of commission on a sale of 6 dozen cards at $1.25 per dozen?

13. David was in school 178 of the 180 school days of this year. Find his per cent of attendance.

14. Mr. Adams planted 60 tomato plants. This was 120% of the number he planted last year. How many did he plant last year?

15. About 60% of type metal is lead. How many pounds of lead are there in 685 lb. of type metal?

16. Will's salary was raised from $.60 per hour to $.85 per hour. Find the per cent of increase.

17. Mr. Barker sells real estate at a 5% commission. For a certain sale, his commission amounts to $392.50. For how much did he sell the property?

18. Mr. Jones paid 35% of a bill. The payment was $2.94. What was the total bill?

19. There is a Federal tax of 5% on airline fares. How much is the tax on a ticket that costs $28.60?

20. In a school of 1,300 pupils, 351 pupils were on the honor roll. What per cent were on the honor roll?

21. Tom's weight of 135 lb. is 108% of what it was a year ago. How much did he weigh a year ago?

22. Richard's team won 24 games out of 35 games played. What per cent of the games were won?

23. A man earned $240 and spent $27 for a suit. What per cent of the money earned did he spend for the suit?

24. Jack sold his camera for $18.40. This was 120% of the cost. What was the cost of the camera?

25. At a sale, Joe bought a $37.50 bicycle for $30. What per cent reduction did he receive?

26. How many pounds of butterfat are there in 1,240 lb. of milk if the milk tests 3.9% butterfat?

27. An investment of $850 yielded an income of $59.50. Find the rate of yield.

28. A factory makes 2,450 sweaters, 12% of which are defective. How many perfect sweaters are in the total?

29. A broker sold 1,000 bales of cotton (400 pounds per bale) at $14\frac{1}{2}$¢ per pound. He charged $\frac{1}{8}$% commission. What was his commission?

30. A team won 12 baseball games and lost 18. What per cent did the team win?

31. A wholesale firm reduced rugs from $72 to $64. What was the per cent reduction on the final selling price?

32. A cow gives 42 lb. of milk a day testing 3.7% butterfat. How many pounds of butterfat does the milk contain?

33. A mechanic earning $120 per week had his wages reduced 15%. What was his salary after the reduction?

34. If a pupil solves 35 out of 40 problems correctly, what is his grade (using 100 as a basis for all problems correct)?

35. Government figures show that, in furniture stores, salaries take about 19.7% of the gross sales; rent, 11.2%; advertising, 6.8%; handling, 3.5%; and delivery, 4%. The total expense is what per cent of the gross sales?

36. The owner of a storeroom receives annually in rent $7\frac{1}{2}$% of his investment on the building. If the value of the building is $26,000, what is the rent per month?

37. What is the monthly income of a family that pays $16\frac{2}{3}$% of its monthly income for rent, which amounts to $840 per year?

38. If a merchant's sales amount to $100,000 and his profit to $5,000, what is the per cent of profit?

39. If a ton of ore yields 70 lb. of iron, what per cent of the ore is iron?

40. With an annual income of \$640,000, the town of Perry spends $37\frac{1}{2}\%$ for schools. How much is spent for schools in one year?

41. A house was sold for 15% less than it cost. The selling price was \$4,250. What was the cost?

42. A farmer bought livestock for \$2,835 and sold it for \$2,430. What per cent did he lose on the selling price?

43. What is the rate of discount on goods listed at \$87.20 and sold for \$65.40?

44. If sea water contains about 3% of salt, how many pounds of sea water, when evaporated, will make 150 lb. of salt?

45. Joe sold a radio set for \$13.20, which was 80% of the cost. What was the cost?

46. A lawyer was paid \$87.50 for collecting a \$262.50 bill. What per cent did he receive?

47. If a house valued at \$8,000 is insured for \$5,500, what per cent of the value of the house is insured?

48. What is the tax on property assessed for \$9,600 with a tax rate of 1.56%?

49. A house is mortgaged for \$6,576 at $5\frac{1}{2}\%$ a year. What is the monthly interest due?

50. A house worth \$7,800 is insured for 80% of its value at an annual rate of $\frac{3}{4}\%$. What will the insurance cost for 2 years?

51. In one month, Murphy sold to 85 customers out of 250 called on. Peters sold to 64 customers out of 120. What is the per cent of efficiency of each salesman for that month?

52. A laundry charges 4¢ per pound and allows 15% discount for cash-and-carry. What will 36 pounds cost delivered at the plant?

53. The construction of 359,000 homes was begun during the four months immediately following the start of the Korean war. This was 101,000 more homes than were begun during the same period of the preceding year. Find the per cent of increase in home construction.

54. (OPTIONAL) Mr. Kemp can have his home insulated against cold weather at a saving of $16\frac{2}{3}\%$ on his heating bill. If he used 12 tons of coal per year at \$9.60 per ton for heating and the total cost of insulation will be \$288, in how many years will he save the cost of the insulation?

55. (OPTIONAL) Mr. King sold a car for \$1260, which was a 5% gain on the cost. What was the cost? *Note:* Cost (100%) + gain (5%) = 105%, selling price.

56. (OPTIONAL) Miss Neil sold a coat for \$68 at a loss of 15% of the cost. What was the cost?

57. (OPTIONAL) Mrs. Head bought a suit for \$29.92. This was 12% less than the original price. What was the original price of the suit?

58. (OPTIONAL) The enrollment of a school is 1,125 pupils, a decrease of 10% from last year. What was the last year's enrollment?

59. (OPTIONAL) Edward has a collection of 840 stamps. This is $33\frac{1}{3}\%$ more than Bob has. How many stamps has Bob? *Note:* Bob's stamps $(\frac{3}{3}) + \frac{1}{3}$ of his stamps = Edward's stamps.

60. (OPTIONAL) Robert has had his salary increased $16\frac{2}{3}\%$, which makes him receive \$42 per week. What was his salary before the increase?

61. (OPTIONAL) Because of illness, Floyd's attendance at school this year is 154 days. This is a $12\frac{1}{2}\%$ decrease from his attendance of last year. Find his last year's attendance.

62. (OPTIONAL) Mr. B. is a salesman. He receives a salary of \$200 per month plus a commission of 5%. In June he received \$350. How much were his sales for June?

63. (OPTIONAL) If water expands 9% when it freezes, how many cubic inches of water will be needed just to fill a gallon jar (231 cu. in.) when the water freezes?

64. (OPTIONAL) A house is insured for $\frac{3}{4}$ of its value at $\frac{2}{3}\%$. If the cost of the insurance is \$120, for how much is the house insured, and what is its value?

CHAPTER V

DENOMINATE NUMBERS; RATIO AND PROPORTION

VOCABULARY

1. ratio 2. proportion 3. means 4. extremes

44. Denominate number tables. Tables showing the relationship between the common units of measure are found in Appendix D. The relationships that are most frequently used should be memorized.

Inventory Test 14 RELATIONSHIP BETWEEN UNITS OF MEASURE

Time—5 minutes.

Express in the specified units:

1. 18 in. = ___ ft. ___ in.

2. 4 ft. = ___ in.

3. 2 ft. = ___ yd.

4. 1 yd. 2 ft. = ___ in.

5. 2 ft. 5 in. = ___ in.

6. 2 yd. = ___ in.

7. $\frac{2}{3}$ yd. = ___ ft.

8. 2 bu. = ___ pk.

9. 9 pk. = ___ bu. ___ pk.

10. 4 hr. 20 min. = ___ min.

11. 2 mi. = ___ rd.

12. $1\frac{1}{2}$ gal. = ___ qt.

13. 3 qt. = ___ pt.

14. 6 pt. = ___ qt.

15. $\frac{1}{2}$ lb. = ___ oz.

16. 48 oz. = ___ lb.

17. 5,000 lb. = ___ tons ___ lb.

18. $\frac{3}{4}$ ton = ___ lb.

19. $\frac{1}{2}$ yd. = ___ in.

20. 9 in. = ___ yd.

21. $\frac{1}{3}$ sq. yd. = ___ sq. ft.

22. $\frac{1}{2}$ sq. yd. = ___ sq. ft.

23. 12 in. = ___ yd.

24. 8 in. = ___ ft.

25. 9 articles = ___ doz.

26. $\frac{1}{4}$ bu. = ___ pk.

27. $\frac{1}{2}$ gal. = ___ qt.

28. 27 in. = ___ yd.

29. 6 qt = ___ pt.

30. $\frac{3}{4}$ ft. = ___ in.

Rating	Correct	Next Step
A	29 or 30	Do Inventory Test 15.
B	27 or 28	Do Inventory Test 15, or do Practice Exercises 14a to 14c.
C	26 or less	Do Practice Exercises 14a to 14c; then do Inventory Test 14 again.

Inventory Test 15 ADDITION OF DENOMINATE NUMBERS

Time—5 minutes.

Add these units of measure. Reduce every sum to its simplest form.

1. 6 ft. 5 in. 3 ft. 2 in.	**4.** 12 lb. 8 oz. 4 lb. 9 oz.	**7.** 18 bu. 5 qt. 1 pt. 13 bu. 7 qt. 1 pt.
2. 6 gal. 2 qt. 1 gal. 3 qt.	**5.** 16 T. 1,400 lb. 7 T. 700 lb.	**8.** 7 hr. 48 min. 9 hr. 32 min.
3. 6 yr. 9 mo. 2 yr. 2 mo. 3 yr. 4 mo.	**6.** 2 qt. 1 pt. 3 qt. 1 pt. 2 qt. 1 pt.	**9.** 6 yd. 2 ft. 9 yd. 2 ft. 4 yd. 1 ft.

Rating	Correct	Next Step
A	8 or 9	Do Inventory Test 16.
B	6 or 7	Do Inventory Test 16, or Practice Exercise 15a.
C	5 or less	Do Practice Exercise 15a; then do Inventory Test 15 again.

EXERCISES

Courtesy Trans-World Airlines

Fig. 1. A Constellation airplane.

1. On Feb. 3, 1946, the *Constellation* flew from Burbank, California, to La Guardia Airport in New York in 7 hr. 27 min. 48 sec., breaking by 4 hr. 27 min. the past record set by the Boeing *Stratoliner*. What was the record of the *Stratoliner*?

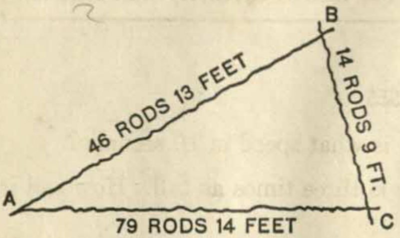
2. Four peach trees yielded 1 bu. 2 pk., 2 bu. 2 pk., 3 bu. 3 pk., and 4 bu. 3 pk., respectively. What amount of peaches was produced?

3. A company marketed 346 tons 756 lb. of lead, 123 tons 263 lb. of copper, 21 tons 202 lb. of zinc, and 28 tons 798 lb. of nickel. Find the total weight marketed.

4. How long should curtain material be cut for a window 5′ 8″ long, if the curtain is to hang 6 in. below the sill and is to have a 3-in. hem at the top and a $2\frac{1}{2}$-in. hem at the bottom? Allow $\frac{1}{2}$ in. for each turn-in.

5. What is the perimeter of a tennis court 25 yd. 2 ft. 8 in. wide and 11 yd. 2 ft. 6 in. long?

6. How many feet is it around a playground that is 24 yd. 2 ft. long and 15 yd. 2 ft. wide?



7. What is the distance from A, through B and C, and back to A in the diagram at the left?

8. On his way to school, Don made four consecutive stops the following distances apart: 15 rd. 9 ft., 17 rd. 6 ft., 14 rd. 8 ft., and 13 rd. 4 ft. How far does Don live from school?

9. May bought three remnants of ribbon of the following lengths: 4 yd. 9 in., 2 yd. 24 in., and 3 yd. 30 in. How much ribbon did she buy?

10. Mr. Mann bought four chickens at the market. Their weights were: 3 lb. 8 oz., 4 lb. 5 oz., 4 lb. 9 oz., and 5 lb. 4 oz. How many pounds of chicken did he buy?

11. Measure the length and width of your teacher's desk. Calculate its perimeter.

12. Measure the length and width of your classroom. What is its perimeter?

13. Measure the length and width of the blackboard. Calculate its perimeter.

Inventory Test 16 — MULTIPLICATION OF DENOMINATE NUMBERS

Time—5 minutes.

Multiply these units of measure and reduce each product to its simplest form:

1. 1 qt. 1 pt.
$\underline{\quad 2 \quad}$

2. 3 bu. 3 pk.
$\underline{\quad 4 \quad}$

3. 5 T. 280 lb.
$\underline{\quad 8 \quad}$

(Inventory Test 16—Continued)

4. 4 ft. 3 in.
3

5. 3 lb. 8 oz.
5

6. 8 hr. 16 min.
6

7. 16 min. 50 sec.
8

8. 2 ft. 7 in.
10

9. 12 gal. 3 qt.
7

Rating	Correct	Next Step
A	8 or 9	Do Inventory Test 17.
B	6 or 7	Do Inventory Test 17, or do Practice Exercise 16a.
C	5 or less	Do Practice Exercise 16a; then do Inventory Test 16 again.

EXERCISES

1. A speed of 8 yd. 2 ft. in 1 second is what speed in 10 seconds?

2. Tom is 1′ 9″ tall and his mother is three times as tall. How tall is Tom's mother?

3. Clara weighs 18 lb. 8 oz. Her father weighs 9 times as much. How much does Clara's father weigh?

4. If it takes 3 hr. 28 min. to make an apron, how long will it take to make 8 such aprons?

5. If a child should drink 1 quart of milk each day, how many gallons should an institution caring for 75 children buy for the children to drink?

6. If 1 gal. of cream contains 1 lb. 10 oz. of fat, how much fat does 20 gallons of such cream contain?

7. Mrs. Barr gave 12 issues of a certain magazine to a school's paper drive. What did the magazines weigh, if one magazine weighed 1 lb. 7 oz.?

8. Ralph worked 5 days after school for 3 hr. 45 min. each day, and on Saturday he worked 8 hr. 30 min. How many hours did he work during the week?

9. Mrs. Starr buys 1 qt. 1 pt. of milk each day. How much does she buy in a 30-day month?

10. How much do 6 boxes of candy weigh, if each box weighs 1 lb. 6 oz.?

11. A beef roast, medium done, requires 20 minutes per pound cooking time. A 5-pound roast requires how long to cook?

12. A turkey requires 22 minutes per pound cooking time. How long will it take to bake an $18\frac{1}{2}$-pound turkey?

Inventory Test 17 SUBTRACTION OF DENOMINATE NUMBERS

Time—5 minutes.

Subtract these units of measure:

1. 14 bu. 3 pk.
10 bu. 1 pk.

2. 7 yd. 2 ft.
3 yd. 1 ft.

3. 8 lb. 15 oz.
3 lb. 9 oz.

4. 9 hr. 30 min.
2 hr. 45 min.

5. 72 gal. 1 qt.
17 gal. 3 qt.

6. 27 ft. 8 in.
18 ft. 9 in.

7. 1951 yr. 11 mo. 1 da.
1776 yr. 7 mo. 4 da.

8. 14 yd. 2 ft. 8 in.
8 yd. 1 ft. 10 in.

9. 15 bu. 1 pk.
8 bu. 3 pk.

Rating	Correct	Next Step
A	8 or 9	Do Inventory Test 18.
B	6 or 7	Do Inventory Test 18, or Practice Exercise 17a.
C	5 or less	Do Practice Exercise 17a; then do Inventory Test 17 again.

EXERCISES

1. A flagpole is 106′ 6″ long. It stands 77′ 5″ above the ground. How far does the pole extend into the ground?

2. A certain plane weighs $25\frac{1}{2}$ tons empty and 37 tons loaded. How many pounds of cargo can it carry?

3. Alex is 4′ 9″ tall and his father is 6′ 1″. Find the difference in their heights.

4. A rough piece of metal weighing 42 lb. 3 oz. is turned in a lathe. What is the loss in weight in finishing if the finished weight is 38 lb. 11 oz.?

5. Sam should weigh 87 lb. 8 oz. for his age and height. How much overweight is he if he weighs 94 lb. 2 oz.?

6. How long has it been since July 4, 1776?

7. Calculate your exact age for today.

8. How long is it since November 11, 1918?

9. What length must be cut from a 6-ft. board to make a 3′ 1″ shelf?

10. When vacation started, James weighed 114 lb. 12 oz. At the close of vacation he weighed 120 lb. 8 oz. How much did he gain during vacation?

11. During the school year Clarence grew in height from 4′ 10″ to 5′ 3″. What was his gain in height?

12. In a scrap-collection drive, each pupil pledged 10 lb. of scrap. Jack has collected 6 lb. 5 oz. How much more does he need to collect to fill his quota?

13. John Adams was born October 19, 1735, and his son John Quincy Adams was born July 11, 1767. Find the difference in their ages.

14. The Statue of Liberty was unveiled October 28, 1886. How long ago was it unveiled?

Inventory Test 18 DIVISION OF DENOMINATE NUMBERS

Time—5 minutes.

Divide:

1. $2\overline{)6 \text{ bu. } 2 \text{ pk.}}$	**4.** $3\overline{)7 \text{ ft. } 9 \text{ in.}}$	**7.** $6\overline{)8 \text{ T. } 180 \text{ lb.}}$
2. $2\overline{)4 \text{ yd. } 2 \text{ ft.}}$	**5.** $4\overline{)9 \text{ ft. } 5 \text{ in.}}$	**8.** $6\overline{)3 \text{ gal. } 2 \text{ qt.}}$
3. $3\overline{)2 \text{ qt. } 1 \text{ pt.}}$	**6.** $3\overline{)7 \text{ hr. } 14 \text{ min.}}$	**9.** $4\overline{)15 \text{ lb. } 3 \text{ oz.}}$

Rating	Correct	Next Step
A	8 or 9	You have completed all the Inventory Tests.
B	6 or 7	Do Practice Exercise 18a if you wish.
C	5 or less	Do Practice Exercise 18a; then do Inventory Test 18.

EXERCISES

1. Charles Lindbergh, on his famous flight to Paris, flew 3,610 mi. in $33\frac{1}{2}$ hours. What was his average speed per hour? (To three decimal places.)

2. An airplane speed of 285 ft. per second is how many miles per hour?

3. Bob and Jerry shared equally the 5 bu. 3 pk. of apples that they gathered. How much did each receive?

4. Alice, Julia, and Naomi shared alike the 14 gal. and 2 qt. of berries that they had picked. How much did each receive?

5. An 8′ 6″ board is to be divided into 3 equal shelves. How long will each shelf be?

6. Mr. Wilson has 3 tons 720 lb. of scrap paper, which he promises to divide equally between Sara and Ruth. How much will each receive?

7. Clara has 2 hr. 40 min. to divide equally among three studies. How much time will she have for each study?

8. Divide 2 qt. 1 pt. of milk equally among four children.

9. Margaret has 3 yd. 20 in. of ribbon out of which to make two ornaments. How much ribbon does she have for each ornament?

10. Twelve magazines of the same issue weigh 18 lb. 12 oz. What does each magazine weigh?

RATIO AND PROPORTION

45. The meaning of "ratio." In a class of 24 girls and 12 boys, we may compare the number of girls and boys in the class in two ways, either by *subtraction* or by *division.* If we make the comparison by subtracting 12 from 24, we say, "There are 12 more girls than boys in the class." If we make the comparison by dividing 24 by 12, we say, "There are twice as many girls as there are boys in the class." When two quantities are compared by division, the result is called the *ratio* of one quantity to the other quantity.

Ratio means relationship. Hence, the ratio between the number of girls and the number of boys in the class is the ratio of 2 to 1, that is, the number of girls is twice as great as the number of boys.

The ratio of one quantity to a similar quantity is their quotient, found by dividing the first quantity by the second quantity.

A ratio may be a fraction, such as $\frac{3}{4}$; an integer, such as 3; a mixed number, such as $1\frac{1}{3}$; or a per cent, such as 50%.

EXERCISES

1. What is the quotient of $8 ÷ $2? What is the ratio of $8 to $2?

2. 18 is how many times 6? What is the ratio of 18 to 6?

3. 8 = ___ times 4. What is the ratio of 8 to 4?

4. 9 = ___ times 6. What is the ratio of 9 to 6?

5. 6 = ___ times 9. What is the ratio of 6 to 9?

6. 12 = ___ times 8. What is the ratio of 12 to 8?

7. 8 = ___ times 12. What is the ratio of 8 to 12?

8. The ratio of 15 to 5 is the same as ___ to 1.

9. The ratio of 21 to 3 is the same as ___ to 1.

10. The ratio of 10 to 3 is the same as ___ to 1.

11. The ratio of 9 to 4 is the same as ___ to 1.

12. The ratio of 8 to 6 is the same as ___ to 3.

13. The ratio of 12 to 10 is the same as ___ to 5.

14. The ratio of 14 to 20 is the same as ___ to 10.

15. The ratio of 6 to 16 is the same as 3 to ___.

16. The ratio of 3 to 5 is the same as ___ to 100.

17. The ratio of 16 to 40 is the same as 2 to ___.

18. The ratio of 15 to 20 is the same as ___ to 100.

19. The ratio of 8 to 10 is the same as ___ to 100.

20. The ratio of 18 to 24 is the same as 3 to ___.

46. **The writing and use of ratios.** We may express the ratio of one quantity to another in the form of a fraction, such as $\frac{2}{3}$. We may also express the ratio as $2 \div 3$, and $2 : 3$.

$$8 \div 10, \quad \text{or} \quad 8 : 10$$

The ratio $\frac{8}{10}$ should be reduced to $\frac{4}{5}$, $4 \div 5$, or $4 : 5$. That is, 8 is to 10 is the same as 4 is to 5.

Both terms (numbers) of a ratio may be multiplied or divided by the same number without the value of the ratio being changed.

EXERCISES

Reduce these ratios to the simplest terms:

1. $\frac{9}{15}$, $\frac{8}{12}$, $\frac{9}{6}$, $\frac{12}{15}$, $\frac{15}{20}$, $\frac{8}{4}$, $\frac{12}{3}$, $1\frac{1}{2}/3$.

2. $6 \div 2$, $12 \div 18$, $15 \div 12$, $8 \div 20$.

3. $8 : 6$, $4 : 16$, $9 : 12$, $25 : 15$.

4. $\frac{2\frac{1}{2}}{5}$, $\frac{2}{6}$, $\frac{4}{18}$, $\frac{4}{2}$, $\frac{3}{9}$, $\frac{2}{8}$, $\frac{3}{12}$.

5. 6 girls to 24 girls.

6. 8 books to 24 books.

7. Irma has 21 books and Sally has 24 books. What is the ratio of Irma's books to Sally's?

8. Helen won 6 games of tennis and Jean won 9 games. What is the ratio of Helen's winnings to Jean's?

9. *A* and *B* formed a partnership. *A* put in \$1,200 capital; *B* put in \$1,800. What is the ratio of *A*'s part of the capital to *B*'s? Of *A*'s part to the total capital?

10. What is the ratio of 6 hours spent in school to a whole day?

11. The North High School has 810 boys and 1,260 girls enrolled. What is the ratio of the number of boys to the number of girls? Of girls to boys? Of girls to the total enrollment? Of boys to the total enrollment?

12. Tony sells 36 of his 40 magazines. Find the ratio of magazines sold to the number received. Express this ratio as a per cent.

13. Carl earns \$4.50 per week and saves \$3. What is the ratio of money saved to money earned?

$$\frac{3}{4.50} \times \frac{100}{100} = \frac{300}{450}$$

(Get rid of the decimal in the denominator by multiplying both numerator and denominator by the same number, 100 in this case. To do so does not change the value of the fraction or the ratio.)

14. Find the ratio of:

(a) \$2.50 to \$4.50
(b) 1.5 to 6
(c) \$7 to \$7
(d) \$2.75 to \$22
(e) \$1.25 to \$12.50
(f) 18 in. to 4 ft.

Numbers that are to be compared must be expressed in the same *unit*. Hence, the ratio of 40 minutes to 1 hour is not 40 to 1; it is 40 to 60 (since 1 hr. = 60 min.), or 2 to 3.

EXERCISES

A. Express these ratios orally in their simplest forms:

1. The ratio of 1 lb. to 8 oz. is ___ oz. to ___ oz. or ___ : ___.

2. The ratio of 1 gal. to 2 qt. is ___ qt. to ___ qt. or ___ : ___.

3. The ratio of 10 min. to 1 hr. is ___ min. to ___ min. or ___ : ___.

4. The ratio of 2 bu. to 2 pk. is ___ pk. to ___ pk. or ___.

5. The ratio of 1 gal. to 1 pt. is ___ ÷ ___ or ___ : ___.

6. The ratio of 2 ft. to 8 in. is ___ ÷ ___ or ___ : ___.

7. The ratio of 4 yd. to 2 ft. is ___ ÷ ___ or ___ : ___.

8. The ratio of 9 in. to 1 yd. is ___ ÷ ___ or ___ : ___.

9. The ratio of 50¢ to \$3 is ___ ÷ ___ or ___ : ___.

10. The ratio of \$2 to 75¢ is ___ ÷ ___ or ___ : ___.

11. The ratio of $1\frac{1}{2}$ ft. to 8 in. is ___ to ___ or ___ : ___.

12. The ratio of $1\frac{1}{4}$ hr. to 45 min. is ___ to ___ or ___ : ___.

B. Write these ratios in their simplest forms:

13. Fred sold 148 magazines last week and 160 this week. What is the ratio of last week's sales to this week's sales? Of this week's sales to last week's?

14. Clara bought 5 doz. cards and sold 42 cards. What is the ratio of cards purchased to cards sold? Of cards sold to cards purchased?

15. Clark earns \$6 per week and saves \$2.50 of it. Find the ratio of his savings to his earnings.

16. A cake recipe calls for $1\frac{1}{2}$ cups of flour and $\frac{3}{4}$ cups of milk. What is the ratio of flour to milk in the recipe?

17. A recipe for ice cream calls for $\frac{3}{4}$ cup of sugar and 2 cups of cream. What is the ratio of cream to sugar in the recipe?

18. The dimensions of a room are in the ratio of 3 to 4. The room is longer than it is wide, and its width is 12 ft. How long is the room?

19. A carpet is 10 ft. wide. What is its length if the ratio of its width to its length is 2 to 3?

20. A garden is 20 ft. long. Find its width if the ratio of its width to its length is 2 to 5.

21. Measure the length and width of your textbook. Write the ratio of its width to its length.

22. Measure the width and length of your notebook. Write the ratio of its width to its length.

23. What is the ratio of the width to the length of your classroom floor?

24. What is the ratio of the heights of the shortest and tallest pupils in your class?

25. Find the ratio of the hours you sleep to the hours in a day.

26. Measure to the nearest inch the length and width of an American flag. What is the ratio of its length to its width?

27. Measure to the nearest inch the height and width of a window in your classroom. Find the ratio of the width to the height.

47. Proportion. Henry earns \$20 per month and saves \$5. Sam earns \$16 per month and saves \$4. They each save $\frac{1}{4}$ of their earnings, since $\frac{5}{20} = \frac{1}{4}$ and $\frac{4}{16} = \frac{1}{4}$. This shows the two ratios to be equal. That is, $\frac{5}{20} = \frac{4}{16}$.

An expression of equality between two ratios is called a "proportion."

The above proportion may also be written

$$5 : 20 = 4 : 16$$

or

$$5 : 20 :: 4 : 16$$

It is read: 5 is to 20 as 4 is to 16. The four numbers used in a proportion are the *terms* of the proportion.

In the above proportion: 5 is the first term; 20 is the second term; 4 is the third term; 16 is the fourth term.

The second and third terms are called the *means*. The first and last terms are called the *extremes*. In the proportion $5 : 20 = 4 : 16$, the 20 and the 4 are means and the 5 and the 16 are extremes.

In any proportion the product of the means equals the product of the extremes. In the proportion $5 : 20 = 4 : 16$

$5 \times 16 = 80$, product of the extremes.
$20 \times 4 = 80$, product of the means.

Likewise, in the proportion $2 : 3 = 4 : 6$

$2 \times 6 = 12$, product of the extremes.
$3 \times 4 = 12$, product of the means.

If three terms of a proportion are known, the missing term can be found.

Example A

$3 : 4 = 6 : ?$
$4 \times 6 = 24$, product of the means.
$3 \times ? = 24$, product of the extremes.
$24 \div 3 = 8$ (which is the missing term), because the product $\div$ the known factor = the missing factor (the missing term).

Example B

$8 : 12 = ? : 6$
$8 \times 6 = 48$, product of the extremes.
$12 \times ? = 48$, product of the means.
$48 \div 12 = 4$ (the missing term). Why?

EXERCISES

Find the missing term in each proportion:

1. $\frac{7}{8} = \frac{14}{?}$

2. $\frac{15}{24} = \frac{?}{8}$

3. $\frac{5}{9} = \frac{?}{27}$

4. $\frac{?}{7} = \frac{8}{28}$

5. $\frac{6}{?} = \frac{2}{24}$

6. $\frac{6}{14} = \frac{?}{21}$

7. $\frac{36}{90} = \frac{?}{75}$

8. $\frac{?}{15} = \frac{12}{18}$

9. $\frac{18}{?} = \frac{27}{30}$

10. $\frac{6}{10} = \frac{15}{?}$

11. $6 : 9 = 16 : ?$

12. $15 : ? = 20 : 24$

13. $? : 12 = 12 : 18$

14. $25 : 15 = ? : 9$

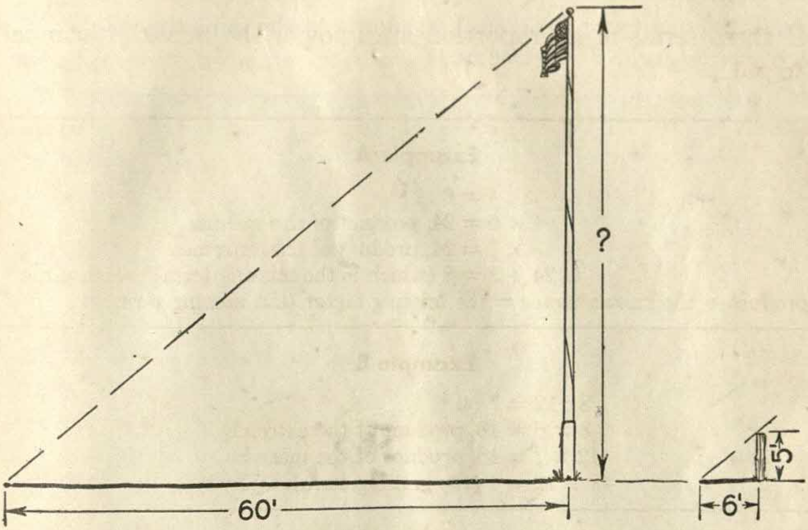
15. $25 : 20 = 5 : ?$

48. Using proportion. Certain kinds of problems can be solved if the numbers in the problems are used as terms in a proportion. In such cases the unknown number (the answer) is one of the four terms

in the proportion. If three of the terms are known, the unknown fourth term can always be found.

Example

It has long been known that, at the same time of day, *the heights of perpendicular objects are in the same ratio as the lengths of their shadows.*



At a certain time of day a 5-ft. fence post casts a shadow 6 ft. long. At the same time, a certain flagpole casts a shadow 60 ft. long. How high is the flagpole?

The ratio of the heights—post to pole—is $\frac{5 \text{ ft.}}{? \text{ ft.}}$, or 5 : ?.

The ratio of the shadow lengths—post to pole—is $\frac{6 \text{ ft.}}{60 \text{ ft.}}$, or 6 : 60.

Let h represent the height of the flagpole. Since the ratios are equal, $5 : h = 6 : 60$. Since the product of the means = the product of the extremes,

$$\begin{aligned} 6 \times h &= 5 \times 60 \\ 6 \times h &= 300 \\ h &= 300 \div 6 \\ h &= 50 \end{aligned}$$

Hence, the height of the flagpole is 50 ft.

Check:

$$\begin{aligned} \tfrac{5}{50} &= \tfrac{6}{60} \\ 5 \times 60 &= 6 \times 50 \\ 300 &= 300 \end{aligned}$$

EXERCISES

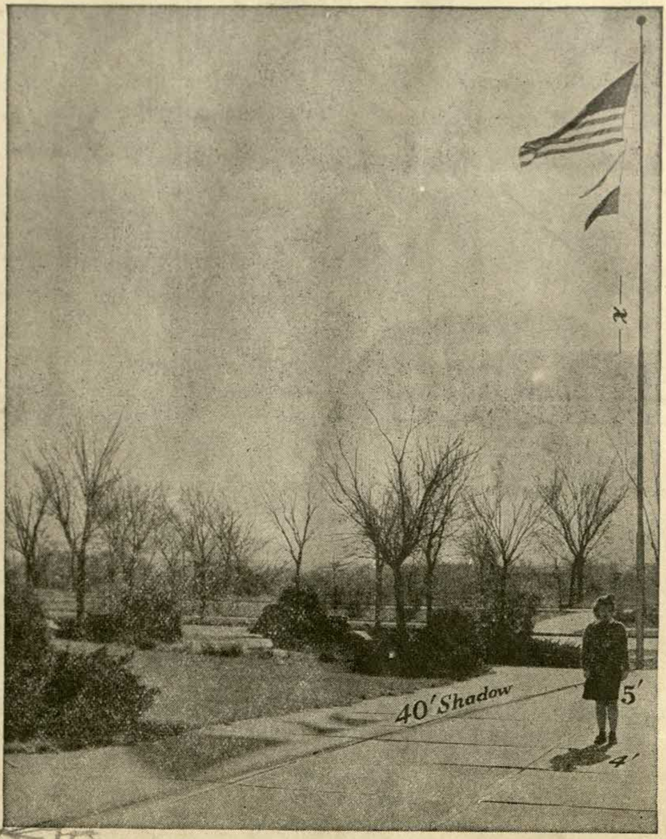
Solve by proportion:

1. A post 4 ft. high casts a shadow 6 ft. long at the same time that a pole casts a shadow 18 ft. long. Form a proportion and find the height of the pole.

2. A 5-ft. boy casts an 8-ft. shadow at the same time that a tree casts a shadow 48 ft. long. How high is the tree?

3. How long is the shadow of a 32-ft. telephone pole, when the shadow of a 6-ft. man is 9 ft. long?

4. A 100-foot tree casts a 30-ft. shadow. At the same time of day, how long a shadow will a 75-ft. tree cast?



HOW HIGH IS THE FLAGPOLE?

5. Jean is 5 ft. tall and at a certain time of day casts a shadow 4 ft. long. At the same time a flagpole casts a shadow 40 feet long. How high is the flagpole?

6. A yardstick casts a shadow 2 ft. long. At the same time a monument casts a shadow 80 ft. long. How high is the monument?

7. How high is an electric light pole that casts a shadow 10 ft. long when a 6-ft. post casts a shadow 4 ft. long? Sketch the triangles and mark the given parts.

8. A telephone pole casts a shadow 20 ft. long when a 3-ft. stick casts a shadow 2 ft. long. What is the height of the pole?

9. If a flagstaff 60 ft. high casts a shadow of 30 ft., how long a shadow will a 40-ft. tree cast at the same time?

10. When a tower 75 ft. high casts a shadow of 60 ft., how tall is a boy who casts a shadow of 4 ft.?

11. Jerry knows that he is 5′ 6″ tall. At the time of day that he casts a 3-ft. shadow, Larry casts a 2-ft. shadow. How tall is Larry?

12. Bert is 5′ 4″ tall and at a certain time of day casts a shadow 2 ft. long. How high is a building which casts a 63′ 4″ shadow at the same time?

13. If 2 doz. oranges cost 85¢, what will 9 doz. cost?

$$\frac{\text{A small amount}}{\text{A large amount}} = \frac{\text{cost of small amount}}{\text{cost of large amount}}$$

$$\frac{2}{9} = \frac{85¢}{?}$$

14. If 6 lb. of meat costs \$8.40, what will 20 lbs. cost at the same rate?

15. A grocer paid \$4.60 for 12 doz. lemons. How much would 15 doz. lemons cost him?

16. If 6 gal. of gasoline cost \$1.56, how many gallons at the same rate can be bought for \$2.21?

17. If 840 bu. of potatoes can be raised on 6 acres, how many acres would be required to produce at the same rate 2.100 bu. of potatoes?

18. In a class, the ratio of the number of boys to girls is 6 to 8. If there are 16 girls, how many boys are in the class?

19. An ordinary tree, about the size of the tree in the picture, gives off 5 barrels of water per day (24 hours). How many barrels of water would it give off in the 7 hours that you are in school? In the 9 hours while you sleep?

20. Walter's car will run 72 mi. on 5 gal. of gasoline. How many gallons will he need to travel 180 mi.?

21. Mr. Clark divided some money between John and Mary in the ratio of 4 to 5. If John received \$128, how much did Mary receive?

22. If a plane travels 1,270 mi. in 10 hours, how long will it take the plane to go 3,175 mi. at the same rate of speed?

23. Two partners, X and Y, share profits in the same ratio as their investments. X invested \$2,500 and Y \$3,500. How much profit will X receive when Y receives \$210?

24. If a \$70 commission is paid on a sale of \$2,100, what commission would be paid on a sale of \$1,500?

25. It required 25 days to dig a trench $1\frac{1}{2}$ mi. long. At the same rate, how long would it take to dig a trench 4 mi. long?

26. Taxes of \$45 were paid a certain year on property assessed at \$2,000. At that rate, find the amount of taxes on property assessed at \$5,000.

27. The ratio of Roy's winnings to Joseph's winnings in a series of games was 8 : 9. Roy won 32 games. How many games did Joseph win?

28. If 6 castings cost \$14.40, what will 5 castings cost?

29. If 5 tons of coal cost \$41.50, what will $\frac{1}{2}$ ton cost?

30. A map that is 8 in. wide and 15 in. long is enlarged so that it is now 20 in. wide. What is the new length?

31. If 15 boys can pick 105 qt. of berries in an hour, how many quarts can 25 boys pick in one hour?

32. A boy is paid 75¢ for 2 hours' work. How much will he be paid for 9 hours?

33. (OPTIONAL) At \$1.70 for 3 sq. ft., what will be the cost of a concrete walk that is $2\frac{1}{2}$ ft. wide and 60 ft. long?

34. (OPTIONAL) If 1 gal. of paint will cover 400 sq. ft., how many gallons will be needed to paint the walls of a hall that is 40 ft. long, 6 ft. wide, and 10 ft. high?

PROCEDURE FOR ACHIEVEMENT TESTS

We have now come to the first of a series of four *Achievement Tests*. The other three tests are at the close of remaining chapters.

After each of these Achievement Tests, record your score on one of the achievement charts found in the back of the book. Record your six separate scores as follows: (a) in the column headed "Add" place a dot on the printed line at the number which is the same as your score in addition; (b) in the same way mark your subtraction score in the column headed "Sub"; (c) likewise record your multiplication, division, percentage, and total scores; (d) with a red pencil* draw a line connecting the six dots. This line graph is a picture of your achievement in the fundamentals of arithmetic at this time.

*Draw the graph lines for the four Achievement Tests all on the same chart, but using a different color for each one.

To help you to improve your score on each of the succeeding tests, *Reviews of Fundamentals* follow the Achievement Tests.

Record the date on which each Achievement Test is taken at the extreme right of the card.

ACHIEVEMENT TEST I

Addition—4 minutes

1. $\begin{array}{r} 8 \\ 9 \\ 6 \\ 5 \\ 4 \\ \underline{7} \end{array}$

2. $\begin{array}{r} 36 \\ 27 \\ 83 \\ \underline{78} \end{array}$

3. $\begin{array}{r} 585 \\ 634 \\ 127 \\ 986 \\ 469 \\ \underline{375} \end{array}$

4. $\begin{array}{r} 18.68 \\ 5.97 \\ 35.06 \\ \underline{7.40} \end{array}$

5. $\begin{array}{r} \frac{1}{10} \\ \underline{\frac{4}{5}} \end{array}$

6. $4.51 + .182 + 2.8$

7. $\frac{1}{3} + \frac{3}{4} + \frac{1}{2}$

8. $\begin{array}{r} 27\frac{5}{6} \\ \underline{19\frac{2}{3}} \end{array}$

9. $\begin{array}{r} 26\frac{7}{8} \\ \underline{5\frac{3}{4}} \end{array}$

10. $\begin{array}{r} \text{8 ft. 9 in.} \\ \underline{\text{7 ft. 8 in.}} \end{array}$

Subtraction—4 minutes

11. $\begin{array}{r} 202 \\ \underline{84} \end{array}$

12. $\begin{array}{r} 5060 \\ \underline{189} \end{array}$

13. $\begin{array}{r} 814.06 \\ \underline{312.18} \end{array}$

14. $7.64 − $.89

15. $8 − 46¢

16. $12½ − $1.25

17. $\frac{9}{10} - \frac{3}{5}$

18. $\begin{array}{r} 48\frac{3}{4} \\ \underline{26\frac{1}{3}} \end{array}$

19. $\begin{array}{r} 76\frac{1}{4} \\ \underline{18\frac{1}{2}} \end{array}$

20. $\begin{array}{r} \text{42 min. 30 sec.} \\ \underline{\text{18 min. 45 sec.}} \end{array}$

Multiplication—4 minutes

21. $\begin{array}{r} 64 \\ \underline{38} \end{array}$

22. $\begin{array}{r} 206 \\ \underline{307} \end{array}$

23. $\begin{array}{r} 180 \\ \underline{70} \end{array}$

24. $\begin{array}{r} 12.6 \\ \underline{.03} \end{array}$

25. $\frac{1}{2} \times \frac{1}{3}$

26. $2\frac{1}{9} \times 63$

27. $\frac{1}{3} \times 18\frac{3}{4}$

28. $\frac{2}{3} \times 90$

29. $\begin{array}{r} 48\frac{1}{2} \\ \underline{28\frac{3}{4}} \end{array}$

30. $\begin{array}{r} \text{2 gal. 2 qt.} \\ \underline{3} \end{array}$

Division—4 minutes

31. $6\overline{)188}$

32. $.18\overline{)4.32}$

33. $28.4 \div 10$

34. $3.5\overline{)21.7}$

35. $\begin{array}{r} 182 \\ 4.6\overline{)83.72} \end{array}$

36. $\begin{array}{r} 182 \\ .46\overline{).8372} \end{array}$

37. $\frac{2}{3} \div \frac{5}{6}$

38. $\frac{1}{2} \div 7$

39. $37\frac{1}{2} \div 6\frac{1}{4}$

40. $\frac{1}{2}$ of 3 qt. 1 pt.

(Achievement Test 1—Continued)

Per Cent—4 minutes

Write as common fractions:

41. 18% **42.** $33\frac{1}{3}\%$

Write as decimal fractions:

43. 83% **44.** $2\frac{1}{2}\%$

Write as per cents:

45. $\frac{4}{15}$ **46.** 3.5

Arrange in order of size, largest first:

47. 37%, $\frac{38}{100}$, .037, 3.6.

48. 80% of 72 = ?

49. 4% of ? = 12

50. ?% of 90 = 36

Record your scores on the achievement chart.

Your graph line will show you in which of the fundamentals you need to improve.

Review of the Fundamentals of Arithmetic—1

Addition and Subtraction

Add and prove:

1.	2.	3.	4.	5.	6.	7.
7	69	81	585	$6367	7.85	475.09
3	58	71	634	1468	19.26	18.65
5	47	45	127	5703	56.79	245.97
6	93	12	986	472	.66	18.59
1	25	23	469	9476	7.4	5.67
2	36	56	375			

8. $518.86
94.78
467.76
8.78

9. 68.5 + 18.097 + 65.47 + .009 + 1.07

10. 5.8 + .87 + 9.583 + 48.5 + 16 + 1.893

Add:

11. $\frac{3}{8} + \frac{2}{3}$

12. $\frac{5}{12} + \frac{2}{3}$

13.	14.	15.	16.	17.
$\frac{7}{10}$	$\frac{7}{8}$	$162\frac{7}{12}$	$16\frac{1}{2}$	$16\frac{3}{4}$
$\frac{3}{4}$	$\frac{5}{12}$	$237\frac{5}{8}$	$37\frac{2}{3}$	$45\frac{3}{8}$
$\frac{2}{5}$	$\frac{3}{4}$		$28\frac{1}{9}$	$37\frac{3}{16}$

18. $67\frac{4}{5} + 28\frac{2}{3} + 67\frac{3}{10}$

19. $178\frac{1}{4} + 209\frac{7}{8} + 64\frac{1}{2}$

20. 8 hr. 10 min. 36 sec.
5 hr. 15 min. 24 sec.

Subtract and prove:

21. 2438 1349	**22.** 54798 48809	**23.** 163809 74386	**24.** 40079 37939	**25.** 690078 508798

26. 14.7 7.3	**27.** 9.071 5.6	**28.** .07 .068 .010	**29.** 46.97 − 26.08 = **30.** 8 − 5.667 =

Subtract:

31. $\frac{2}{3} - \frac{1}{2}$ **32.** $\frac{5}{8} - \frac{1}{2}$	**33.** $17\frac{1}{7}$ 15	**34.** 38 $27\frac{1}{4}$	**35.** $138\frac{5}{8}$ $93\frac{7}{16}$	**36.** $64\frac{1}{4}$ $44\frac{1}{2}$	**37.** $48\frac{5}{8}$ $26\frac{3}{4}$

38. $381\frac{3}{4} - 79.125$	**39.** 72 gal. 1 qt. 17 gal. 3 qt.	**40.** 21 T. 800 lb. 16 T. 1000 lb.

Review of the Fundamentals of Arithmetic—2

Multiplication and Division

Multiply and prove:

1. 485 7	**2.** 236 49	**3.** 5500 97	**4.** 406 504	**5.** 1607 49
6. .508 63	**7.** 5.72 .85	**8.** 6.67 .038	**9.** .0859 .206	**10.** 197.2 5.09

Multiply:

11. 84.65 × 10 **13.** 6.99 × 100 **15.** 6.72 × 1000

12. 10 × .5675 **14.** 100 × .423 **16.** 10 × 6.7986

17. $5 \times 1\frac{1}{2}$ **18.** $\frac{5}{8} \times \frac{4}{10} \times \frac{1}{2}$ **19.** $9\frac{1}{3} \times 9$ **20.** $2\frac{1}{4} \times 3\frac{2}{3}$

21. $42 \times 76\frac{3}{7}$ **22.** $14\frac{1}{3} \times 8\frac{2}{5}$ **23.** $44\frac{2}{5} \times 64\frac{1}{2}$ **24.** $18\frac{2}{3} \times 9\frac{1}{4}$

25. 16 yd. 2 ft. 7 in. 10	**26.** 12 gal. 3 qt. 1 pt. 12

Divide and prove:

27. 2765 ÷ 48	**31.** 192 ÷ .04	**35.** 57.68 ÷ 10
28. 7283 ÷ 76	**32.** 2.1432 ÷ .38	**36.** 54.43 ÷ 100
29. 52,272 ÷ 54	**33.** 424.08 ÷ 9.3	**37.** 711.27 ÷ 10
30. 45,724 ÷ 67	**34.** 40.608 ÷ 9.6	**38.** 45 ÷ 1000

Divide:

39. $\frac{1}{2} \div \frac{3}{4}$

40. $\frac{5}{16} \div 2$

41. $25 \div 4\frac{1}{6}$

42. $10 \div 3\frac{1}{3}$

43. $1\frac{1}{8} \div \frac{3}{8}$

44. $\frac{7}{12} \div \frac{7}{18}$

45. $15\frac{5}{8} \div 6\frac{1}{4}$

46. $18\frac{2}{5} \div 3\frac{3}{5}$

47. $6\frac{2}{9} \div 19\frac{1}{5}$

48. $7\frac{1}{9} \div 26\frac{2}{3}$

49. 4)15 lb. 3 oz.

50. 3)8 ft. 7 in.

Review of the Fundamentals of Arithmetic—3

Percentage

Express each of the following numbers as (1) a common fraction, (2) a decimal, (3) a percentage:

1. $\frac{4}{5}$

2. $\frac{2}{25}$

3. .3

4. $3\frac{1}{2}$

5. 75%

6. $33\frac{1}{3}\%$

7. $37\frac{1}{2}\%$

8. .25

9. $\frac{1}{2}\%$

10. $\frac{7}{50}$

11. 125%

12. .12

13. $166\frac{2}{3}\%$

14. .7

15. $.12\frac{1}{2}$

16. $\frac{1}{400}$

17. .136

18. $\frac{1}{7}$

19. $2\frac{3}{4}$

20. 72%

Solve the following:

21. 75% of 852

22. $66\frac{2}{3}\%$ of .434

23. 175% of 24,600

24. $6\frac{2}{3}\%$ of \$45.90

25. 38% of 756.74

26. 8% of \$47.50

27. $5\frac{1}{2}\%$ of \$250.87

28. 9% of 67,475

29. 14% of 555.64

30. 26% of .4206

31. 2% of \$1,256.85

32. $1\frac{3}{4}\%$ of 5,268

33. 12% of \$88.50

34. $16\frac{2}{3}\%$ of 4,728.6

35. $3\frac{1}{3}\%$ of 93,684

36. $\frac{1}{2}\%$ of \$687

37. $8\frac{1}{2}\%$ of 268

38. $12\frac{1}{2}\%$ of $12\frac{1}{2}$

Fill in the following:

39. ?% of 75 = 9

40. 5% of ? = 84

41. ?% of 12 = 42

42. 8% of ? = 72

43. 3% of ? = 78

44. ?% of 15 = 80

45. 24% of ? = 16

46. 1% of ? = 54

47. $2\frac{1}{2}\%$ of ? = 75

48. ?% of 1 = $\frac{1}{2}$

49. ?% of 35 = 140

50. 25% of ? = 100

Solve the following:

51. 4 is what per cent of 12?

52. 10 is what per cent of 40?

53. What per cent of 16 is 2?

54. 9 is what per cent of 18?

55. 30 is what per cent of 80?

56. What per cent of 25 is 7?

57. 6 is what per cent of 9?

58. What per cent of 10 is 8?

CHAPTER VI

DIRECT MEASUREMENT OF LINES AND ANGLES

DIRECT MEASUREMENT OF LINES

VOCABULARY

1. approximate	4. thickness	7. subdivision
2. linear foot	5. dimension	8. measurement
3. width	6. length	9. accuracy

49. The origin of our measurements. Man has always found it necessary to measure the world about him. The Egyptians undoubtedly made much use of measurement at the time of the building of the pyramids, about 3000 B.C. As early as 1400 B.C., some type of measure was used to relocate land boundaries after the annual overflow of the River Nile. The British Museum contains a work on methods of measurement that was written by an Egyptian about 1550 B.C.*

Early measurements were not made by present-day methods. Early man knew no standards of measurement. He often measured distances by comparing the distance to be measured with the length of some part of his body. Fingernails, fingers, feet, and arms all became units of length.

The first known measurement, the *cubit*, was the length of a forearm from point of elbow to end of the middle finger. As you might expect, there were various cubits differing somewhat in length, since men's forearms and middle fingers are not all the same length. Two outstanding cubits were the Royal Egyptian cubit, averaging about 20.62 inches, and the Olympic cubit, averaging 18.24 inches. The Olympic cubit was subdivided into 2 spans (length between the tips of the thumb and the little finger of the outstretched hand

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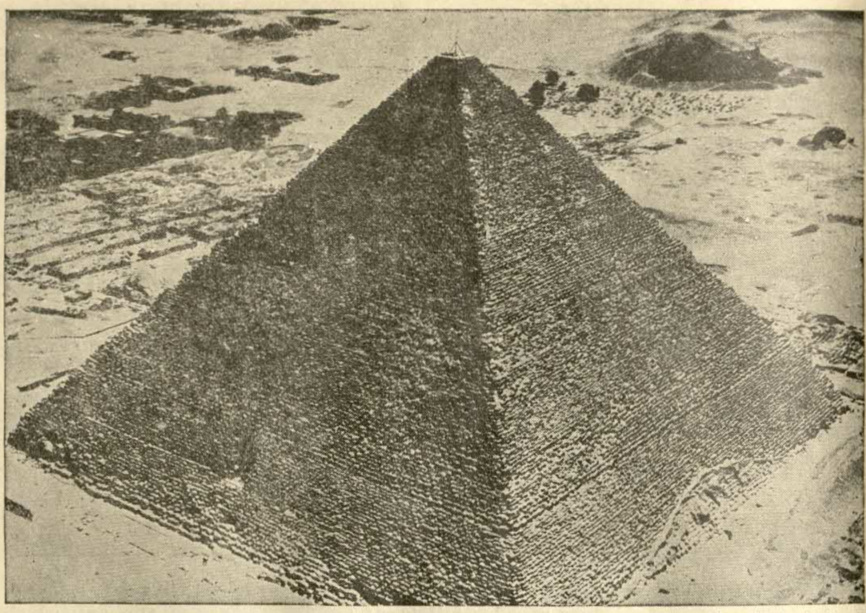
*The history of many of the early measurements is told in *The Story of Weights and Measures*, prepared under the auspices of the Committee on Materials of Instruction of the American Council on Education, Washington, D. C.



From Ewing Galloway

—about 9 inches each), 6 palms (3 inches each), or 24 *digits* ($\frac{3}{4}$ inch each). Later, two-thirds of an Olympic cubit was called a "foot." The foot was subdivided by the Greeks into 12 thumb-breadths. All measurements of the Egyptian pyramids (above) are in multiples or fractions of cubits.

In 1324 Edward II decreed that three barleycorns taken from the center of the ear of barley, placed end to end, should equal an *inch*; a foot ranged from $9\frac{3}{4}$ inches to 19 inches in length. England's King Henry I decreed that the distance from the point of his nose to the end of his thumb on his outstretched arm was the lawful *yard*.

To standardize the yard, a bronze bar one yard in length was kept as the Standard of Reference in the King's Exchequer in England.

In 1855 two copies of the standard yard were presented to the United States. Later these copies were accepted by the Office of Weights and Measures as the standards of the United States.

In 1500 the English mile was established as 8 furlongs—a furlong being 40 rods.

50. Measurement. The purpose of measurement is to determine size. The degree of accuracy required depends upon the use to which the measurement is to be applied. There are measuring tools capable of measuring to any degree of accuracy needed. The simpler

—DIGIT—

Equal to the breadth of a finger. From .72 to .75 inch.

—CUBIT—

About 20 inches. Length from point of elbow to end of the middle finger.

and less accurate measuring tools are the ruler, yardstick, tapeline, and protractor.

LARRY MORRES,3

51. The ruler. The ruler is the simplest and most common measuring instrument. A steel ruler is called a *scale*. Rulers and scales make straight-line measurements and, when carefully used, will measure to within $\frac{1}{100}$ of an inch. They may be subdivided into eighths, sixteenths, thirty-seconds, and sixty-fourths of an inch; they may be divided also into tenths and hundredths, or into units of the metric system.

It is important in the use of the ruler (or scale) to follow these suggestions:

(a) Use only rulers and scales with square, clean-cut ends. A ruler or scale that is battered, rounded, or worn leads only to inaccurate measurement.

(b) When measuring rectangular pieces, make sure that the ruler or scale is placed parallel to the length being measured, not at an angle. See figures 1 and 2.

(c) Instead of laying the ruler flat upon the piece being measured, stand it on edge. The flat position leads to inaccuracy, as in this position the markings on the ruler are too far from the object being measured for the dimension to be estimated accurately. See figure 3.

(d) Curved measurements can be made with a flexible ruler.

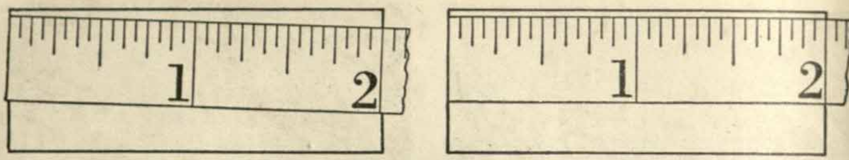


Fig.

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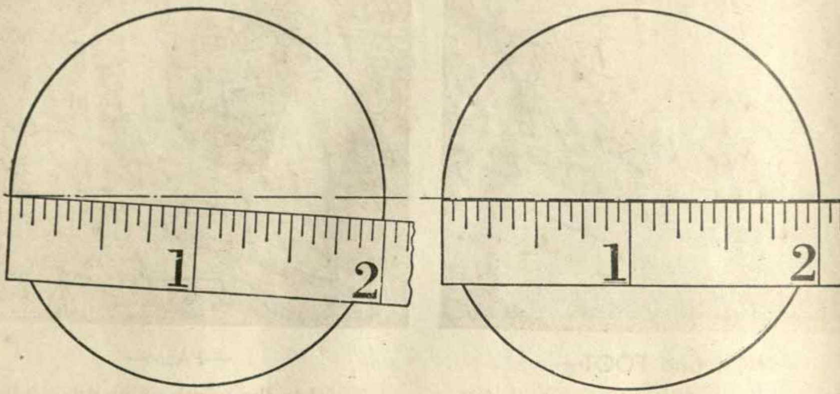

Fig.

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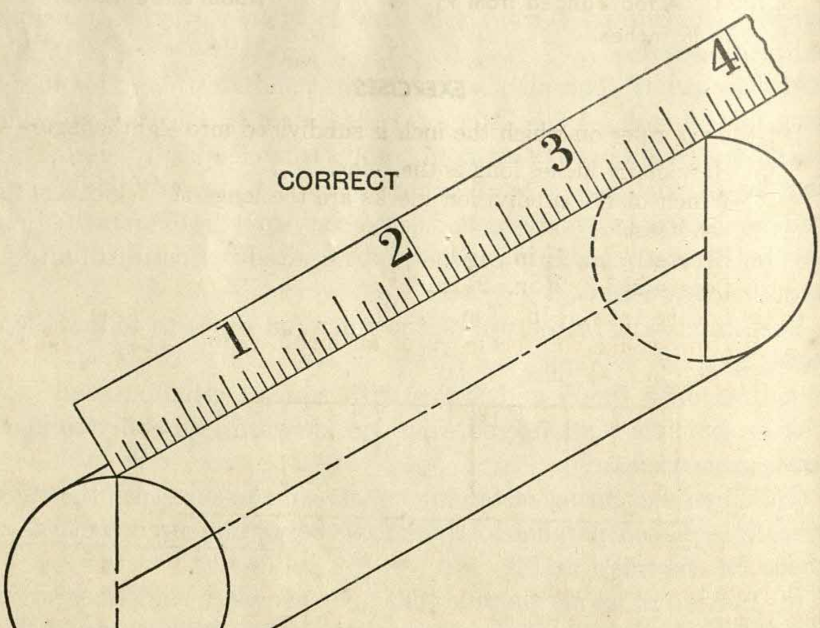
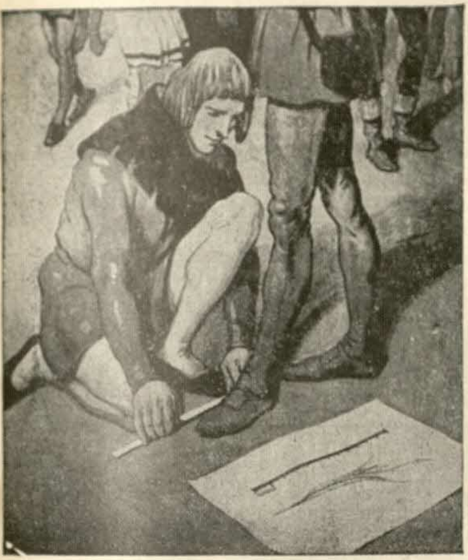


Fig.



—INCH and FOOT—

Three barley corns from the center of the ear placed end to end equalled one inch. (Edward II, 1324.) A foot ranged from $9\frac{3}{4}$ to 19 inches.

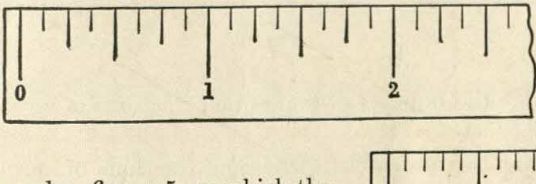
—PALM—

Equal to the width of an open hand at the base of the fingers. About three inches.

EXERCISES

1. Note the ruler on which the inch is subdivided into eighths, figure 4:
   (a) How many inches long is the ruler?
   (b) Which of the subdivision marks are the longest? Which are the shortest?
   (c) Locate $1\frac{1}{2}$ in., $1\frac{1}{4}$ in., $1\frac{3}{4}$ in.
   (d) Locate $2\frac{1}{4}$ in., $2\frac{1}{8}$ in., $2\frac{3}{8}$ in.
   (e) Locate $1\frac{5}{8}$ in., $\frac{7}{8}$ in., $\frac{3}{8}$ in.
   (f) How would you locate $\frac{1}{16}$ of an inch? $\frac{3}{16}$ in.? $\frac{7}{16}$ in.? $2\frac{5}{16}$ in.? $3\frac{9}{16}$ in.? $4\frac{15}{16}$ in.?

Fig. 4



2. Note the ruler, figure 5, on which the inch is subdivided into tenths.
   (a) Which of the subdivision marks are longest? Shortest?

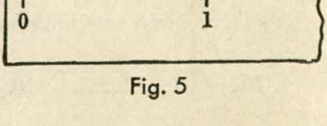
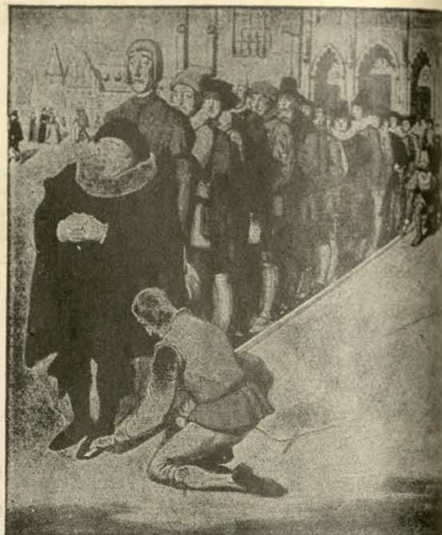


Fig. 5

—YARD—

King Henry I decreed that the distance from the point of his nose to the end of his thumb was the lawful yard.



—ROD—16th CENTURY—

The lawful rod was the length of the left feet of 16 men lined up as they left church on Sunday morning.

(b) Locate .1 in., .3 in., .5 in., .6 in., and .9 in.

(c) Locate 1.8 in., 1.6 in., 1.1 in., and 1.5 in.

3. On the ruler, figure 6:

(a) Determine the number of divisions per inch.

(b) What is the size of the smallest division?

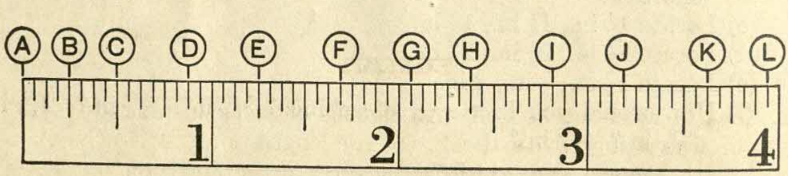


Fig. 6

(c) Determine the number of inches and fractions of an inch from A to each of the other letters.

(d) Determine the number of inches and fractions of an inch from B to each of the other letters in figure 6.

(e) Determine the distance from C to each of the other letters.

4. Measure accurately 10 or more distances with a ruler or scale.

52. How to measure a line with a ruler. Place the zero mark of the ruler directly on one end of the line to be measured. Keep the edge of the ruler parallel with the line. Note the division mark on which the other end of the line falls. How long is each line?

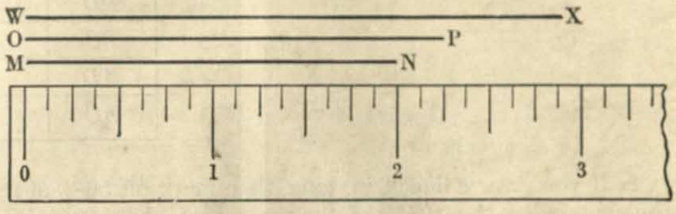


Fig. 7. Measuring with a ruler.

In figure 8, line AB measures 1.7 inches. The point D of the line CD does not fall directly on one of the tenth marks. It falls between the 3.3-in. and 3.4-in. marks, making the line CD greater than 3.3 in. and less than 3.4 in. Since it falls nearer to the 3.3-in. mark, the line is said to be approximately 3.3 in. long, or 3.3 + in. long. If D fell halfway between 3.3 in. and 3.4 in., the line would be 3.35 in. long. To the nearest tenth of an inch, line CD is 3.3 in. long.

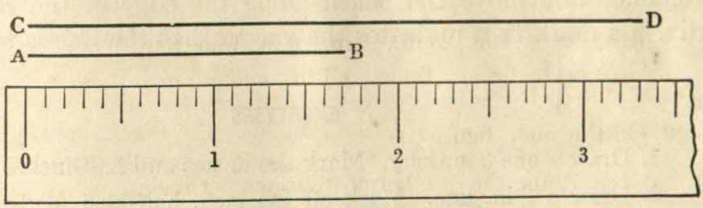


Fig. 8

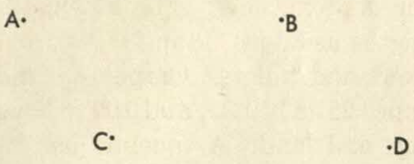
EXERCISES

1. Using a ruler marked off into eighths of an inch, measure: (a) the length of a page in your textbook, (b) the length of a printed line in your book, (c) the length and width of the first 20 pictures in your book to the nearest eighth of an inch.

2. Using a ruler marked off into tenths of an inch, measure the length and width of the second 20 pictures in your book to the nearest tenth of an inch.

3. Draw 10 straight lines. Measure each accurately (a) to the nearest eighth of an inch; (b) to the nearest tenth of an inch.

4. Measure the distances between points A, B, C, and D to the indicated degree of accuracy:



Distance	To Nearest	
	16th	32nd
AB		
BD		
BC		
AD		
AC		

5. If you draw a line 96 in. long, then mark off 50% of it, then $33\frac{1}{3}\%$ of the remainder, then 25% of the remainder, then 50% of the remainder, how long is the last piece?

6. (OPTIONAL) A machinist cut off a steel rod $16\frac{1}{2}$ in. long into 5 consecutive pieces $2\frac{1}{8}$ in. long, $4\frac{15}{16}$ in., $3\frac{1}{2}$ in., and $1\frac{3}{4}$ in., respectively. How long was the last piece, if $\frac{1}{16}$ in. is allowed for each cut?

53. How to draw a line with a ruler. To draw a line, place the zero mark (or the end of the ruler) at the point where you wish one end of the line to fall. Place the pencil point directly over the zero (or end) and move the pencil along the edge of the ruler to the division mark that indicates the length that the line is to be.

EXERCISES

1. Draw a line 3 in. long. Mark the inches and half-inches on the line.

2. Draw a 2-in. line. Mark off the inch, half-inch, and quarter-inch divisions.

3. Draw a line 1 in. long. Mark it off into eighths.

4. Mark off a 1-in. line into sixteenths.

5. Mark off a 1-in. line into tenths.

6. Draw lines of the following lengths: $2\frac{3}{4}$ in., $3\frac{7}{8}$ in., $4\frac{5}{8}$ in., and $3\frac{3}{8}$ in.

7. Draw the following lines: 1.4 in., 2.7 in., 3.8 in., and 4.9 in.

8. Using a straightedge that is not a ruler, draw lines which you estimate to be of the following lengths: $2\frac{1}{2}$ in., $3\frac{1}{4}$ in., $3\frac{7}{8}$ in., and $4\frac{5}{8}$ in. Measure these lines with a ruler to the nearest eighth of an inch. How much of an error was made in each estimate?

9. In the same way draw lines whose lengths you estimate to be 1.5 in., 2.7 in., 3.4 in., and 4.8 in. Measure the lines to the nearest tenth of an inch. Find the amount of error in each estimate.

10. Draw a line $3\frac{5}{16}$ in. long.

11. Draw a line $4\frac{15}{16}$ in. long.

54. Studying the yardstick, tapeline, and steel tape. The yardstick is more convenient than the ruler for measuring lengths of more than a foot, for the yardstick is 36 in. long. The tapeline is made in various lengths. Such lengths as 36 in., 48 in., 60 in., and 72 in. are most useful to dressmakers and tailors. Carpenters and gardeners use tapelines and steel tapes 25 ft., 50 ft., and 100 ft. long to measure the lengths of buildings and land. A tapeline has its divisions marked off on a narrow strip of cloth or steel and is usually attached to and wound around a spring-controlled rod in a case. Surveyors and farmers use a chain tape for measuring land. Each link is exactly one foot in length.

EXERCISES

1. What reasons would explain why a carpenter uses a tapeline rather than a ruler or yardstick in measuring the length of a lot, a house, or a room?

2. Let 6 pupils measure the length of the classroom to the nearest inch from three different points at the same time, 2 pupils using a ruler, 2 using a yardstick, and the other 2 using a tapeline. Note which 2 finish first. Compare results. Account for any difference in results.

3. Estimate the width of the classroom in feet. Measure the width in order to verify your estimate.

4. Estimate and then measure to the nearest inch the lengths of such objects as the blackboard, the bulletin board, a desk, and a bookcase. Find the error in inches in each case.

5. Estimate the height of two members of the class. Verify your estimates by measurement. How much is the error in inches? In per cent?

6. (OPTIONAL) Check measurements for the various events held on the athletic field.

7. (OPTIONAL) Take measurements in the gymnasium.

8. (OPTIONAL) Take accurate measurements of the school grounds so that a scale drawing may be made.

9. (OPTIONAL) Measure the school cafeteria floor for linoleum; the size of windows for screens or curtains.

55. The metric system of measurement. Much of the civilized world uses a simpler method of measuring than the one commonly used in England and the United States. It is the method known as the *metric system.* The basic metric units are the *meter* for length, the *liter* for capacity, and the *gram* for weight. There is a definite

relationship between the meter, the liter, and the gram, as shown in figure 9. One liter of water will fill (at 20° centigrade) 1,000 cubic centimeters and weigh 1,000 grams (1 kilogram).

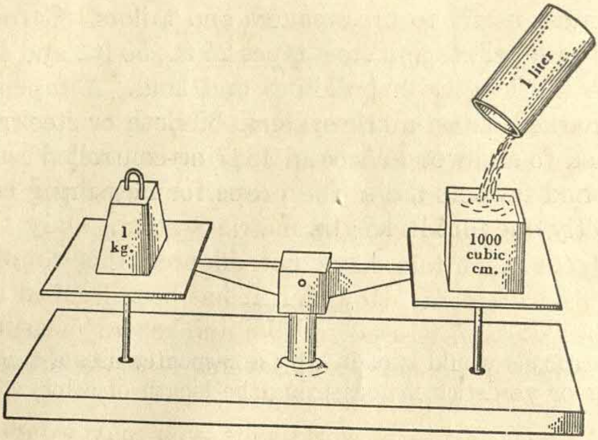


Fig. 9

The unit of length, the meter, is one ten-millionth part of the distance from the equator to either pole. The original meter, a platinum bar made in 1799, is kept in France. Only a slight difference exists between the actual and the intended length.

The meter is divided into 10 equal parts called *decimeters*. Each decimeter is made up of 10 *centimeters*; each centimeter, of 10 *millimeters*. A millimeter, therefore, is 1/1,000 of a meter.

The meter, the liter, and the gram are subdivided decimally, the Latin prefixes *deci*, *centi*, and *milli* being used to indicate the order of decimal divisions of the units, while the Greek prefixes, *deka*, *hecto*, *kilo*, are used to indicate the order of decimal multiplication of the units.

Less than one meter:			More than one meter:		
Latin Prefixes			*Greek Prefixes*		
milli	means	.001	*deka*	means	10
centi	"	.01	*hecto*	"	100
deci	"	.1	*kilo*	"	1000

The table of metric-length measurement then becomes:

1 *milli*meter	=	.001	meter
1 *centi*meter	=	.01	"
1 *deci*meter	=	.1	"
1 meter	=	1	"

1 *deka*meter	=	10 meters
1 *hecto*meter	=	100 "
1 *kilo*meter	=	1000 "

In the same way:

1 *milli*liter	=	.001 liter, and so on.
1 *milli*gram	=	.001 gram, and so on.

56. Uses made of the metric system. The metric system of measurement was adopted in France in 1837 after years of struggle for and against the method. At the present time 82 per cent of the population of the world use the metric system. Only the United States and Great Britain have not adopted this simple decimal method of measurement. However, it has been legal in the United States since 1866 and is used to take and record measurements in medicine, radio, armaments, and several other fields where great accuracy of measurement is necessary.

Since its use is so nearly world-wide, you may wonder why the United States has not discarded the English system and adopted the metric system. In the main, the objections seem to rest upon the confusion that would naturally exist for a short time after the change and upon the cost to manufacturing concerns resulting from such a change.

LENGTH

10 millimeters (mm.)	= 1 centimeter (cm.)
10 centimeters	= 1 decimeter (dm.)
10 decimeters	= 1 meter (m.) = 39.37 in.
10 meters	= 1 dekameter (dkm.)
10 dekameters	= 1 hectometer (hm.)
10 hectometers	= 1 kilometer (km.)

VOLUME

10 milliliters (ml.)	= 1 centiliter (cl.)
10 centiliters	= 1 deciliter (dl.)
10 deciliters	= 1 liter (l.) = 1.0567 liquid qt.
10 liters	= 1 dekaliter (dkl.)
10 dekaliters	= 1 hectoliter (hl.)
10 hectoliters	= 1 kiloliter (kl.)

WEIGHT

10 milligrams (mg.)	= 1 centigram (cg.)
10 centigrams	= 1 decigram (dg.)
10 decigrams	= 1 gram (g.)
10 grams	= 1 dekagram (dkg.)
10 dekagrams	= 1 hectogram (hg.)
10 hectograms	= 1 kilogram (kg.) = 2.2046 lb.

From the table, you can see that the following rules may be used to change from one metric unit to another:

(1) To change a metric unit to the next larger metric unit, divide by 10.

Examples

Length: 39 millimeters = 3.9 centimeters.
Weight: 39 milligrams = 3.9 centigrams.
Volume: 39 milliliters = 3.9 centiliters.

(2) To change a metric unit to the next smaller metric unit, multiply by 10.

Examples

Length: 39 centimeters = 390 millimeters.
Weight: 39 centigrams = 390 milligrams.
Volume: 39 centiliters = 390 milliliters.

EXERCISES

1. Estimate the height in meters of some tall person in your class. Do the same for a short person. Now measure the height of each in meters.

2. Estimate the length of the classroom in meters. Measure it in meters.

3. Estimate the length and width of your textbook in centimeters. Measure it.

4. What part of a dollar is a cent? What part of a meter is a centimeter?

5. What part of a dollar is a mill? What part of a meter is a millimeter?

6. How many grams is 16 kg.?

7. Express 4 cm. as millimeters. Four millimeters are how many centimeters?

8. Change 467 mm. (a) to centimeters, (b) to decimeters, (c) to meters, (d) to dekameters, (e) to hectometers, (f) to kilometers.

9. Change 9.75 km. (a) to hectometers, (b) to dekameters, (c) to meters, (d) to decimeters, (e) to centimeters, (f) to millimeters.

10. How many pieces 1 mm. long can be cut from a bar of metal 5 m. long?

11. A distance of 8.5 m. is how many millimeters? How many kilometers?

To express in inches a measurement given in millimeters, multiply the number of millimeters by .03937. To change meters to inches, multiply by 39.37. To change meters to feet, multiply by 3.2808. To change meters to yards, multiply by 1.0936.

Example

Change 10 meters to inches:
1 meter = 39.37 inches
10 meters = 10 × 39.37 inches = 393.7 inches.

EXERCISES

1. Change 68 mm. (a) to inches; (b) to feet.
2. One meter is about what fraction of a yard?
3. A distance of 17.8 meters is (a) how many feet? (b) how many inches?
4. One-half meter is how many inches?
5. A thousand meters is (a) how many feet? (b) how many yards?
6. One millimeter is about what fraction (a) of an inch? (b) of a foot?
7. A 75-millimeter shell is how many inches thick?

To change inches to millimeters, multiply by 25.4.
To change feet to meters, multiply by .3048.
To change yards to meters, multiply by .9144.

EXERCISES

1. Change 15 in. (a) to millimeters, (b) to centimeters, (c) to meters.
2. The 100-yard dash is how many meters?
3. A building 40 ft. long and 35 ft. wide is how long and wide in meters?
4. One mile is how many meters long?

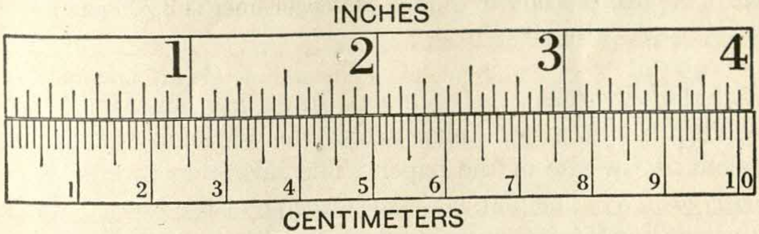


Fig. 10

5. In figure 10, note the length of 1 cm. How does it compare with the thickness of your little finger? Give an example of some article that is approximately 1 cm. in length or width. How does it compare with 1 in.?

6. Locate the 10-cm. mark. What are 10 cm. called? Name some article that is approximately 1 dm. long.

7. About how many centimeters are there in an inch? In 4 in.?

8. What is each of the 10 divisions of the centimeter called? Which of our coins has a thickness of about 1 mm.?

9. Examine a meter stick. (If none is available, draw one on the blackboard.) How many inches longer is the meter stick than the yard-stick?

10. Name an object about a meter in length.

11. Change 10 in. to millimeters; 10 ft. to meters; 10 yd. to meters.

12. A 50-foot tree is how many meters high?

13. A record of 7.445 meters was made one year in the broad jump at the Olympic Games. How many yards does the record represent?

14. From Paris to Calais is a distance of 295 kilometers. Express the distance in miles (1 mile = 1.6094 km.).

15. An aviator flew 160 km. per hour. How many miles is that per hour?

16. The height of the Eiffel Tower in Paris is about 300 m. How many feet high is it?

17. The earth is approximately 25,000 miles in circumference. Express this distance in kilometers.

18. If the distance from Denver to New York is 1620 miles, how many kilometers is it?

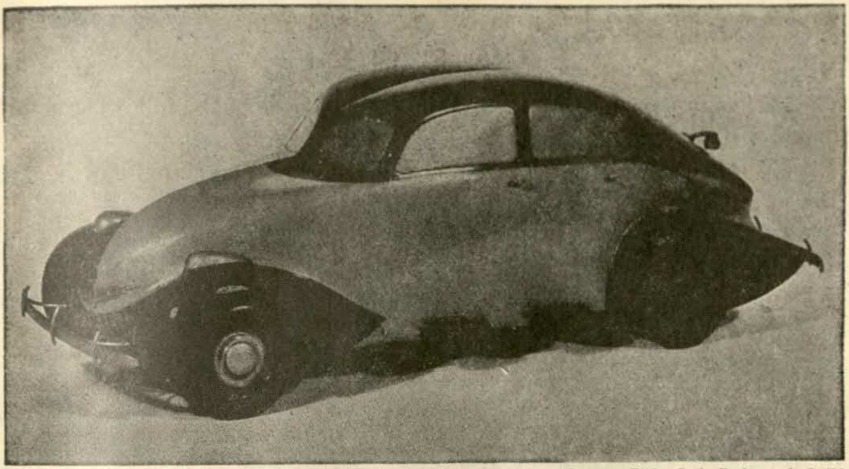
19. The Woolworth Building in New York is 792 ft. high. Express this height in meters.

20. A French schoolboy is 1.6 m. in height. Express his height in feet.

21. A kilogram is approximately 2.2 lb. How many pounds does a bomb load of 1,250 kg. weigh? A load of 1,750 kg.?

22. If the fuel capacity of a large passenger liner is 3,200 gallons, what is its fuel capacity in liters?

57. Scale drawing. An architect draws his house plan greatly reduced in size from the house itself. He "shrinks" the size of the house to fit the size of the paper. The shrinking process is called *drawing to scale.* The plan is a *scale drawing* of the house. Each line in a scale drawing is some definite fractional part of the length of the line that it represents. Such a picture reproduces the object on a smaller *scale.* Each line in the scale drawing may be one-half of the length of the actual line, one-eighth of it, one-hundredth, one-thousandth, or some other definite ratio. All maps are scale drawings. An architect draws all his house plans to scale. He may let 1 in. represent 8 ft., 10 ft., 12 ft, or some other number of feet suitable to the dimensions which he is to represent. If a garage 12 ft. by 18 ft. is to be erected, several scales might conveniently be used;



Courtesy Fisher Body Craftsmen's Guild

A 30-INCH SCALE MODEL OF AN AUTOMOBILE designed and built by a 14-year-old boy. Every dimension on this model is one-seventh the dimension of the full-size automobile.

for instance, 1 in. might represent 12 ft., making the scale drawing 1 in. by $1\frac{1}{2}$ in. Again, $\frac{1}{3}$ in. might represent 1 ft., making the scale drawing 4 in. by 6 in. If $\frac{1}{6}$ in. represents 1 ft., what will be the dimensions of the rectangle drawn to scale?

On a scale drawing the scale should always be given. Here are some commonly used scales:

Scale	*Ratio*
6″ to 1′	$\frac{1}{2}$ full size
3″ to 1′	$\frac{1}{4}$ full size
$1\frac{1}{2}$″ to 1′	$\frac{1}{8}$ full size
1″ to 1′	$\frac{1}{12}$ full size
$\frac{3}{4}$″ to 1′	$\frac{1}{16}$ full size

Example

If the scale on a drawing is 1″ to 1′, how long would 96 in. on the object be on the scale drawing?

A scale of 1″ to 1′ (1″ to 12″) is a ratio of 1 in. to 12 in. or $\frac{1}{12}$. $\frac{1}{\cancel{12}_{1}} \times \frac{\cancel{96''}^{8''}}{1} = 8''$.

Therefore, 96 in. on the object becomes 8 in. on the drawing, since all lengths on the drawing are given in a ratio of $\frac{1}{12}$. Any of the other scale lengths can be found in this way.

EXERCISES

A. Oral

1. Using the scale 1″ to 1′, find the scale lengths of the following actual lengths: 3′, 8′, 12′, 9′, $6\frac{1}{2}$′, $5\frac{3}{4}$′.

2. Let $\frac{1}{4}$″ represent 1′. Find the scale lengths when actual lengths are: 12′, 24′, 18′, 30′, 8′, 32′, 16′, 10′.

3. If a scale is $\frac{1}{2}$″ to 1′ and a scale drawing is 4″ long, what is the actual length of the drawing?

4. Using the same scale. find the actual lengths of the following scale lengths: 10″, 7″, 8″, 9″, 12″, 15″, 6″, 14″.

B. Written

5. When the scale $\frac{1}{8}$″ to 1′ is used, how long are the scale drawings for the following actual lengths: 1′, 8′, 12′, 16′ 6″, 17′ 4″?

6. Using the scale $\frac{1}{16}$″ to 1′, find the scale dimensions for a house 24′ by 32′.

7. If $\frac{1}{16}$″ represents 1′, what are the scale dimensions of a building 30′ by 42′?

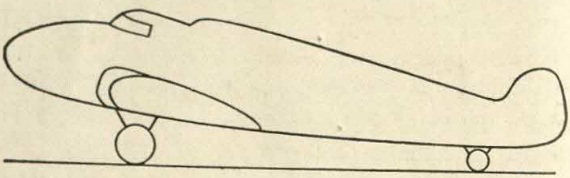
8. Using the scale $\frac{1}{8}$″ to 1′, find the actual lengths of the following scale lengths: 1″, $\frac{1}{2}$″, $\frac{1}{4}$″, 2″, $2\frac{1}{2}$″.

9. What is the actual size of a room drawn on a scale $\frac{1}{12}$″ to 1′ if the scale size is 2″ by $2\frac{1}{2}$″?

10. Determine the actual size of a ground plan whose scale size is 3″ by $4\frac{1}{2}$″. Scale used is $\frac{1}{16}$″ to 1′.

11. What is a convenient scale to use on tablet paper for a house 30′ by 36′? For a rug 9′ by 12′?

Fig. 11



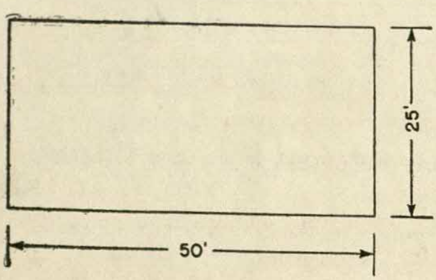


Fig. 12

12. If the airplane in figure 11 is 24′ from nose to tip of tail, what is the scale of this drawing?

Using this scale: (a) find the diameter of the front wheel; (b) the diameter of the rear wheel, (c) the height of the plane from the front wheel up.

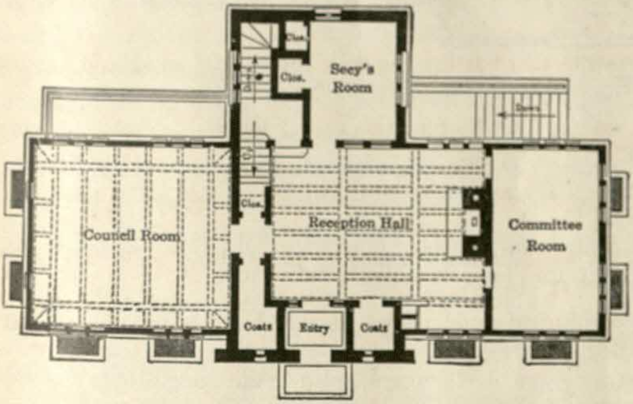


Fig. 13. First-floor plan of a Y.M.C.A. building.

13. Figure 12 is drawn to what scale?

14. If the wing span of a bombing plane is 72′, what will be the wing span of a model if the scale is 1″ = 16′?

15. The scale on a drawing is 1″ to 1′. What is the actual size of a room that appears 1′ by 2′ on the drawing?

16. The first-floor plan of a Y. M. C. A. building is shown in figure 13. One inch in the scale drawing represents 20′ in the building. What are the approximate dimensions of the committee room, reception hall, council room, and secretary's room? Determine by measurement.

17. Using the scale $\frac{1}{10}$″ to 1′, make a drawing of a house with the following dimensions:

Width across front and back, 40′; depth front to back, 28′.
Living room (S.E. corner), 16′S. by 18′E.
Library (N.E. corner), 16′N. by 10′E.
Hall (front to back), 8′ wide.

Dining room (S.W. corner), 16′S. by 16′W.
Kitchen (N.W. corner), 16′N. by 12′W.
Leave an opening for an outside door 4′ wide in the front hall.
Leave openings for doors 3′ wide at convenient places in the plan.

18. Reproduce figure 14. Use the scale 1″ represents 8′.

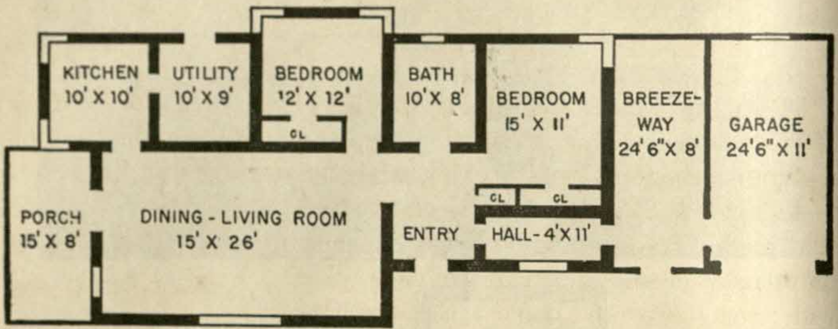


Fig. 14. Ranch-type floor plan, 27′ x 72′.

19. (OPTIONAL) Draw your classroom to scale, locating cupboards, windows, and doors.

20. (OPTIONAL) Draw your school grounds to scale, locating buildings, playgrounds, and so forth. Take actual measurements.

21. (OPTIONAL) Study a floor plan of a house. Draw the plan, making desirable changes in room arrangement, built-in features, location of doors and windows, and location of stairways; also make suggestions for the addition of rooms, porches, ventilation and heating equipment, and so forth.

22. (OPTIONAL) Draw to scale the floor plan of a model kitchen, including the cupboard, stove, table, and refrigèrator. State the scale used and the actual length and width of the kitchen and each piece of furniture.

23. (OPTIONAL) Draw to scale the floor plan of a model workshop, including a workbench, tool rack, and stain table. State the scale used and the actual length and width of the shop and its contents.

24. (OPTIONAL) A railroad station has two tracks. The distance from the center of one track to the center of the other is 15 feet. The front of the station building is 33 feet north of the center of the first track, and a waiting shed across the tracks from the station is 16 feet south of the center of the second track. How far is the waiting shed from the front of the station building? Make a sketch to show how the station and the two tracks and the waiting shed are located.

DIRECT MEASUREMENT OF ANGLES

VOCABULARY

1. angle	4. acute angle	7. protractor
2. vertex	5. obtuse angle	8. vertical angles
3. right angle	6. reflex angle	9. degree

10. $\angle$ symbol for "angle" 11. $\measuredangle$ symbol for "angles"

58. Angles. The study of angles is important because of the use of angles in surveying, navigation, airplane piloting, gunnery, toolmaking, carpentry, metalworking, and engineering of all kinds.

An angle is the opening between two lines that meet. The opening between the blades of a pair of scissors illustrates an *angle.* The wider the scissors are opened, the larger the angle becomes. Another illustration of an angle is as follows: Rotate a straight line OW in a plane about a point O in the direction indicated by the arrows in figure 15. When OW reaches the position indicated by OY, it has turned through the angle WOY.

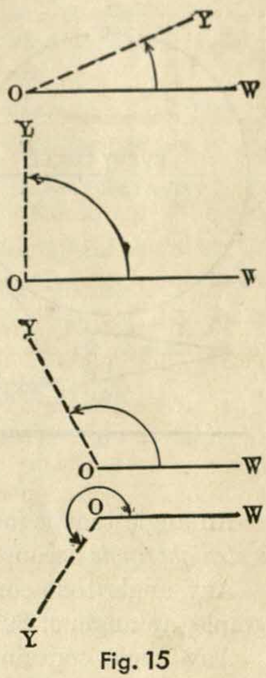


Fig. 15

The fixed point O is called the *vertex* of the angle; the lines OW and OY form the *sides* of the angle.

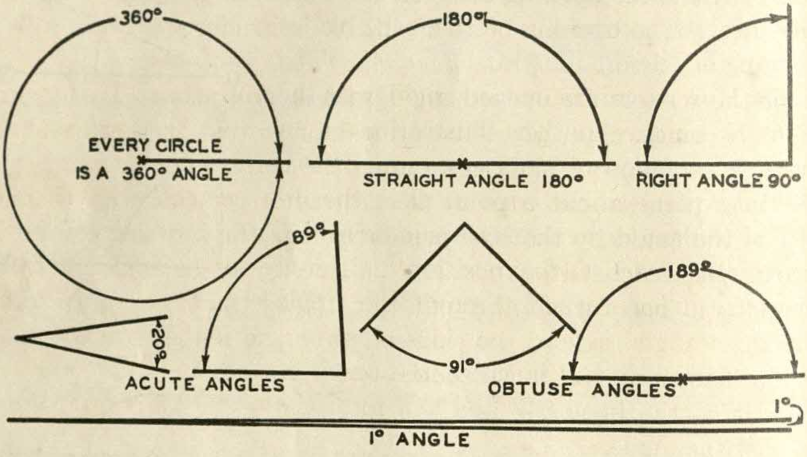
59. Lettering and reading angles. Angles are usually lettered in one of two ways. One method is to place a capital letter at the vertex and at the end of each side of the angle, as in figure 15. The point at the vertex is often labeled O and is read between the other two letters. In figure 15, the angle is read $\angle O$ (angle O), $\angle YOW$, or $\angle WOY$. The other method of lettering an angle is to place a small letter in the angle near the vertex, as in figure 18. Such angles are read $\angle a$, $\angle b$, and so forth.

60. Unit of measurement of angles. Divide the space around the center of a circle into 360 equal parts. Each of those small divisions is one *degree*. Every degree may be divided into 60 equal parts, called *minutes*. Every minute may be divided into 60 equal parts,

called *seconds*. Degrees, minutes, and seconds are the units commonly used to measure angles.

60 seconds ($''$)	= 1 minute ($'$)
60 minutes	= 1 degree ($°$)
360 degrees	= 1 circle.

61. Kinds of angles. The most common angle is the *right angle.* Since four right angles can be grouped around the center of a circle, each right angle contains 90°. The right angle is found in the square corners of pictures, sheets of paper, window panes, rugs, packing cases, and so forth.



An angle that is formed by one-half of a complete circle is called a *straight angle* because its sides form a straight line. It contains 180°.

Any angle that contains less than 90° is an *acute angle*; for example, an angle of 89°, 75°, 4°, 6°.

Any angle containing more than 90° but less than 180° is an *obtuse angle*; for example, an angle of 91°, 100°, 165°, 179°.

An angle containing 180° is called a *straight angle.* Any angle containing more than 180° is called a *reflex angle.*

The lengths of the sides of an angle have nothing to do with the size of the angle.

62. Studying the protractor. The protractor is an instrument used to measure and construct angles. It is usually made in the shape of a semicircle. Examine a protractor. There are usually two rows of numbers on the semicircular scale, one row numbering

180° from left to right, and one row numbering 180° from right to left. This arrangement makes it easy to measure or draw angles pointed in any direction. At what number of degrees on the scale are the degrees the same? Why at this point? Find 60°, 170°, and 30°. How do you locate 65°? 45°? 42°? 38°? 134°?

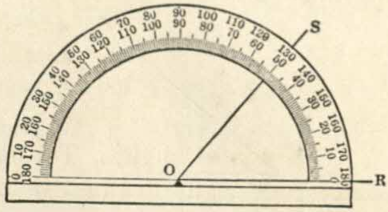


Fig. 16

Notice the triangle in the straight edge. The vertex of this triangle is on the straight line connecting the 0° and the 180° marks. It is very important in measuring or drawing angles to see that the vertex of this triangle is directly on the vertex of the angle and that the line connecting the vertex with 0° coincides with one side of the angle.

63. How to measure an angle with a protractor. To measure $\angle ROS$ in figure 16, place the protractor so that the vertex of the triangle on the protractor rests upon the vertex O of the angle.

Make the straight edge of the protractor coincide with the side OR of the angle, so that OR passes through the zero degrees on the protractor scale. Observe where the side OS of the angle intersects the curved portion of the protractor. Read the degrees. In measuring an angle, extend the sides of the angle if they are too short to be crossed by the curved edge of the protractor.

EXERCISES

1. Measure $\measuredangle AOB$, MON, and XOY in figure 17. How do these angles compare in size? Upon what does their size depend? Upon what does it *not* depend?

2. Measure the angles in figure 18. *Note:* You may extend the sides of the angles to reach the scale of degrees on the protractor by placing a straight edge of paper along the side of the angle. Read the degrees where the straight edge of the paper crosses the scale of degrees.

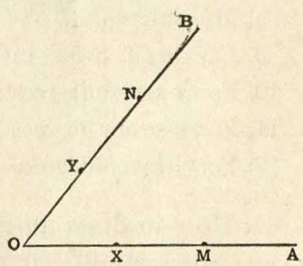


Fig. 17

3. In figure 19 measure angles r, s, t, u, and v. What is the sum of these angles?

4. In figure 20 measure angles a, b, c, and d. What do you find true about $\angle a$ and $\angle b$? about $\angle c$ and $\angle d$? Angles a and b are called *vertical*

angles because they are on the opposite sides of two lines that cross. Angles c and d are also vertical angles. Why are none of the angles in figure 19 vertical angles?

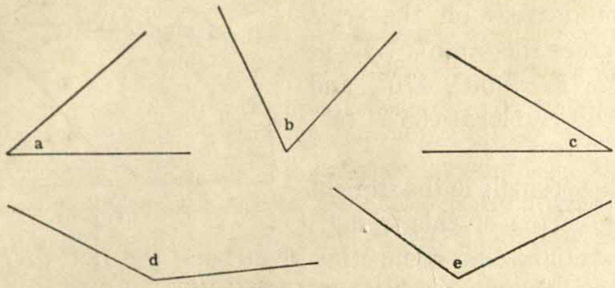


Fig. 18

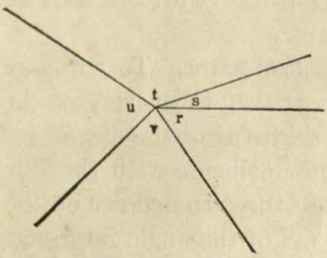


Fig. 19

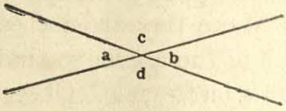


Fig. 20

5. Draw 10 different angles and measure each.

6. Add: 39° 58′ 38″ and 46° 36′ 19″. Change to a higher denomination when possible.

7. Subtract 149° 53′ 49″ from 180° 32″.

8. Multiply 48° 37′ 41″ by 3.

9. Divide 53° 5′ 52″ by 2.

10. Forty-seven degrees plus how many degrees make a right angle?

11. Forty-seven degrees plus how many degrees make a straight angle?

12. Forty-seven degrees plus how many degrees make a circle?

64. How to draw an angle with a protractor. To draw an angle of any size, say 40°, first draw a straight line OY to represent one side of the angle. Place the protractor so that the vertex of the triangle on the straight edge of the protractor falls upon O, the vertex of the angle, and the side OY passes through the 0° on the scale. Then find the 40° mark on the protractor scale. Now draw from this point to the vertex of the angle the line OX. $\angle XOY$ will be the required angle.

EXERCISES

1. Draw an angle of 20°; 40°; 60°; 10°; 30°; 80°; 70°.

2. Draw the following angles: 100°; 120°; 140°; 110°; 150°; 170°.

3. Draw an angle of 15°; 45°; 125°; 5°; 65°; 95°; 35°; 75°.

4. Draw an angle of 38°; 86°; 147°; 93°; 16°; 47°; 187°; 89°.

5. How would you draw, with a protractor, an angle of 189°? 240°? 325°?

6. Draw angles in this order: 5 acute angles, 2 right angles, 5 obtuse angles, 2 straight angles, and 5 reflex angles. Label the number of degrees in each.

SUMMARY QUESTIONS

1. Why are more accurate measurements needed at the present time than were needed by early civilization?

2. What measuring instruments have you seen used?

3. Give an example of where a "rough estimate" is sufficient in measurement.

4. Give an example of wheie a measurement must be accurate to at least one-thousandth of an inch.

5. Explain the statement, "All measurements are approximate."

6. What is a scale drawing?

7. List five examples of scale drawings you have seen.

8. What does a scale of 1″ to 1′ mean?

9. What is it that makes the metric system easier to learn than the English system of measurement?

10. What is the use of a protractor?

11. What are vertical angles? What is true of them?

CHAPTER VII

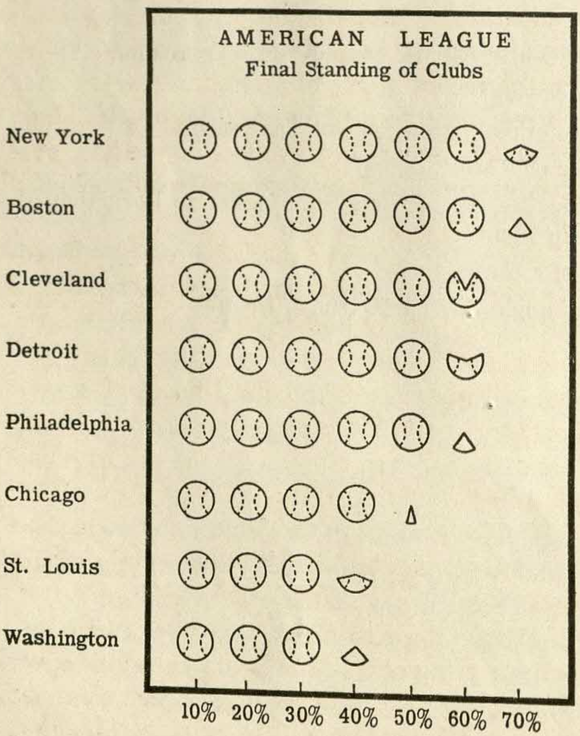
GRAPHS

VOCABULARY

1. pictograph 2. bar graph 3. line graph 4. circle graph

65. Graphs. Lists of figures representing certain facts can be made meaningful at a glance by the use of pictures and drawings.

Table 1 lists the final standings of the eight teams in the American League in a recent year. Graph 1 pictures the final standings. For purposes of record, the table is more satisfactory, as the standing of each team is correctly given to thousandths. The graph is accu-



Graph 1. A Pictograph.

TABLE 1

AMERICAN LEAGUE
FINAL STANDING
OF CLUBS

New York	.630
Boston	.623
Cleveland	.578
Detroit	.565
Philadelphia	.526
Chicago	.409
St. Louis	.344
Washington	.325

rate to tenths only, as smaller fractional parts are difficult to interpret. However, for most purposes the graph is more quickly comprehended and the facts are presented in a more pleasing form.

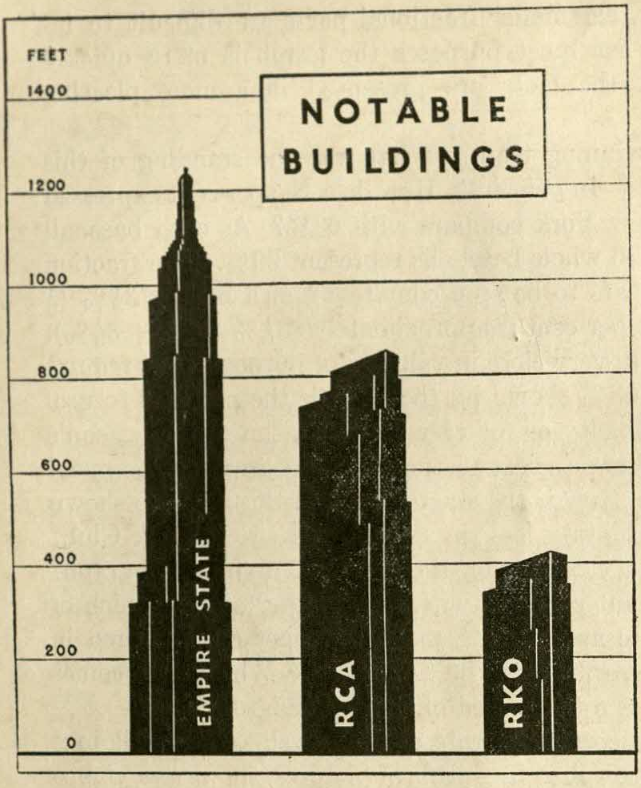
Which was the winning team? What was the standing of this team in thousandths? In per cent? How does the per cent expressed in the graph for New York compare with 63%? As each baseball represents 10%, the 6 whole baseballs represent 60%. The fraction of the baseball appears to be approximately $\frac{1}{3}$ of a ball or $3\frac{1}{3}$% ($\frac{1}{3}$ of 10%). Thus, the per cent is approximately $63\frac{1}{3}$% (60% + $3\frac{1}{3}$%). The two per cents are very close in value. For purposes that require close accuracy, the 63% shown by the table is the per cent to use. For purposes of publicity or for casual comparison of the season's results, the graph shows a clear picture that has greater eye-appeal than does the table. How is the standing of the Boston Club shown in the graph to be slightly less than that of the New York Club? What per cent do you calculate for the Boston Club from the graph? How does the per cent compare with that of the table? Which of the per cents is more accurate? Compare the per cents figured in the table and in the graph for the St. Louis Club. Which team comes nearest to having twice the record of the Washington team?

Table 2 gives the correct heights of three well-known buildings of New York City. Graph 2, by means of pictures, shows at a glance the approximate heights of the buildings and the relationship of their heights to each other.

What is the height given in the table for the Empire State Building? Is the height as represented on the graph apparently the same? Do the heights of the RCA and RKO buildings as pictured on the graph appear to agree with the heights recorded in the table?

In the following questions, study the graph first. Then check your answers by using the heights given in the table. How does the height of the RKO Building compare approximately with the height of the Empire State Building? Which building is about 100 per cent higher than the RKO Building? The Empire State Building is about one and one-half times as high as which building? Which building appears to be about twice as high as one of the other buildings?

Table 3 is a record of the paying passengers that were carried by the United States railways over a period of 45 years. Graph 3 gives the same record in picture form. The number of passengers carried for each year is rounded off to the nearest million. The graph shows very clearly the great difference in railway travel over the 45 years.



Graph 2. A Bar Graph.

TABLE 2

NOTABLE BUILDINGS OF NEW YORK CITY

Empire State . . .	1250 ft
RCA	850 ft.
RKO	409 ft

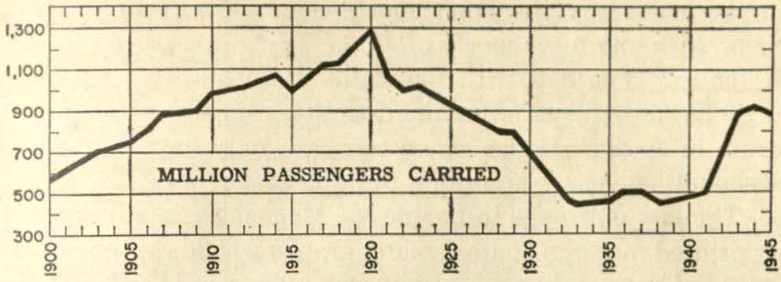
Check the numbers carried as shown by the graph with the numbers given in the table. Then answer the following questions from the graph, checking against the table where possible: (a) During which year were most passengers carried? (b) During which year were the least number of passengers carried? (c) How many million passengers were carried in each of the following years: 1900, 1905, 1910, 1920, 1930, 1940, and 1945? (d) During which years were the following numbers of passengers carried: 700,000,000; 1,000,000,000;

TABLE 3

REVENUE PASSENGERS CARRIED

(United States Railways, Classes 1, 2, and 3)

Year	*Million*	*Year*	*Million*
1900	575	1925	900
1905	725	1930	700
1910	990	1935	450
1915	1,000	1940	450
1920	1,275	1945	900



Graph 3. REVENUE PASSENGERS CARRIED (United States Railways—Classes 1, 2, and 3).

450,000,000; 900,000,000? (e) How many more passengers were carried in 1915 than in 1930? (f) How many fewer passengers were carried in 1900 than in 1925? (g) Did the number of passengers carried increase or decrease from 1910 to 1915? From 1920 to 1925? From 1940 to 1945? (h) Name another year in which about the same number of revenue passengers were carried as in 1945.

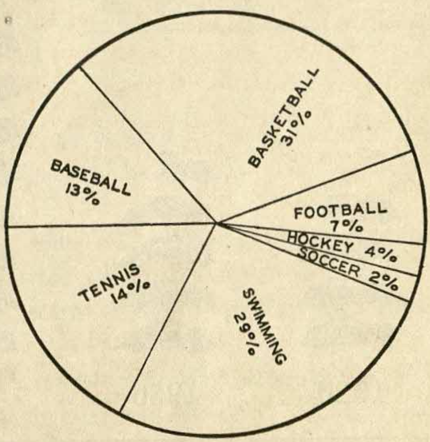
Table 4 gives the reaction of the pupils of a certain school toward the sports offered. Graph 4 pictures the rating of these sports.

The whole circle (100 per cent) represents the whole student body. Thirty-one per cent of the students voted basketball as their favorite sport, while only 2 per cent favored soccer. At a glance the athletic director of the school can see that basketball and swimming are by far the favorite sports and that soccer and hockey are the least liked of the sports. By division he can discover that twice as many prefer tennis as football and that swimming is about four times as popular as football.

TABLE 4

FAVORITE SPORTS OF A LARGE HIGH SCHOOL

Basketball	31%
Swimming	29%
Tennis	14%
Baseball	13%
Football	7%
Hockey	4%
Soccer	2%



Graph 4.

In the same way the other sports can be compared with each other or with the total sports program. How does soccer compare with tennis? Football with baseball? Baseball with basketball? How do basketball and swimming together compare with the whole program of sports? If the school can have only four sports, which ones should be chosen, according to pupil preferences?

66. The use of graphs in business. Magazines, newspapers, and other printed material contain many graphs which at a glance show the trend of business in a most vivid way. The word "graph" is from a Greek word which means *to make clear*. The executive studies graphs representing expenditures, sales, and growth of his business. He may use the graphs to make a comparison of facts that may be presented to the public in the quickest and most appealing way. From some graphs it is even possible to predict future volume of business. The businessman can make a fair prediction of the extent of his future business, if nothing unusual happens to upset the general trend of the past few years.

The four most commonly used graphs are *pictographs, bar graphs, line graphs*, and *circle graphs*. Each kind of graph is adaptable to a particular type of comparison.

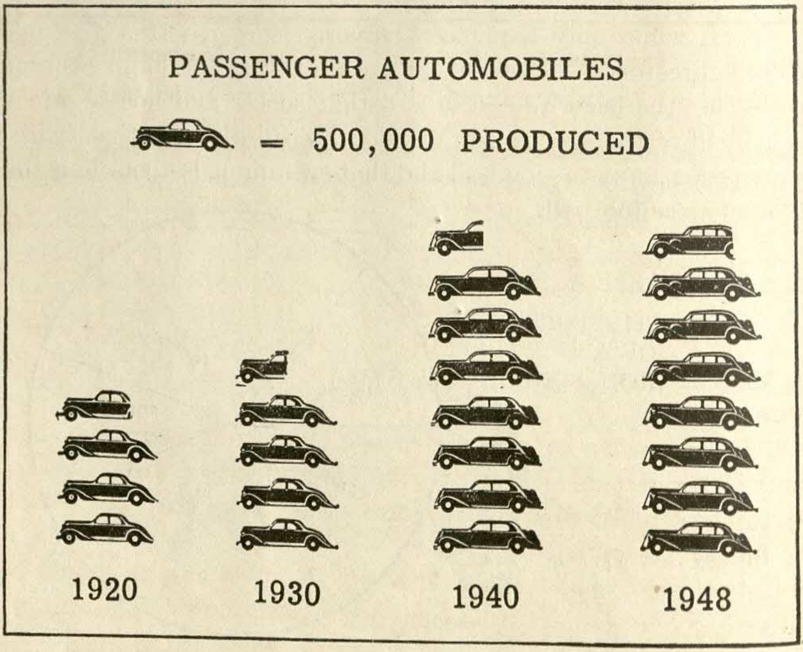


Fig. 1. A Pictograph.

67. Pictographs. Pictures of objects are frequently used in representing growth and in making comparisons. Such representations are often called *pictographs*. Each object in the graph represents a definite quantity. The greater the number of objects pictured, the greater the quantity represented.

In figure 1, (a) one automobile represents how many automobiles produced? (b) In which year was passenger automobile production the greatest? (c) Which year was a very close second? (d) Between these dates there was a period in which practically no passenger cars were produced. Why were none produced? (e) Approximately how many passenger cars were produced each year pictured? (f) How does the graph show that in the year 1948 slightly less than 4,000,000 passenger cars were produced? Judging by the graph, what prediction could be made for the future production of passenger automobiles?

Aids in making a pictograph.

1. *Determine how large a quantity each object is to represent.* Take the largest quantity to be represented and divide it by the number of objects that will fill the longest row that you wish on your graph. If the numbers are quite large, it is usually better to round them off to a more convenient size.

Example

In figure 1, the largest quantity to be graphed is 3,909,270 cars, which can be rounded off to 4,000,000 cars. Using not more than 8 cars to the row, each car will represent 500,000 cars produced (4,000,000 ÷ 8).

Example

In figure 1, the 3,909,270 cars ÷ 500,000 cars = 7.8 (division carried to tenths). Hence, 7.8 is the number of automobiles used for the 1948 column.

2. *Determine how many objects are needed for each row*—by dividing the quantity to be represented by the quantity that each object represents.

3. *Decide on the order of the rows.* The arrangement may be by consecutive periods of time; by the order of size, as from the shortest to the longest row; or by some other order that will best tell the facts represented.

4. *Note how the objects should be placed.* The objects should be a uniform distance apart from each other in the rows. The space between the rows should be about the same width as the rows.

5. *Label each graph with a self-explanatory title.*

6. *Take the utmost care to make the graph accurate and neat*—so that it will serve the purpose for which it was intended.

EXERCISES

A. Oral

1. Suggest how a pictograph might represent increased sales of airplanes if the sales doubled each year for three consecutive years.

2. Suggest how the production of wheat in the United States and the rest of the world may be pictured if for every 9 bushels produced in the United States, 6 bushels are produced in the rest of the world.

3. Suggest two pictographs to represent the number of people in the United States compared with the number in the rest of the world, if the rest of the world has 16 times as many persons as the United States.

B. Written

4. Show by means of a pictograph that the United States produces 11 barrels of oil to every 5 barrels produced in the rest of the world.

5. Make a pictograph showing the production of television sets for the following years:

1947	210,000
1948	1,050,000
1949	2,700,000

6. By means of a pictograph represent the approximate heights of: Mt. Whitney, 14,898 (15,000) feet; Mt. McKinley, 20,464 (20,500) feet; and Pike's Peak, 14,108 (14,000) feet.

7. Represent by a pictograph the following educational attainments for a recent year of persons of age 25 years and over of the United States population:

30.3% had completed the 8th grade.
20.5% had completed 4 years of high school.
5.4% had completed 4 years, or more, of college.

8. (OPTIONAL) Merchant X, believing that salespeople should be paid on the basis of what they sell, rather than for hours worked, made a pictograph of the sales made by clerks A, B, C, D, and E. Make such a picture if in a given time clerk A sold $200 worth of merchandise; clerk B, $150 worth; clerk C, $250; clerk D, $275; and clerk E, $325.

9. (OPTIONAL) Make a pictograph showing interesting facts from your civics or science studies, or from some magazine or newspaper.

10. (OPTIONAL) Cut a pictograph from a newspaper or magazine. Interpret it to the class.

11. (OPTIONAL) Find a pictograph in a newspaper or magazine and summarize the facts represented.

68. Bar graphs. The bar graph is much used in business to compare unrelated facts, such as the amount of production of an article over a period of years; the amount of business done in dollars for a series of years; and the cost of various items over a period of time.

The bars may be placed in either a vertical or a horizontal position. Figure 2 is a vertical bar graph that depicts the comparative costs for an average family for gas, soft coal, oil, and hard coal.

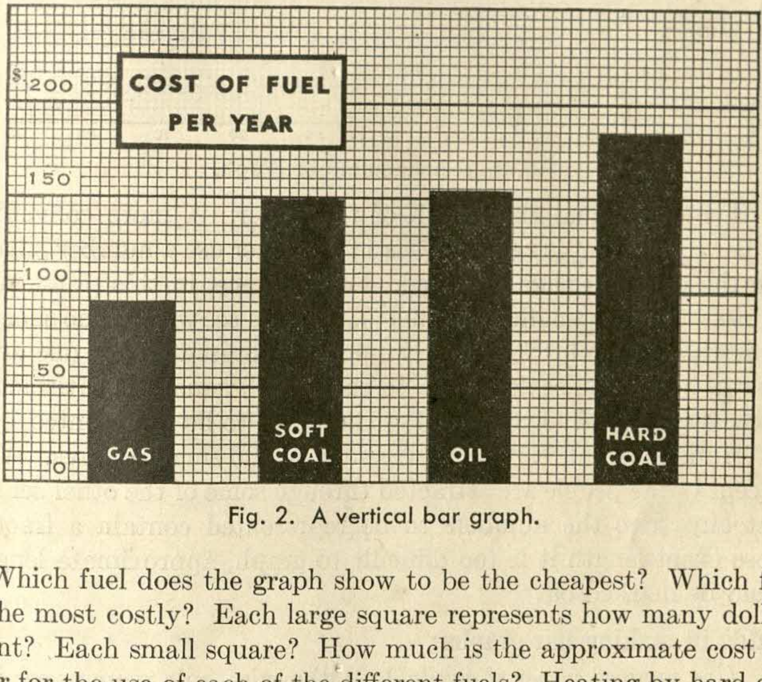


Fig. 2. A vertical bar graph.

Which fuel does the graph show to be the cheapest? Which fuel is the most costly? Each large square represents how many dollars spent? Each small square? How much is the approximate cost per year for the use of each of the different fuels? Heating by hard coal costs how much more than heating by gas? How many times as much? Compare the cost of gas and soft coal and of gas and oil in the same way.

Figure 3 is a horizontal bar graph. If the merchant is to do successful advertising, he must make use of the information pictured in figure 3. One large square represents what per cent? One small square represents what per cent?

The lengths of the bars are determined by a scale suitable to the size of the paper upon which the graph is to be drawn. Because the paper is only 90 squares in length, the longest bar must not be over 90 squares long. The longest bar is to represent 87%. If we divide 87% by 90 (the number of squares in the length of the paper), we see that we can allow $.9\frac{2}{3}\%$ to each small square. The largest bar

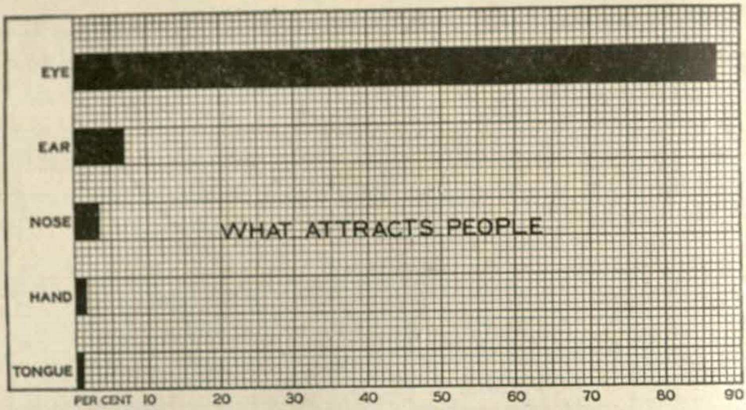


Fig. 3. A horizontal bar graph.

will then just equal the length of the paper. A more convenient number to use that is near $.9\frac{2}{3}\%$ is 1%. Never use a number smaller than the quotient thus obtained. For instance, in this problem do not use a number smaller than $.9\frac{2}{3}\%$. Do you see why? The shortest bar is to represent 1%. This would be represented by one small square; hence the scale of 1% to each small square is satisfactory for the largest and for the smallest numbers to be represented. For the "Hand" appeal, 1.5% is represented by 1.5 small squares. What per cent of the people are attracted through some of the other senses? If at any time the numbers to be represented contain a fraction whose exact length it is too difficult to graph, approximate lengths should be marked off.

Aids in making bar graphs:

1. *Use squared paper if available.* Otherwise, rule your paper into squares of convenient size.

2. *Arrange the numbers* to be graphed in a row or table in the order in which they are to appear on the graph.

3. Usually the *bottom line* and the *line at the left side* of the paper are used for the guide lines.

4. *Choose and mark off a workable scale* on the guide lines, remembering that the scale must be both small enough to represent on the paper the largest quantity in the table, and large enough for the smallest quantity to be visible.

5. *To make the bars* of the graph, determine the required length of the bars and draw shaded bars of uniform width, usually leaving the space between the bars the same width as the bars.

6. *Give a title to every graph.*

EXERCISES

1. The rate of climb of a Boeing *Stratoliner* is:

With 4 engines, 1,200 ft. per minute for the first 5,000 feet.
With 3 engines, 600 ft. per minute for the first 5,000 feet.
With 2 engines, 125 ft. per minute for the first 5,000 feet.

Make the graph on 6″ x 9″ squared paper. If the bars are vertical and placed the long way of the paper and 1 in. is allowed at the bottom for the title, 8 in. will be left for the length of the longest bar. The longest bar is to represent 1,200 ft. in the 8-in. space on the paper. Then 150 ft. will be represented by 1 in. on the paper. The 600 ft. will be represented by how many inches? The 125 ft.?

2. Lawrence is to make a bar graph on 6″ x 9″ paper divided into half-inch squares which are subdivided into 5 parts. Using lengths to the nearest 500 miles, he is to compare the lengths of several rivers: the Mississippi-Missouri, 4,194 (4,000) miles; the St. Lawrence, 2,100 (2,000) miles; the Amazon, 3,300 (3,500) miles; and the Yangtze-Kiang, 3,000 miles. He turns his paper sideways and allows 1 in. on the left edge for the label. How long can the longest bar be? How many miles must it represent? This allows how many miles per inch? Per half-inch or one large square? per each small square? How long should he make each bar?

3. Make a bar graph of the following data selected from certain manufacturing industries in the "Occupational Study of Kansas City, Missouri":

Abilities To Be Developed in School	*Number of Times Named*
Following directions	317
Arithmetic fundamentals	254
Mental alertness	251
Logical thinking	244
Memory	122
Physical alertness	120
Pleasant speech	78

4. Make a bar graph showing the approximate per cent of voters who went to the polls as shown in the following table:

1942	32%
1944	54%
1946	38%
1948	53%

(a) How do you account for the increase of 1944 and 1948 over 1942 and 1946?

5. (OPTIONAL) Compare by means of a bar graph the horsepower of various automobile engines.

6. (OPTIONAL) Cut a bar graph from a newspaper or magazine and interpret it to the class.

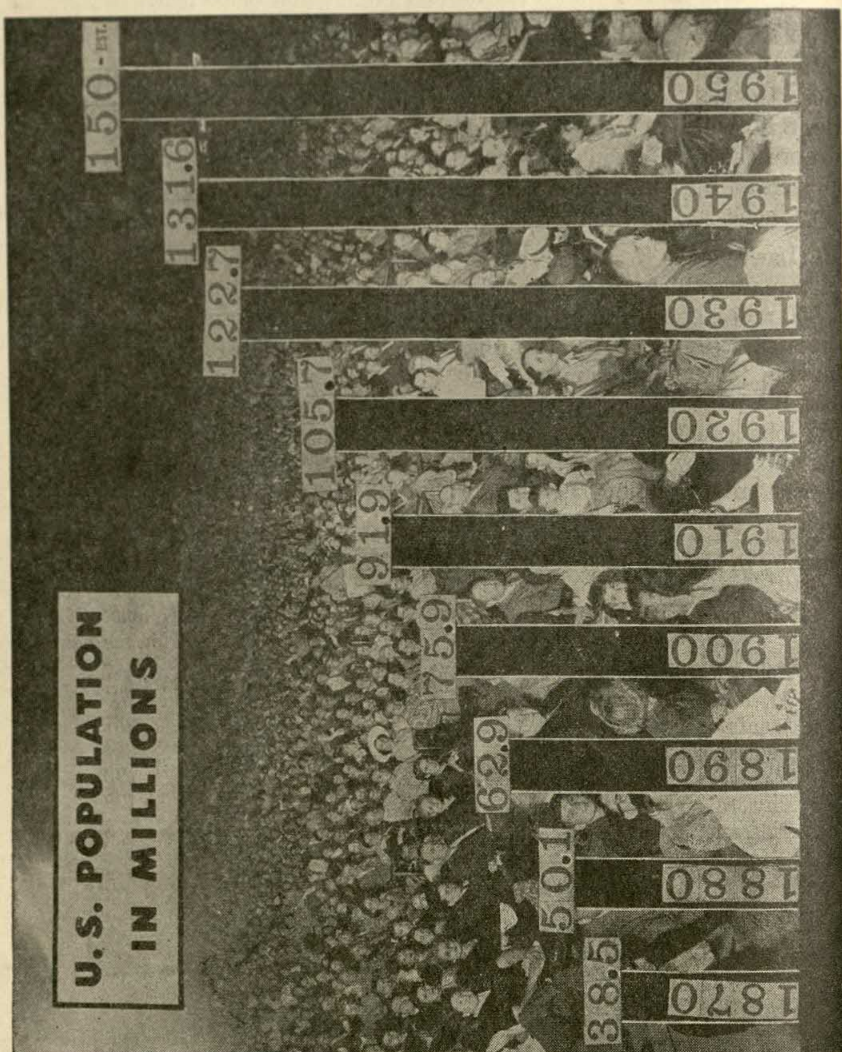


Fig. 4. Growth of United States Population, 1870–1950.

7. Interpret figure 4: (a) What is the total population for the years represented? (b) The estimated population for 1950 is how many times the population (1) of 1900? (2) of 1870?

8. (OPTIONAL) In a paragraph summarize the facts represented in a bar graph cut from a paper or magazine.

69. Line graphs. The line graph is most commonly used to picture related facts.

A broken-line graph represents changes of an irregular nature, such as variations in price of commodities in different years; profits made in a firm year by year; or variations in daily sales.

The straight-line graph is used to represent regular changes taking place, such as the cost of several pounds of sugar at the same price per pound.

The visibility over the ocean, on a clear day, from various heights is given in the following table:

TABLE 5

Height	100′	150′	200′	250′	300′	350′
Visibility (*in nautical miles,* 6,080 *feet each*)	12	14.1	16.2	18.2	19.9	21.5

Figure 5 is a broken-line graph. The graph pictures the facts given in the preceding table which gives the visibility over the ocean from various heights. The graph line is broken because the visibility does not increase the same amount for each 50-foot rise. If the increase in visibility were the same for each 50-foot rise, the graph line would be straight. (a) What is the distance seen at a height of 100 ft.? Of 150 ft.? Of 200 ft.? (b) How many miles farther can be seen as a result

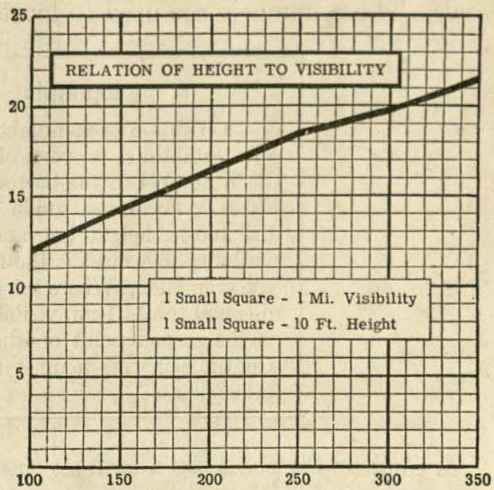


Fig. 5. A Line Graph.

of these 50-ft. rises? (c) What is true of the graph line between the 100-ft. and the 200-ft. heights? Why? (d) What is the approximate visibility at the 250-ft., the 300-ft., and the 350-ft. heights? (e) How does the approximate distance of visibility in the graph compare with the distance given in the table? (f) At what point does the graph line make a definite break? Why? (g) Does the graph line rise faster before or after the 250-ft. height? Why?

Aids in making a line graph:

1. *Choose two scales*—a horizontal scale and a vertical scale, being careful that the graph will be long enough and wide enough to represent the data to be used and still not be too long or too wide for the paper.

> **Example**
>
> In figure 5 one vertical small square represents one mile of visibility and one horizontal small square represents ten feet in height.

2. *Label the guide lines.*

3. *Make a table of data arranged by pairs.* One number of each pair is for the horizontal scale; the other number is for the vertical scale. Those numbers are used to locate points on the graph.

> **Example**
>
> In Table 5 each number in the top row establishes a vertical guide line. Each number in the bottom row establishes a horizontal guide line. Where the 100-ft. height line and the 12-mi. visibility line meet is the first point on the graph line. Where the 150-ft. height line and the 14.1-mi. visibility line meet is the second point on the graph line. Continue in this manner to locate the other points.

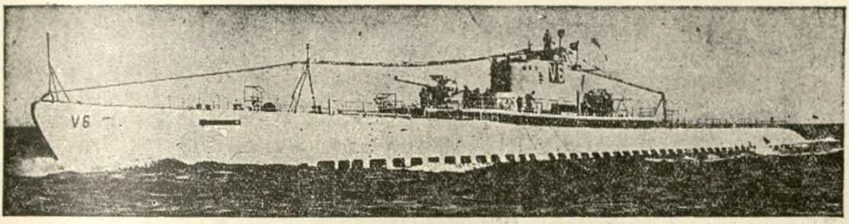
4. *Connect the points by a straight line.*

EXERCISES

1. The cost of airplane fabric varies with the width of the cloth. Draw a line graph showing the increase in price as width increases:

Width	2″	3″	3½″	4″	5″	7½″
Price per yd.	$.50	.75	87½	1.00	1.25	1.87½

(*Hint:* Locate the highest point to be represented by the width and cost.) Represent the inches up the left-hand side of the graph and the dollars along the bottom line. Locate each number on the graph with a point. Connect the points with a line. Label the graph.



Courtesy U. S. Navy

THE AMERICAN SUBMARINE V6 REACHED A RECORD DEPTH OF 336 FEET

Fig. 6. Water pressure increases 43 lb. per square inch of exposed surface for every 100 ft. of depth. What was the water pressure on each square inch of the surface of the submarine V6 at the 336-ft. depth?

2. Construct a line graph illustrating the facts of figure 6.

3. Make a line graph of your grades for several days, or of your grades on several tests.

4. Make a line graph using the following data:

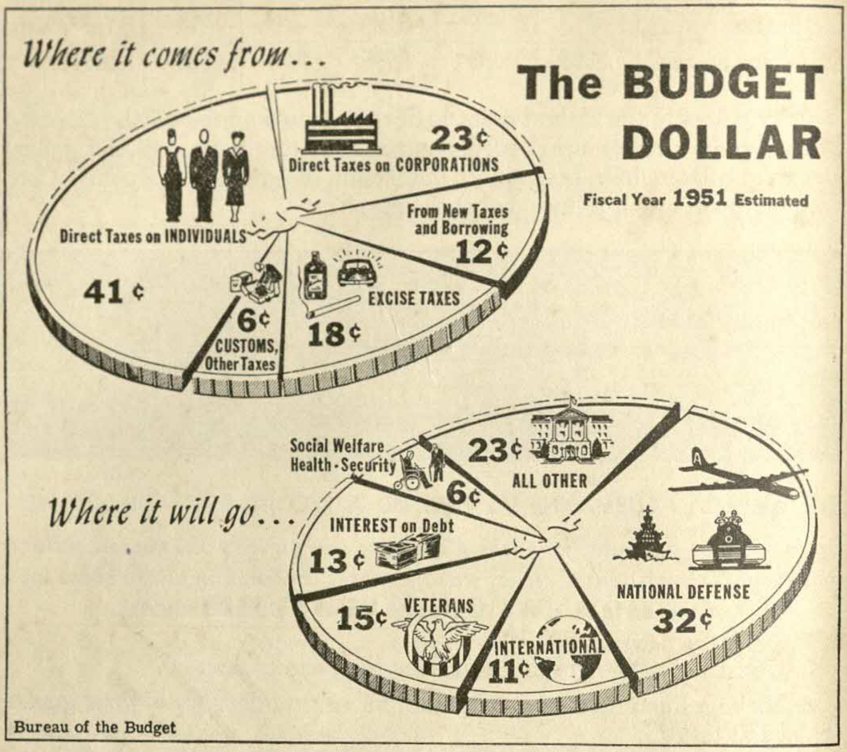
Altitude of Plane	Ground	1,000′	2,000′	3,000′	4,000′	5,000′	6,000′	7,000′	8,000′
Temperature of Air (Fahrenheit)	81°	80°	75°	70°	60°	56°	43°	29°	6°

5. (OPTIONAL) Make a line graph showing interesting facts from your civics or science studies, or from some magazine or newspaper.

70. Circle graphs. The circle graph is especially useful for showing the relation of one item to another and of one item to the whole number of items.

Figure 7 is a circle graph made up of two circles viewed from the side. Each circle represents one dollar. One circle pictures the number of cents in one dollar that are received from the several sources of income of the United States Government. The other circle shows how many cents of each dollar received by the United States Government are spent for various items.

Reading from the graphs: (a) What is the source of most of the United States Government's income? (b) What taxes do individuals



Courtesy Bureau of the Budget

Fig. 7. The budget dollar as estimated for the fiscal year 1951.

pay as a direct tax to the United States Government? (c) This tax represents what per cent of the income of the United States Government? (d) The least amount of income for the Government is from what source? (e) What source of income is taxed three times as much as the smallest source? (f) Two times as much? (g) By what method does the United States Government borrow money? (h) What item represents the largest expenditure for the Government? (i) Express the amount in (h) as a per cent. (j) What is the total per cent spent for past wars and to maintain peace? (k) This leaves what per cent for all other expenditures? (l) How do the per cents in (j) and (k) compare with the per cents shown in figure 7?

The portion of a circle enclosed between two radii and part of the circumference (an *arc*) is called a *sector* of a circle. Where have you seen that shape? The size of the sector is determined by the size of the angle formed by the two radii. The larger the angle, the larger the sector.

In this circle the sector between the two radii setting off "Veterans" is marked "15¢," which is $\frac{15}{100}$ or 15 per cent of the dollar.

Aids in making a circle graph:

1. *Draw a circle* large enough to make a clear drawing of the facts to be pictured but not so large as to be unwieldy.

2. *Express the number facts* to be graphed as per cents. Then arrange them in a table.

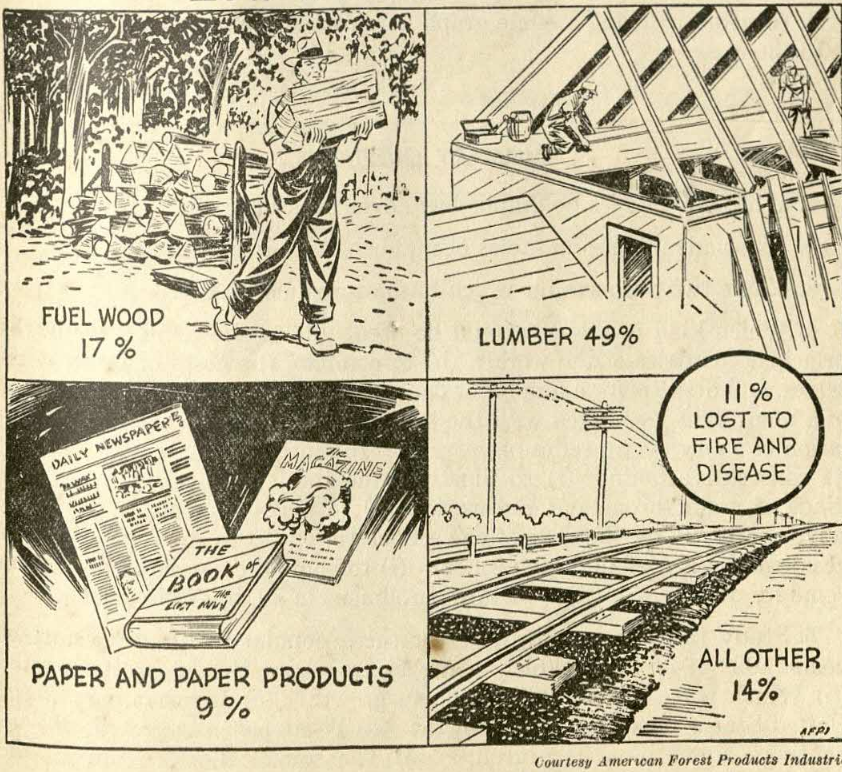
3. *Mark off sectors* of the circle corresponding to the required per cents.

Example

The "Veterans" sector is 15 per cent of the area of the circle. Hence, the angle between the two radii is 15 per cent of 360°, or 54°.

4. *Give a title to the graph.*

HOW WOOD IS CONSUMED



Courtesy American Forest Products Industries

Fig. 8. Various uses of wood.

EXERCISES

1. Figure 8 pictures the use of wood. (a) For which item is wood mostly used? (b) Which item pictured uses the smallest amount? (c) The 14 per cent pictures what uses? (d) What other uses are there that are not pictured in the 14 per cent? (e) What can you do to decrease the 11 per cent circle? (f) Arrange a table of the uses in the order of the per cents—largest first. (g) Prepare a circle graph of these facts.

2. Make a circle graph illustrating the facts in each of the following:

(a) Out of every \$100 received, a certain manufacturer spends \$62 for raw materials and \$20 for operating expenses. The remaining \$18 is his net profit.

(b) Manufacturer *A* finds that 60% of his selling price is spent for raw material and 24% for operating expenses. Sixteen per cent remains as net profit.

(c) A store made a survey of the ages of its employees, with the following results: 10.3% were under 40 years of age, 50.1% between 40 and 50 years, 39.6% over 50 years.

Hint: Use the nearest whole per cent, as, under 40 years = 10%.

3. Illustrate by a circle graph how you divide the 24 hours of the day. Use these activities, adding any others needed: Sleep, School, Study at Home, and Recreation.

4. (OPTIONAL) Bring a circle graph cut from a newspaper and interpret it to the class.

SUMMARY QUESTIONS

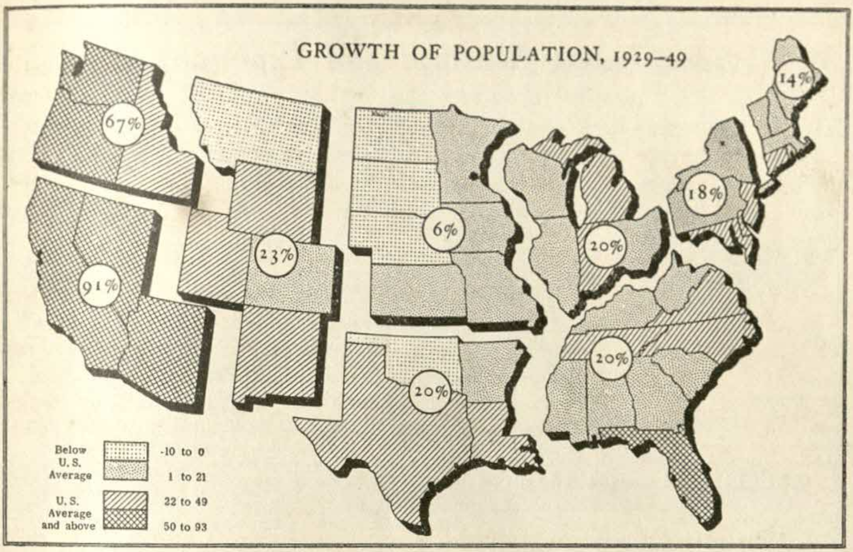
1. What advantages do graphs have over tables of numbers?

2. What advantages do tables of numbers have over graphs?

3. Name two purposes for which businessmen use graphs.

4. Which kind of graphs should be used: (a) to show the areas of the principal countries of the world; (b) to compare the costs of meat, vegetables, and desserts for a family for one month; (c) to compare the per cent of a monthly salary saved with the per cent spent; (d) to show the population of a city over a series of years; (e) to represent the temperature for 24 consecutive hours; (f) to illustrate the total production of different kinds of grain throughout the world; (g) to make a record of problems solved correctly for the days of one school week; (h) to show the record of accidents by grades in a high school; (i) to show the source of annual income for a business; (j) to picture enrollment in all grades in high school?

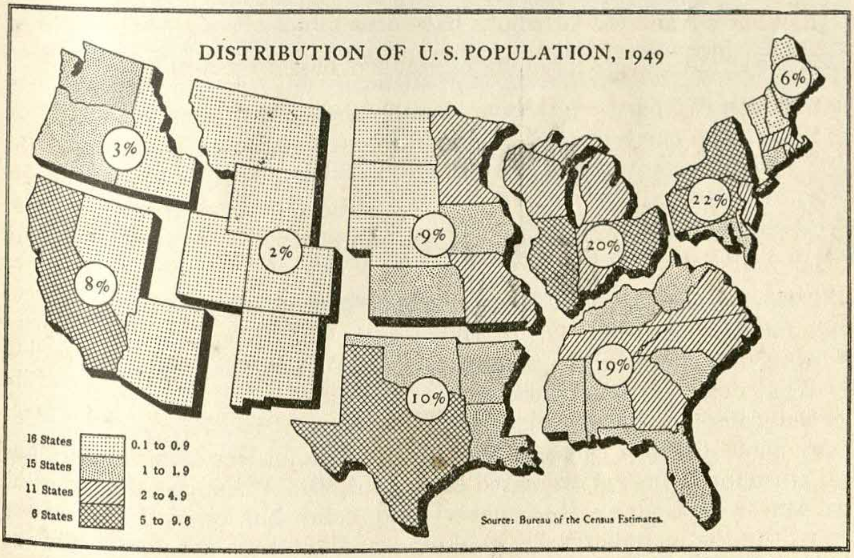
5. Study figure 9 and then answer these population questions for the years 1929–1949: (a) Which three states show the greatest growth? (b) Which three states rank second in growth? (c) In what part of the United States was there least growth? (d) What per cent growth did the Southern states have as a group? (e) The square checks represent the individual states having the most growth. Name these states.



Courtesy Bureau of the Census

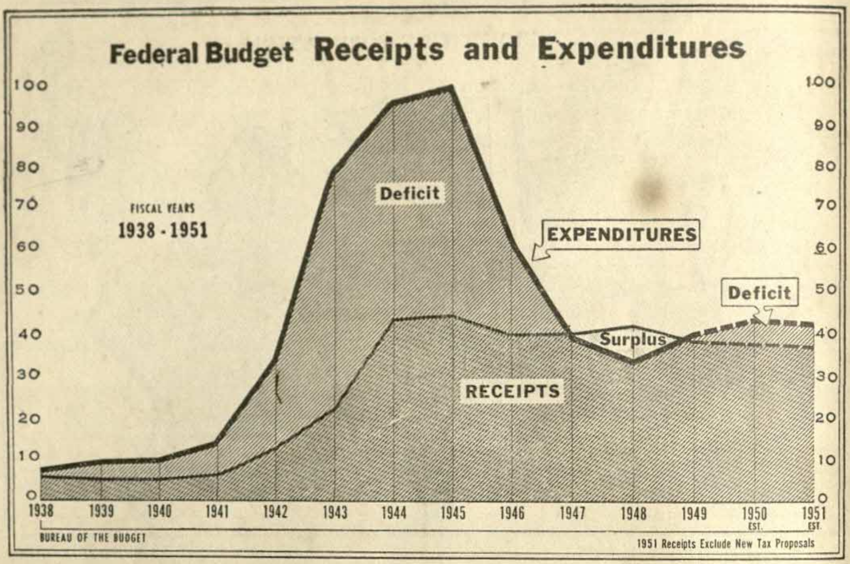
Fig. 9. Per cent change in population, 1929–1949. (U. S. average increase, 22%.)

6. Study figure 10 and then write a paragraph on "Distribution of Population in the United States in 1949."



Courtesy Bureau of the Census

Fig. 10. Per cent distribution of United States population.



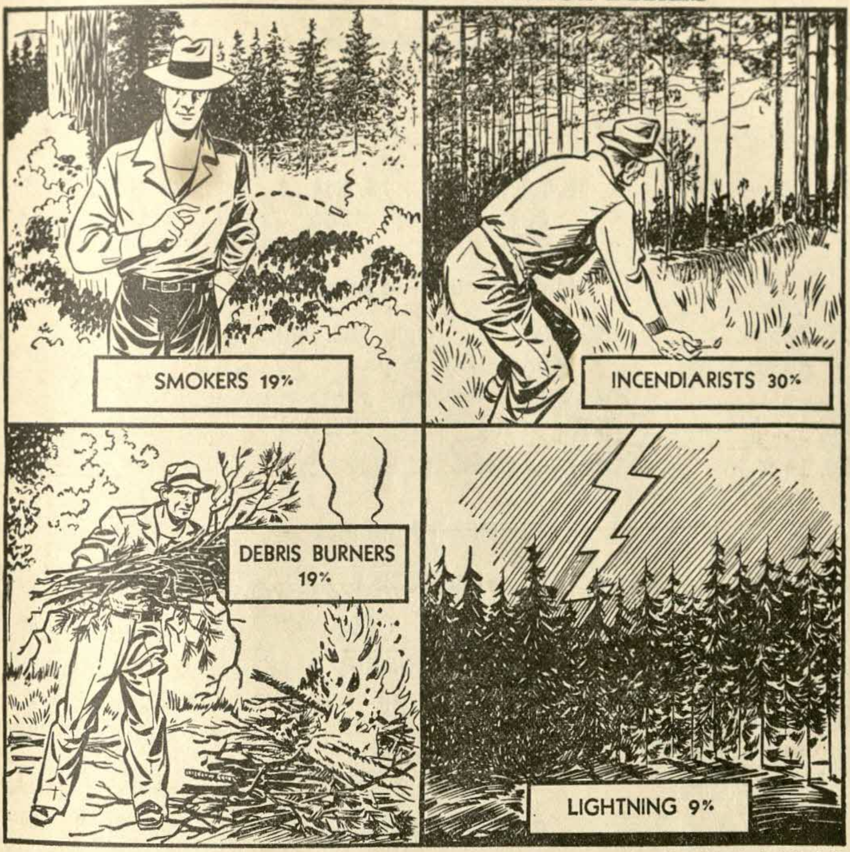
Courtesy Bureau of the Budget

Fig. 11. Federal budget for the fiscal years 1938–1951 in billions of dollars.

7. Figure 11 is made up of 2 broken-line graphs. One line shows the receipts of the United States Government and the other line the expenditures over a period of years. (a) Which years show a surplus? (b) Which year shows the greatest deficit? (c) Between what years was the greatest climb in expenditures? (d) Why the rapid climb at this time? (e) Between what years was the decline in expenditures? (f) Why the rapid decline between these years? (g) Approximately how much were the receipts in 1945? The expenditures? The deficit? (h) Estimate the receipts for 1948. The expenditures. The surplus. (i) What was the relationship between receipts and expenditures in 1947? In 1949? (j) Read the amount of receipts and expenditures for 1942. What is the amount of the deficit? (k) In 1943 what was the relationship of receipts to expenditures?

8. During a recent year fires burned 16,556,780 acres of national and state forests. See figure 12. (a) What is the total per cent of the causes of forest fires shown in the picture? (b) What is the per cent not shown? (c) What might be some of the causes not pictured? (d) What is the total per cent caused by the carelessness of smokers and debris-burners? (e) How many acres did they cause to burn? (f) The total loss from all national and state forest fires is estimated at $32,461,804. What was the value of the forests burned by the smokers and debris-burners? (g) Of those burned by the incendiarists? (h) What was the total loss caused by these three types of individuals? (i) How many $10,000 houses could have been built with the money value of the forests burned by these three causes?

MAJOR CAUSES OF FOREST FIRES



Courtesy American Forest Products Industries

Fig. 12. Most forest fires could be prevented.

ACHIEVEMENT TEST II

Addition—4 minutes

1.	2.	3.	4.	5.
7	28	247	23.79	$\frac{3}{8}$
6	49	629	.56	$\frac{1}{3}$
8	63	324	24.38	
5	77	482	8.77	
3		138		
9		275		

6. $7.5 + 2.16 + .7$

7. $\frac{3}{5} + \frac{1}{10} + 2$

8. $45\frac{3}{4}$
 $18\frac{1}{2}$

9. $12\frac{1}{6}$
 $7\frac{2}{3}$

10. 4 yd. 2 ft.
 5 yd. 2 ft.

(Achievement Test II—Continued)

Subtraction—4 minutes

11. $\begin{array}{r} 409 \\ \underline{36} \end{array}$ **12.** $\begin{array}{r} 6003 \\ \underline{278} \end{array}$ **13.** $\begin{array}{r} 722.8 \\ \underline{528.2} \end{array}$ **14.** \$3.22 – \$.76

15. \$6 – 47¢

16. $72\frac{1}{4} - 18.5$

17. $\frac{7}{8} - \frac{1}{4}$

18. $\begin{array}{r} 62\frac{7}{10} \\ \underline{50\frac{3}{5}} \end{array}$ **19.** $\begin{array}{r} 84\frac{1}{3} \\ \underline{28\frac{1}{2}} \end{array}$ **20.** $\begin{array}{r} \text{8 hr. 45 min.} \\ \underline{\text{2 hr. 55 min.}} \end{array}$

Multiplication—4 minutes

21. $\begin{array}{r} 72 \\ \underline{45} \end{array}$ **22.** $\begin{array}{r} 306 \\ \underline{408} \end{array}$ **23.** $\begin{array}{r} 240 \\ \underline{90} \end{array}$ **24.** $\begin{array}{r} 9.7 \\ \underline{.06} \end{array}$

25. $\frac{1}{3} \times \frac{1}{3}$

26. $1\frac{1}{2} \times 16$

27. $\frac{1}{5} \times 18\frac{1}{3}$

28. $\frac{3}{4} \times 76$

29. $\begin{array}{r} 36\frac{1}{4} \\ \underline{84\frac{2}{3}} \end{array}$ **30.** $\begin{array}{r} \text{5 qt. 1 pt.} \\ \underline{4} \end{array}$

Division—4 minutes

31. $8\overline{)196}$ **32.** $.16\overline{)28.8}$ **33.** $7.65 \div 10$

34. $4.5\overline{)11.7}$ **35.** $\begin{array}{r} 156 \\ .38\overline{)592.8} \end{array}$ **36.** $\begin{array}{r} 156 \\ 38\overline{)5.928} \end{array}$

37. $\frac{4}{5} \div \frac{3}{10}$ **38.** $8 \div \frac{1}{4}$ **39.** $26\frac{1}{4} \div 4\frac{1}{6}$

40. $3\overline{)\text{4 gal. 1 qt.}}$

Per Cent—4 minutes

Write as common fractions:

41. 24% **42.** $37\frac{1}{2}\%$

Write as decimal fractions:

43. 27% **44.** $3\frac{1}{3}\%$

Write as per cents:

45. $\frac{2}{3}$ **46.** .5

Solve:

47. 30% of 84 = ?

48. 6% of ? = 18

49. ?% of 60 = 24

Arrange in order of size—smallest first:

50. $12\frac{1}{2}\%$, .013, 1.2, $\frac{12}{100}$

Record your scores on the achievement chart.

Which of your scores show an improvement over your scores on Achievement Test I? There will be another chance for improvement of scores next quarter.

Review of the Fundamentals of Arithmetic—4

Addition and Subtraction

Add and prove:

1.	2.	3.	4.	5.	6.	7.
4	37	78	326	8067	\$46.17	\$734.59
8	46	18	714	49	5.64	17.97
6	68	32	528	88593	17.59	5.76
5	77	35	712	97700	6.08	564.56
7	29	12	501	49975	43.58	57.48
	18	31	623			

8. \$87.96
88.09
12.86
76.57

9. $2.6 + 8.06 + 9.066 + 6.6 + 66.6 + .6$

10. $.97 + .875 + .304 + .004 + 4.44 + .87$

Add:

11. $\frac{2}{3} + \frac{3}{4}$

12. $\frac{1}{7} + \frac{2}{3}$

13.	14.	15.	16.	17.
$\frac{1}{2}$	$\frac{5}{12}$	$376\frac{5}{16}$	$9\frac{1}{6}$	$45\frac{5}{8}$
$\frac{5}{8}$	$\frac{1}{4}$	$478\frac{3}{4}$	$31\frac{3}{8}$	$5\frac{5}{24}$
$\frac{3}{4}$	$\frac{1}{3}$		$66\frac{3}{4}$	$6\frac{2}{3}$

18. $107\frac{2}{3} + 26\frac{4}{5} + 17\frac{3}{10}$

19. $83\frac{6}{7} + 98\frac{3}{4} + 75\frac{1}{2}$

20. 18 bu. 1 pk. 2 qt.
13 bu. 2 pk. 3 qt.

Subtract and prove:

21.	22.	23.	24.	25.
40053	75214	56083	369006	5206031
29785	68509	38795	170997	4307945

26.	27.	28.
84.7	78.84	8
9.3	55.16	4.62

29. $73.4 - 36.872 =$

30. $8.7 - .864 =$

Subtract:

31. $\frac{11}{12} - \frac{3}{4}$

32. $\frac{11}{20} - \frac{11}{30}$

33.	34.	35.	36.	37.
83	$40\frac{5}{9}$	$499\frac{3}{8}$	$38\frac{3}{20}$	$468\frac{5}{6}$
$56\frac{5}{18}$	$25\frac{7}{36}$	$307\frac{1}{2}$	$19\frac{9}{10}$	$429\frac{1}{9}$

38. $156\frac{3}{4} - 77.5$

39. $67\frac{3}{8} - 49.125$

40. 18 hr. 10 min. 20 sec.
17 hr. 42 sec.

Review of the Fundamentals of Arithmetic—5

Multiplication and Division

Multiply and prove:

1. 541	2. 960	3. 1607	4. 709	5. 5709
7	39	49	207	56
6. 5.93	7. .978	8. 4.26	9. 1.006	10. 5.063
.29	67	51.4	.209	.46

Multiply:

11. $100 \times .16$
12. 485×10
13. $100 \times .69842$
14. 100×43.855
15. 10×63.89
16. $1000 \times .68\frac{3}{4}$
17. $\frac{5}{9} \times \frac{3}{4} \times \frac{2}{15}$
18. $6\frac{2}{3} \times 3$
19. $\frac{9}{10} \times \frac{2}{5} \times \frac{1}{2}$
20. $7\frac{4}{11} \times 4\frac{8}{9}$
21. $27\frac{4}{5} \times 15$
22. $14\frac{3}{4} \times 24$
23. $15\frac{2}{5} \times 12\frac{1}{2}$
24. $48\frac{5}{6} \times 18\frac{2}{3}$
25. $\frac{5}{8} \times \frac{3}{4} \times \frac{8}{9} \times \frac{7}{15} \times \frac{3}{7} \times 12$
26. 5 T. 280 lb.
8

Divide and prove:

27. $7585 \div 93$
28. $26240 \div 26$
29. $74188 \div 73$
30. $28440 \div 48$
31. $4.5 \div .005$
32. $2.0822 \div 71$
33. $2.9088 \div 3.6$
34. $8.075 \div .95$
35. $62.85 \div 10$
36. $75.6 \div 1000$
37. $.0468 \div 100$
38. $6 \div 1000$

Divide:

39. $\frac{3}{8} \div \frac{5}{9}$
40. $\frac{2}{9} \div \frac{1}{3}$
41. $\frac{5}{6} \div \frac{5}{8}$
42. $12 \div 1\frac{1}{2}$
43. $4\frac{1}{2} \div 2\frac{1}{4}$
44. $14 \div 3\frac{1}{2}$
45. $10\frac{1}{8} \div 6\frac{3}{4}$
46. $9\frac{3}{5} \div 8\frac{4}{7}$
47. $13\frac{5}{7} \div 4\frac{4}{7}$
48. $45\frac{5}{6} \div 9\frac{1}{6}$
49. 5)1 qt. 1 pt.
50. 4)6 mi. 180 rd.

Review of the Fundamentals of Arithmetic—6

Percentage

Express each of the following numbers as (1) a common fraction, (2) a decimal, (3) a percentage:

1. 40%
2. $66\frac{2}{3}\%$
3. 75%
4. $\frac{3}{10}$
5. $\frac{1}{6}$
6. $\frac{1}{2}$
7. $\frac{7}{10}$
8. .09
9. .4
10. 125%
11. .16
12. 38%
13. $\frac{1}{3}$
14. 5%
15. .125
16. $4\frac{1}{2}\%$
17. .016
18. 200%
19. $133\frac{1}{3}\%$
20. $1\frac{1}{2}$

Solve the following:

21. 18% of 5
22. 60% of 30
23. 120% of 25
24. $66\frac{2}{3}$% of 15
25. $1\frac{1}{2}$% of 16
26. 5% of 25
27. 18% of 2,074
28. 7% of 16.3
29. $\frac{1}{2}$% of 6,090
30. $12\frac{1}{2}$% of 608
31. 25% of 1,604
32. 200% of 14.68
33. $4\frac{1}{2}$% of 7.26
34. $3\frac{1}{3}$% of 9.9
35. 5% of .25
36. 18% of 5,847
37. $\frac{3}{4}$% of 6000
38. 1% of 160

Fill in the following:

39. 1% of 19 = ?
40. 1% of ? = 3
41. ?% of \$6 = \$3
42. 3% of 12 = ?
43. ?% of 8 = 8
44. 3% of ? = 15
45. ?% of 12 = 24
46. $4\frac{1}{2}$% of \$80 = ?
47. 12% of ? = 7.2
48. ?% of 40 = 16
49. $\frac{1}{4}$% of 8 = ?
50. 8% of ? = 44.8

Solve the following:

51. 16 is what per cent of 36?
52. 7 is what per cent of 42?
53. What per cent of 16 is 3?
54. 21 is what per cent of 70?
55. 27 is what per cent of 9?
56. What per cent of 24 is 2?
57. 16 is what per cent of 80?
58. What per cent of 25 is 9?

CHAPTER VIII

INSURANCE

VOCABULARY

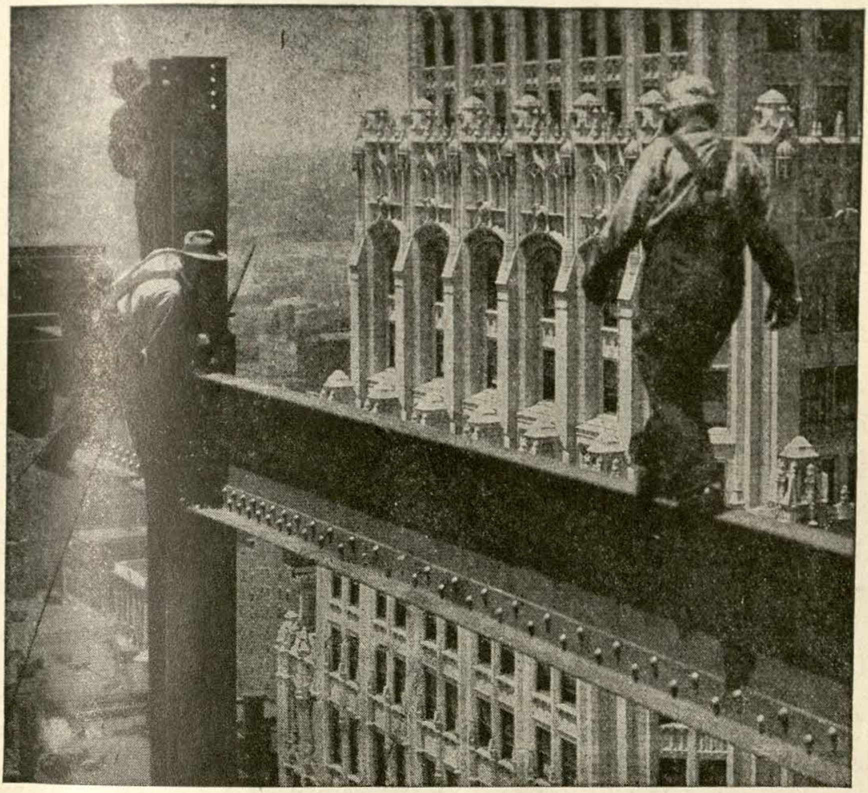
1. insurance
2. insurer
3. insured
4. policy
5. face of policy
6. beneficiary
7. premium
8. dividend
9. participating
10. non-participating
11. annuity
12. social security

71. Need for insurance. FIRE SWEEPS VILLAGE—DIES OF CRASH INJURIES—HAIL DESTROYS WHEAT CROP—ACRES OF FARM LAND FLOODED—RETIRED AT 65—FIFTY EMPLOYEES LOSE JOBS—AUTOMOBILE STOLEN. Such are headlines seen in the daily papers over and over again. How combat the risks of life? How replace the losses sustained?

Nothing can replace such losses as a life, a pair of eyes, or even a treasured family possession. *Insurance* is meant to replace a loss, and in a financial sense it does, in part, at least. A building can be replaced. A sum of money can be paid to the jobless, to the sick, to the retired, and to the family in the loss of a member. Insurance is not a preventative but it is a protection in case of loss or damage.

72. Terms of insurance. The company assuming the risk is called the *insurer*; the person protected is called the *insured*; the contract between the company and the insured is called the *policy*; the amount of the *risk* is called the *face of the policy*; the person to whom the face of the policy is to be paid in case of loss or damage is called the *beneficiary*; the fixed amount of the annual cost as stated in the policy is called the *premium*; and the portion of the profits of the company allotted to the insured is called the *dividend.*

73. Kinds of insurance. Life, accident, health, hospital, fire, marine, and automobile insurance are common forms of protection. Musicians insure their hands; a merchant insures his plate-glass windows; a singer insures her voice; a contractor insures the men working for him; an automobile owner insures his car; a business-



MEN IN DANGEROUS OCCUPATIONS MAKE POOR LIFE INSURANCE RISKS.

man insures his health; and a large circus insures against stormy weather.

74. Life insurance. The number of policyholders in United States life insurance companies has increased very rapidly in the 1900's. In 1925 the policies in force numbered more than 97,000,000. In 1948 the number had jumped to more than 169,000,000 policies.

In life insurance, the insurance company agrees to pay the beneficiary named in the policy a specified sum of money at the time of the death of the insured. In certain policies, if the insured is alive at a time named in the policy, the face of the policy may be paid to him. The insured agrees to pay the yearly premium charged by the insurance company.

The *premium* on all forms of insurance must be paid in advance by the insured to the insurer. The annual premium rate on life insurance varies with the age of the insured, the kind of insurance, the number of annual premiums, and the face of the policy. These

different premium rates are listed in tables, as on page 135. Each premium represents the cost of a $1,000 policy for 1 year. To find the annual cost of a $2,000 policy, multiply the premium quoted by 2.

Investing in life insurance is one of the safest financial investments anyone can make because strict laws control insurance company operations and the insurance departments of all the states supervise them.

75. Dividends. Some insurance companies divide a portion of their net earnings among their policyholders at the close of each year. If the policy states that the insured is entitled to a part of the earnings of the company, it is called a *participating policy*. The amount of the earnings that each insured receives is called his *dividend*. A policy that does not entitle the insured to share in the dividends is a *nonparticipating policy*.

The insured may use his dividend each year in one of the following ways:

(a) The dividend may be presented to the insured as a cash payment.

(b) The dividend may be applied as part payment on the premium due.

(c) The dividend may be applied to the purchase of more insurance.

(d) The dividend may be left with the company to draw interest.

The last method is a form of savings, for nearly all insurance companies pay interest compounded annually. The dividends and the interest may be withdrawn at any time.

EXERCISES

1. A man received the following dividends on his participating policy. Give orally the total amount received for the first and second years, for the second and third years, and so on. 1st yr., $14.70; 2nd yr., $15.43; 3rd yr., $14.56; 4th yr., $15.17; 5th yr., $14.97; 6th yr., $15.49; 7th yr., $14.72; 8th yr., $14.21; 9th yr., $15.06; 10th yr., $14.53.

2. If the dividends of exercise 1 were all paid in cash to the insured, what was the total amount of cash that he received?

3. If the amount of the 5th dividend in exercise 1 is 15% of the amount of his annual premium, what does this man's insurance cost each year?

4. If, in exercise 3, the premium on the man's life insurance is $49.90 annually for each $1,000 of insurance, what is the face of his policy?

5. If the dividends on a policy are $3.38 per $1,000 of insurance for 1 yr., what would be the dividends on an $18,000 policy? On a $4,500 policy? On a $6,700 policy? On a $500 policy?

76. Kinds of life insurance. The three most common forms of life insurance are: *ordinary, limited payment,* and *endowment.* Each type varies slightly with different insurance companies. The insured should select the kind of life insurance that is best suited to his needs.

77. Ordinary life insurance. Originally, ordinary life was the only form of life insurance written. The wage earner wished to provide protection for his family in case of his death.

The annual premium rate for this policy is less than that for any other type of life insurance, but the premium must be paid each year during the entire life of the insured. After the death of the insured, premiums are no longer paid and the face of the policy is paid to the beneficiary.

This type of insurance is especially desirable for a person who has a number of dependents and only a small amount of money available to pay for insurance, as it offers the most protection for the least cost per year.

78. Limited payment life insurance. In limited payment life insurance there is a shortening of the number of premium payments. The insured may not want to contract to pay premiums for life. In that case the insured and the company agree upon a plan of payment. The plan might be for the insured to pay over a period of 10 years, 15 years, 20 years, or some other period of years. If the insured lives beyond that period of years, no more premiums need be paid and the insurance is said to be *paid up.* If the insured dies before the policy is paid up, premium payments stop and the face of the policy is paid to the beneficiary. *Twenty-payment life* means that the premium on the insurance is to be paid each year for 20 years before the insurance is paid up. The greater the number of years before the policy is paid up, the smaller the amount of each annual premium. Why is the annual premium rate on a limited payment life policy more than that on an ordinary life policy?

This kind of insurance is desirable for a person who desires to leave some money to dependents and who wishes to pay his insurance premiums while his earning power is at its peak.

79. Endowment life insurance. Endowment insurance offers protection for others and an investment for the insured. The insured must pay the premiums on an endowment policy for a stated number of years, as in a limited payment policy. An endowment policy states that at the end of this period of years, the insured is to be paid the amount of the face of the policy; but if he should die before the end of this period, the face of the policy is to be paid to his beneficiary. No premiums are paid after the death of the insured in any form of life insurance. While endowment insurance is usually written for a period of 10, 20, or 30 years, it is sometimes written so as to be paid at a stated age; as, for example, an endowment to be paid at age 65 or age 85.

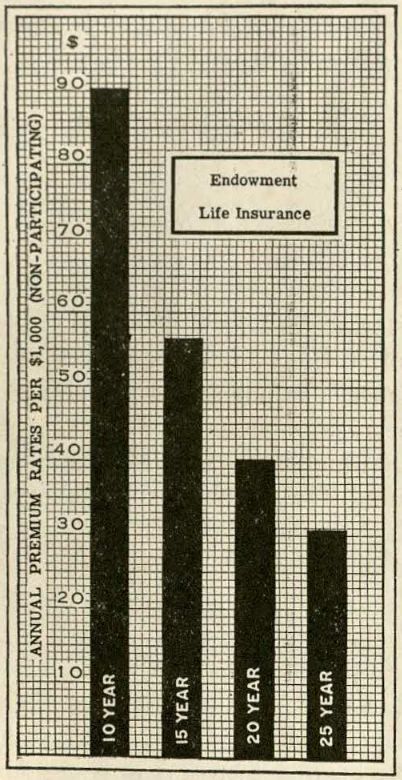


Fig. 1. Annual premium on endowment life insurance at the age of 20 years.

Endowment insurance has the combined advantages of life insurance and a savings account. For this reason, the annual premiums are higher than for other common types of life insurance. Endowment insurance is desirable for a person who wishes to secure protection for his dependents and to provide savings for business deals or support in old age.

Figure 1 illustrates how very much less the annual premiums are when the payments are spread over a greater number of years. (a) The annual premium on a 10-year endowment is approximately how many times as much as on a 25-year endowment? How many times as many payments are made? (b) Make the same comparisons for the 10-year and the 20-year endowment policies. (c) What is the advantage of the 10-year plan? The disadvantage? (d) Of the

25-year plan? (e) At age 20, which of the plans would likely be the best plan to take out? Why? (f) At age 50? Why?

80. Life annuities. There are various types of annuities. However, the chief purpose of each is an assured income for old age. An annuity is an investment for the purchaser.

An *Immediate Life Annuity* is paid for in one lump sum. In return the insurance company starts paying a monthly, quarterly, semiannual, or annual income to the purchaser as long as he lives. There is no beneficiary, as at the death of the purchaser the payments stop.

A *Refund Annuity* pays to a beneficiary any money remaining from the purchase price. It may be paid in a lump sum or in payments.

A *Deferred Life Annuity* may be purchased in one payment or in payments extending over a number of years.

81. Group insurance. Group insurance is a form of ordinary life insurance taken out by a group of workers in the same business organization or in similar occupations. The policy is usually issued without medical examination. The rate of premium for group insurance is less than the regular individual rate. The firm may pay all, part, or not any of each premium. The policy may be canceled if the employee leaves the company; however, an exchange may then be made for an ordinary life or endowment policy with the same insurance company.

EXERCISES

A. Oral

1. Using short cuts, find the total amount of premium for 1 yr. on each of the following:

(a)	\$10,000	policy	@	\$67.00	per	\$1,000
(b)	5,000	"	"	16.00	"	1,000
(c)	8,000	"	"	40.00	"	1,000
(d)	12,000	"	"	25.00	"	1,000
(e)	16,000	"	"	12.50	"	1,000
(f)	12,000	"	"	75.00	"	1,000
(g)	500	"	"	24.00	"	1,000
(h)	10,000	"	"	42.00	"	1,000

B. Written

2. Find the annual premium on each of the following:

(a)	$12,000	policy	@	$56.30	per	$1,000
(b)	7,500	"	"	45.73	"	1,000
(c)	3,500	"	"	78.07	"	1,000
(d)	3,750	"	"	47.54	"	1,000
(e)	7,250	"	"	76.06	"	1,000
(f)	13,750	"	"	50.80	"	1,000
(g)	500	"	"	48.26	"	1,000
(h)	11,000	"	"	51.83	"	1,000
(i)	10,500	"	"	66.78	"	1,000
(j)	25,000	"	"	43.32	"	1,000

3. Complete the following:

	Face of Policy	*Annual Premium*	*Premium Rate per $1,000 Insurance*
(a)	$13,000	$_____	$42.50
(b)	5,000	105.00	_____
(c)	_____	162.00	27.00
(d)	500	_____	68.42
(e)	4,750	231.61	_____
(f)	_____	245.25	32.70

Solve the following by using the table of rates:

4. Find the premiums on a $5,000 ordinary life policy at each of the following ages: 46, 36, 56, 24, 29, 40, 59.

5. Find the premiums on a $5,000 ten-year endowment policy at each of the following ages: 24, 44, 38, 17.

6. Find the annual premiums on a $12,000 policy, limited payment plan, for 10 yr., 15 yr., and 20 yr. at each of the following ages: 38, 18, 57.

7. Find the premium for each plan on a $2,000 limited payment life insurance policy issued to an insured at age 42.

8. Find the premium for each plan on an endowment insurance policy for $5,000 issued to a person 35 years of age.

9. A woman 38 years old took out a 15-year endowment policy. How much would she have saved on the premium each year if she had taken out the insurance one year earlier? What would she have saved on premiums for the 15 years?

10. If you take out a 20-payment life insurance policy at the age of 19, at what age will the insurance be paid up? How much will you pay in premiums during the 20 years on $1,000?

11. At the age of 24, a man took out a 20-year endowment policy for $7,000. He died after he had paid 11 annual premiums. How much had he paid into the company? How much was paid to his beneficiary? How much more did his beneficiary receive than the amount that had been paid into the company?

ANNUAL PREMIUM RATES PER $1,000
(NON-PARTICIPATING)

Age	Ordinary Life, $5,000	Limited Payment Life			Endowment				Term Policies	
		10-Payment	15-Payment	20-Payment	10-Year	15-Year	20-Year	25-Year	10-Year	20-Year
	$	$	$	$	$	$	$	$	$	$
15	......	33.34	24.48	20.16	90.07	56.46	40.09	30.63	.....	.....
16	......	33.82	24.84	20.45	90.09	56.49	40.12	30.68	.....	.....
17	......	34.32	25.21	20.76	90.11	56.52	40.15	30.74	.....	.....
18	......	34.84	25.59	21.08	90.13	56.56	40.19	30.80	.....	.....
19	......	35.38	25.99	21.41	90.15	56.59	40.22	30.87	.....	.....
20	68.85	35.94	26.40	21.76	90.18	56.62	40.26	30.94	9.70	10.12
21	70.40	36.51	26.83	22.11	90.21	56.65	40.30	31.01	9.78	10.22
22	72.05	37.11	27.28	22.48	90.25	56.69	40.35	31.09	9.85	10.34
23	73.75	37.73	27.74	22.87	90.28	56.73	40.41	31.17	9.94	10.47
24	75.50	38.38	28.22	23.27	90.31	56.77	40.46	31.27	10.03	10.60
25	77.40	39.04	28.71	23.68	90.34	56.82	40.51	31.36	10.13	10.75
26	79.40	39.73	29.23	24.12	90.38	56.87	40.57	31.47	10.23	10.92
27	81.45	40.45	29.76	24.57	90.42	56.92	40.64	31.59	10.34	11.09
28	83.65	41.19	30.32	25.03	90.48	56.98	40.72	31.71	10.47	11.29
29	85.95	41.96	30.89	25.52	90.53	57.04	40.80	31.85	10.59	11.52
30	88.40	42.76	31.49	26.02	90.58	57.12	40.89	32.00	10.74	11.75
31	90.95	43.59	32.11	26.55	90.63	57.19	41.00	32.16	10.90	12.03
32	93.65	44.45	32.76	27.10	90.71	57.27	41.11	32.34	11.08	12.34
33	96.50	45.34	33.43	27.67	90.77	57.37	41.25	32.54	11.27	12.68
34	99.55	46.26	34.13	28.27	90.85	57.47	41.42	32.76	11.48	13.07
35	102.75	47.22	34.85	28.89	90.94	57.58	41.61	33.00	11.70	13.50
36	106.10	48.21	35.61	29.54	91.03	57.71	41.82	33.27	11.97	13.98
37	109.70	49.25	36.39	30.23	91.13	57.86	42.05	33.56	12.25	14.52
38	113.50	50.31	37.21	30.94	91.25	58.01	42.30	33.89	12.58	15.13
39	117.50	51.42	38.07	31.69	91.37	58.19	42.59	34.25	12.93	15.79
40	121.80	52.57	38.96	32.47	91.52	58.39	42.90	34.66	13.34	16.54
41	126.30	53.76	39.89	33.30	91.77	58.73	43.25	35.11	13.79	17.37
42	131.15	55.00	40.86	34.17	92.04	59.10	43.64	35.60	14.32	18.29
43	136.30	56.30	41.88	35.09	92.33	59.50	44.08	36.16	14.90	19.32
44	142.10	57.64	42.95	36.06	92.66	59.95	44.56	36.77	15.57	20.47
45	148.25	59.03	44.07	37.09	93.01	60.43	45.10	37.46	16.32	21.72
46	155.10	60.49	45.24	38.17	93.50	60.96	45.70	38.00	17.30	23.30
47	162.50	62.00	46.48	39.32	94.03	61.56	46.46	38.81	18.42	25.03
48	170.30	63.58	47.78	40.55	94.60	62.22	47.30	39.72	19.65	26.96
49	178.70	65.22	49.14	41.84	95.22	62.94	48.23	40.72	21.04	29.09
50	187.55	66.99	50.64	43.27	95.91	63.74	49.25	41.83	22.58	31.42
51	196.95	69.15	52.44	44.99	97.02	64.86	50.52	.....	24.12	33.72
52	207.00	71.40	54.34	46.81	98.20	66.07	51.90	.....	25.82	36.23
53	217.70	73.75	56.35	48.76	99.46	67.40	53.41	.....	27.69	38.94
54	229.10	76.19	58.47	50.85	100.81	68.83	55.08	.....	29.77	41.87
55	241.20	78.76	60.72	53.08	102.26	70.40	56.91	.....	32.07	45.03
56	254.05	81.01	62.78	55.19	103.26	71.73	.....	.....	.....	.....
57	267.75	83.35	64.96	57.43	104.36	73.20	.....	.....	.....	.....
58	282.30	85.80	67.29	59.84	105.59	74.83	.....	.....	.....	.....
59	297.85	88.39	69.76	62.44	106.95	76.64	.....	.....	.....	.....
60	314.40	91.11	72.41	65.24	108.47	78.64	.....	.....	.....	.....

For semi-annual premium, multiply by .52; for quarterly premium, multiply by .265.

Maximum amount carried on one life, $15,000.

Courtesy of Equitable Life Insurance Company.

12. Mr. X has his life insured for $15,000. He is 44 years old and has a 20-year endowment policy. What will the policy cost him for 5 years, beginning at this age? If he pays his premiums semiannually, what is the amount of each installment? If he pays the premiums quarterly, how much is each installment?

13. A girl takes out a $10,000 ordinary life policy at the age of 24. What does the insurance cost her each year? For how long must she pay the premium?

14. $5,000 20-year endowment policies are issued to insureds of ages 15 and 25. What is the difference in the annual premiums on the two policies? What will be the total difference for the 20 years?

82. Health, accident, and hospital insurance. *Health insurance* pays the insured a sum of money for time lost from work on account of sickness. The number of payments made for continued illness, and the amount of each payment, are stated in the policy.

Accident insurance pays a stated sum of money to the insured in case of injury resulting from an accident. In some cases, a fixed sum is paid for the loss of both arms, both legs, or both eyes, or half that amount for the loss of one arm, one leg, or one eye.

Hospital insurance has grown rapidly during the past few years. It offers specified hospital care for a specified number of days.

Health, accident, and hospital insurance are written by a great many different plans. This makes it necessary for the insured to read understandingly his policy coverage. These policies are very often sold as group policies.

83. Workmen's compensation insurance. The laws of practically every state hold an employer liable for the injury or death of an employee resulting from an accident while at work. *Workmen's compensation insurance* protects the employer from injured workmen's lawsuits for damages. The insurance company assumes this risk and pays the injured person a sum of money, usually each week. The amount varies in different states and with the degree of the injury. In case of accidental death, the dependents are paid a sum of money, as in life insurance.

84. Social Security. *Social Security* is a type of insurance sponsored by the Federal Government. It grants old-age pensions and certain benefits for survivors. As first enacted, Social Security covered all employment except those occupations specifically excepted. The exceptions were fairly numerous and were broad in extent. A person who is included in the Social Security plan must obtain a Social Security card which will bear a number that belongs to that

person only. The card and number are to be kept by the employee throughout his working years, as a Social Security account is opened for him by the Federal Government.

In most cases the employer and the employee each pay an equal amount into the Social Security fund of the employee.

In 1950, Congress made several important changes in the Social Security law. The changes include:

(a) **Number of persons covered.** Coverage was broadened considerably by narrowing some of the former exemptions, such as those for domestic servants and agricultural workers, and by extending coverage under certain circumstances to others, like employees of non-profit organizations and to self-employed persons.

(b) **Amount of wages subject to tax.** Beginning in January, 1951, the maximum amount of wages taxable was set at \$3,600 a year. An employee earning \$4,000 pays the tax on \$3,600 of the \$4,000 earned. An employee earning less than \$3,600 pays the tax on his total salary.

The amount of employee taxes due are withheld from the wages of employees. If the employer should fail to withhold the taxes due, the employee is personally responsible for payment of the taxes.

(c) **Tax rate.** Through 1950–1953 the tax rate on wages is $1\frac{1}{2}\%$. Thereafter the rate of tax increases as follows:

1954–1959	2%
1960–1964	$2\frac{1}{2}\%$
1965–1969	3%
1970 and after	$3\frac{1}{4}\%$

Through 1951–1953 a covered self-employed person pays $2\frac{1}{4}\%$ of his *net* income from his business. He does not pay on an income less than \$400 or on the part of his net income that is over \$3,600. After 1953 the rate of tax on such a person increases as follows:

1954–1959	3%
1960–1964	$3\frac{3}{4}\%$
1965–1969	$4\frac{1}{2}\%$
1970 and thereafter	$4\frac{7}{8}\%$

At the age of 65 years a retired employee receives a certain sum each month for the rest of his life. The amount received depends upon his wage record, and in some cases upon the number of years that he has been employed in covered employment. Under certain conditions benefits are also payable to the wife and children of a retired person. After his death his family will continue to receive a sum of money every month.

EXERCISES

1. Jack, an employee who is covered by the Social Security law, earned \$2,400 in 1951. What amount was deducted from his wages to cover his Social Security tax?

$$1\tfrac{1}{2}\% \text{ of } \$2{,}400 = ? \text{ Jack's tax.}$$

2. Mr. Black is employed and covered by the Social Security law. During 1952 his salary was \$4,800. (a) On how much of his salary did Mr. Black pay the Social Security tax? (b) How much was held from Mr. Black's salary for his Social Security tax? (c) How much did his employer contribute toward Mr. Black's Social Security account? (d) What was the total amount added to Mr. Black's Social Security account in 1952?

3. Mr. Cox, an employee, has Social Security coverage. He received a salary of \$5,000 in 1952. (a) How much of his salary was taxable for Social Security? (b) How much did Mr. Cox contribute to his Social Security account in 1952? (c) How much did his employer contribute? (d) Determine the total amount contributed in 1952.

4. John Marks, a student whose earnings are taxable for Social Security, earned \$850 during 1952. How much was added to his Social Security account for the year 1952?

5. If Jane Jackson, an office worker, earns \$2,500 in 1954 and her wages are subject to Social Security tax, how much would be added to her Social Security fund in 1954?

6. Mr. Ault is self-employed (earns no wages) and pays the Social Security tax. His net income for 1952 was \$6,000. Calculate the Social Security tax paid by Mr. Ault in 1952.

7. Mr. Baker, who is self-employed (earns no wages) and is covered by the Social Security law, had a net income of \$7,500 from his shoe shop in 1952. How much did he pay into his Social Security fund in 1952?

8. Mr. Boyd has both self-employment income and a salary subject to Social Security tax. In 1952 his salary amounted to \$1,000 and his net income from his self-employment was \$4,000. Calculate his Social Security tax.

Explanation: No person is taxed on more than \$3,600 for any year. If the salary earned for a year is \$3,600 or more, the tax is all calculated on the salary and the self-employment income is not taxed. If the salary is less than \$3,600, the tax on the salary is figured first; then the tax is found on that part of the self-employment income that is needed to be added to the salary to total \$3,600. If both earnings do not total as much as \$3,600, the tax is calculated on the full amount of both earnings.

Solution: All of Mr. Boyd's salary of \$1,000 must be taxed. As Mr. Boyd has total earnings of more than \$3,600, only part of his self-employment income will be taxed—the part that when added to his \$1,000 salary will total \$3,600.

$1,000 @ $1\frac{1}{2}\%$ = $______ tax on salary
$2,600 @ $2\frac{1}{4}\%$ = $______ tax on income
$______ total tax

9. Mr. Long has both self-employment and wages subject to Social Security tax. If in 1954, Mr. Long's wages are $1,200 and his income from self-employment is $5,000, how much Social Security tax will Mr. Long pay?

10. If in 1960 Carl Case receives a $3,000 salary and a net income of $2,000 from a business, and both earnings are subject to the Social Security tax, figure the amount of Social Security tax to be paid by Mr. Case.

FIRE INSURANCE

85. Fire protection. Fire insurance applies only to damage or destruction of the particular property named in the policy. In case of a fire, the insurance company is not liable for the destruction of furniture, clothes, and so forth, unless they were insured with the building. Fire insurance covers destruction by the fire itself, the damage from water and chemicals, and the breakages made by the firemen while fighting the flames. Unless so stated in the policy, damage by lightning, wind, hail, tornado, earthquake, and crashes into a building by an automobile or airplane is not included in the fire insurance. These and other risks can be covered in the fire insurance policy for a small increase in the premium. A fire insurance policy should be carefully read, as there are many provisions stated in it, some of which might cause the insured to lose his protection in case he was unaware of their inclusion.

The premium on fire insurance is quoted as a certain amount for each $100 of insurance bought, as $1.20 per $100. Sometimes the rate is quoted at a certain per cent of the face of the policy, as $1\frac{1}{4}$ per cent. Fire insurance policies are usually made out for a period of 1, 3, or 5 years. The greater the number of years for which the policy is issued, the less the premium averages per year.

86. Eighty per cent coinsurance clause. To encourage property owners to carry sufficient insurance, fire insurance companies include a coinsurance clause in their policies, the *80 per cent coinsurance clause* being the most common. It provides that the insured shall carry insurance on his property to 80 per cent of its value, and then in case of fire the company will pay the full damage, provided the loss does not exceed the face of the policy.

Examples

(a) Find the amount of fire insurance required on a house valued at $6,000, if the policy is to contain an 80 per cent coinsurance clause.

80% of $6,000 is $4,800, the required insurance.

(b) If the owner of the above property had only $4,000 fire insurance, how much of a $2,400 fire loss would the insurance company pay him?

$\frac{4000}{4800}$, or $\frac{5}{6}$, of $2,400 is $2,000, the amount of loss paid.

EXERCISES

A. Oral

1. Find the amount of insurance carried on the following buildings if each building is insured to 80 per cent of its value: (a) $5,000; (b) $2,500; (c) $10,000; (d) $1,500; (e) $1,200; (f) $50,000; (g) $20,000; (h) $9,000.

B. Written

2. Find the amount of fire insurance on each of the following values of property if the policy in each case contains an 80 per cent coinsurance clause: (a) $4,550; (b) $5,250; (c) $15,000; (d) $32,500; (e) $7,500.

3. Complete the following (each policy contains an 80 per cent coinsurance clause):

	80% of Property Value	*Amount of Insurance*	*Fire Loss*	*Amount of Loss Paid by Insurance Company*
(a)	$8,000	$7,000	$2,400	____
(b)	7,500	4,000	675	____
(c)	3,600	3,600	1,374	____
(d)	5,450	4,500	218	____
(e)	2,680	1,340	769	____
(f)	6,675	4,550	1,335	____
(g)	9,750	8,425	3,900	____
(h)	9,680	9,680	1,210	____
(i)	8,740	5,250	6,000	____
(j)	6,800	4,600	697	____

4. Complete the following:

	Value of Property	*Amount of Insurance*	*Per Cent of Insurance*
(a)	$13,500	$____	$33\frac{1}{3}$
(b)	8,000	3,000	____
(c)	5,500	4,400	____
(d)	4,800	____	$62\frac{1}{2}$
(e)	____	10,500	75
(f)	____	7,500	50

5. What is the amount of the premium on a house valued at $8,500, insured for $\frac{1}{2}$ its value at 50¢ per $100?

6. What would be the loss to the owner if a fire damaged the above house to the amount of $4,000, and the policy contained an 80 per cent coinsurance clause?

7. A farmer had his property insured as follows:

House insured for $4,500 at 47¢ per $100
Barn " " $3,200 " 97¢ " $100

Find the total cost of the insurance for the house and barn.

8. A merchant took out insurance on his store building, valued at $18,000, and on his stock of goods, valued at $7,450. What is the amount of insurance carried on each if both are insured to the full extent of the 80 per cent coinsurance clause?

9. If the fire insurance premium on the store in exercise 8 is 69¢ per $100, and that on the stock $1.12 per $100, what is the cost of the insurance? If a fire damages the store to the extent of $5,750, and the stock to the extent of $4,500, what settlement will be made with the merchant?

10. What is the premium for 1 yr. on a $6,400 fire insurance policy, if the rate is $1\frac{1}{8}$ per cent of the face of the policy?

11. Tornado insurance on a garage costs $1.65 per $100 annually. What is the amount of annual premium on such a policy for $12,500?

Fire insurance is usually paid in advance for a term of years. The amount of the premium for a 3-year policy is usually $2\frac{1}{2}$ times the premium for a 1-year policy; the premium on a 5-year policy is usually 4 times the premium for a 1-year policy.

12. What is the premium for (a) a 3-year policy, (b) a 5-year policy, if the rate on a 1-year policy is 34¢? 62¢? $1.14? $1.46?

13. What is the premium for (a) a 3-year policy, (b) a 5-year policy, if the premium on a 1-year policy is $14.42? $18.68? $39.80? $126.14?

14. (OPTIONAL) Complete the following (each policy contains an 80 per cent coinsurance clause):

	Value of Property	*80% of Property Value*	*Amount of Insurance*	*Fire Loss*	*Amount Paid by Insurance Company*
(a)	$10,000	_____	$7,000	$240	_____
(b)	5,000	_____	3,200	1,495	_____
(c)	8,000	_____	6,400	1,760	_____
(d)	15,000	_____	8,500	480	_____
(e)	1,500	_____	750	96	_____
(f)	9,000	_____	7,200	1,260	_____

15. (OPTIONAL) Find the premiums on each of the following buildings insured to 80 per cent of their valuation:

Type of Building	*Valuation*	*Annual Premium per $100 Insurance*	*Annual Premium*	*3-Year Premium*	*5-Year Premium*
House, wood........	$4,000	$.48	$——	$——	$——
stone........	6,000	.38	——	——	——
brick	5,000	.39	——	——	——
Store, wood........	15,000	1.16	——	——	——
stone	20,000	.98	——	——	——
brick	18,000	1.05	——	——	——

87. Automobile insurance. The wise automobile owner protects himself against the risk of accident and theft by purchasing automobile insurance. He may purchase the following kinds of protection:

(a) *Liability Insurance.* If his automobile kills or injures another person, the automobile owner is protected up to the amount stated in his policy.

(b) *Property Damage Insurance.* If his automobile damages property, such as another automobile, a building, or a wall, he is protected up to the amount stated in his policy.

(c) *Fire Insurance.* If his automobile catches fire, he is protected for the fire loss.

(d) *Theft Insurance.* If his car is stolen, he is protected for the loss.

(e) *Collision Insurance.* If his own car is damaged in a collision, he is protected for the amount of damage to the car.

The rates for the different kinds of automobile insurance vary with the size of the city, the number of past accidents in the locality, and the horsepower of the car. Why should the various premiums on automobile insurance be higher in a city than in a small town? Why should the number of past automobile accidents in a locality affect the rates of insurance? In what way should the horsepower of an automobile affect the rates of insurance? Which kinds of automobile insurance would have a higher rate on an old car? Why? On a new car? Why?

EXERCISES

1. Jack Glenn insured his automobile against fire for $1,800 at the rate of 45¢ per $100 per year. How much did the insurance cost him?

2. Ben Craig took out automobile insurance against the following risks: (a) fire — $1,500, at 35¢ per $100; (b) theft — $1,500, at $1.10 per $100; (c) property damage — $1,200, at $22.50 per $1,000. Find the total premiums.

3. Harold Cooper bought a new car for $2,500. He insured it for 90 per cent of its value against fire at 40¢ per $100 and against theft at $1.20 per $100. Property damage cost him $33.75, collision insurance cost him $48 and liability insurance cost $36. Find the total of his insurance premiums.

4. Dick Wylie bought a used automobile for $1,200 and insured it for $\frac{3}{5}$ of its value. The liability insurance cost him $58, fire insurance cost $1.08 per $100, and the property damage cost him $8.64. What did his insurance cost him per year?

5. What does the insurance on Dick Wylie's car cost on the average per month?

SUMMARY QUESTIONS

1. Why is it better to take out life insurance in youth?

2. Why is the annual premium on a 10-year endowment policy higher than on a 20-year endowment policy?

3. Why is ordinary life the cheapest form per year of life insurance?

4. What is the effect of the number of premiums to be paid on the amount of each premium?

5. Why should the premium be paid annually, if possible, instead of quarterly?

6. Compare ordinary life, limited payment life, and endowment policies as to: (a) annual cost per $1,000, (b) total number of payments to be made, (c) time of settlement by the insurance company.

7. Why are the rates on health, accident, and hospitalization insurance lower than the rates on life insurance?

8. What are some outstanding advantages of group insurance?

9. What is an advantage of a monthly income settlement over a lump-sum settlement from an insurance company? What disadvantage?

10. Whom does workmen's compensation insurance cover?

11. What damage does fire insurance cover?

12. Explain the 80 per cent coinsurance clause.

13. Fire insurance on a garage is more expensive than fire insurance in the same amount on a house. Why?

14. Would a house with wooden shingles have a higher or lower fire insurance rate than the same house with composition shingles? Why?

15. List five conditions that cause fire insurance rates to be lower.

16. List five conditions that cause fire insurance rates to be higher.

17. Which kinds of automobile insurance protect:
   (a) The owner's car?
   (b) The other fellow's car or his other property?

CHAPTER IX

BANKS, SAVINGS, AND INVESTMENTS

VOCABULARY

1. commercial	5. depositor	9. payee
2. security	6. currency	10. indorsement
3. signature	7. drawer	11. balance
4. deposit	8. drawee	12. reconciliation

88. Banks. What is a safe place to keep money? Many people, to their sorrow, have found that money is not safe hidden in the home. Most citizens have come to believe in the safety of banks. Especially is this true since the Federal Deposit Insurance Corporation created by Congress in 1934 now protects each bank depositor against loss of deposits up to $10,000 in banks which are members of the Corporation. Banks perform a great variety of useful and necessary services for the community. Modern banks are highly organized and highly specialized for the purpose of selling to you and to me but one article—financial service.

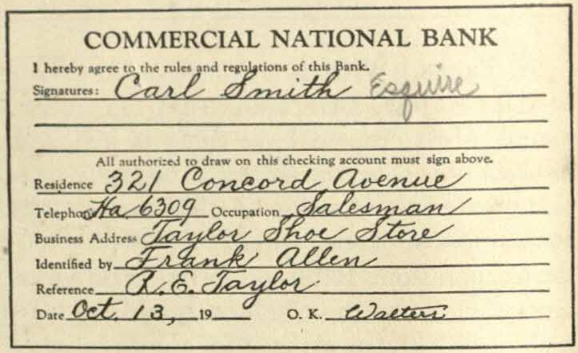
89. Commercial banks. The commercial bank is the most common form of bank. Its services are as follows: (a) to receive money for deposit from its customers, and to pay back the money on demand, (b) to make loans on good security, (c) to buy mortgages, notes, and other business papers, (d) to advise customers regarding investment of their money, (e) to collect money due on papers deposited for collection, and (f) to rent safe-deposit boxes.

90. Opening a checking account. Checking accounts are opened in commercial banks. Before the first deposit can be made in a commercial bank, an introduction to one of the bank officials is necessary. If one is not well known in the community, the banker may ask for several references. By requiring this introduction, the bank is enabled to reject the accounts of undesirable persons.

After filling out a signature card, the customer is ready to make his first deposit and to receive a checkbook.

At least $50 is necessary to open a checking account in most small banks, while in larger banks a minimum of $100 is required. The customer is expected to keep the average of his account equal to or more than the minimum sum. If in any month the average of the customer's account falls below the required minimum, many banks charge a fee for that month.

91. Signature card. Every pupil, early in his school life, should adopt a definite way of writing his name and thus establish a signature for himself. Some persons prefer to run the letters of their names together, making them unreadable. In this way they hope to prevent forgery, but a well-written, legible signature is considered safer by banks.



COMMERCIAL NATIONAL BANK

I hereby agree to the rules and regulations of this Bank.

Signatures: Carl Smith Esquire

All authorized to draw on this checking account must sign above.

Residence 321 Concord Avenue

Telephone Ha 6309 Occupation Salesman

Business Address Taylor Shoe Store

Identified by Frank Allen

Reference A. E. Taylor

Date Oct. 13, 19__ O. K. Walters

Fig. 1.
Signature card.

The bank, in order to know how the depositor always signs his name, must have a copy of his signature. This is written on a *signature card*, which remains in the bank to be used in the future for comparison with the signatures on checks and other business papers handled by the bank. It is made out at the time of the introduction to the bank officials. (See figure 1.)

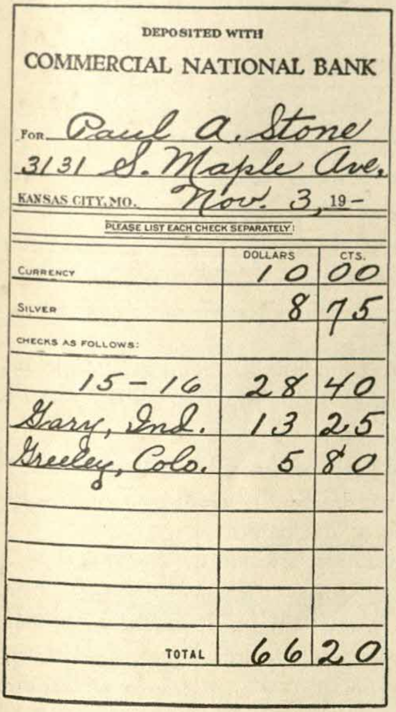
92. Deposit tickets. A deposit ticket is a blank form provided by the bank to be filled out by the depositor when making a deposit. Deposit tickets vary in details, but the name and address at the top of the ticket should always be written in the same way as on the signature card. *Currency* means paper money. *Silver* includes all metal money. The amount of each check must be listed separately on the deposit ticket.

Banks differ as to the method of listing checks on deposit tickets. Figures 2 and 3 illustrate two methods in common use. In figure 2 the number of the local bank and the town in which the out-of-town

banks are located are written directly to the left of the amount of the check. Numbers are assigned to each bank by the American Bankers' Association. The number of the bank is found on the face of the check. Each number consists of two parts separated by a dash, as *15—8*. This number indicates the eighth bank in the fifteenth banking district. No two banks have the same number.

For an out-of-town bank, list on the deposit ticket the name of the town; and for a local bank, list either the name of the bank or the number of the bank.

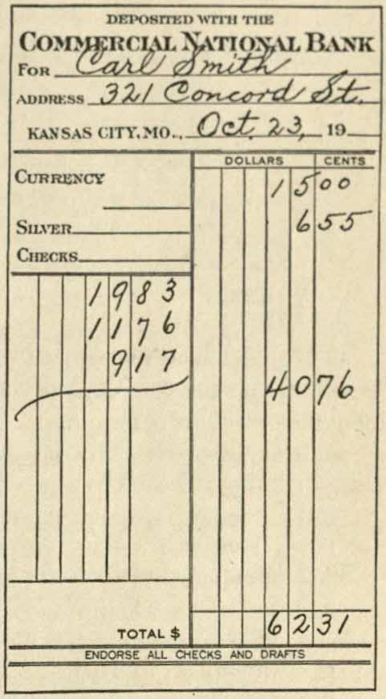
Other banks prefer the listing merely of the amount of the check, as in figure 3. The deposit tickets of such banks have a separate column for dollars and cents on the left side of the deposit ticket below the word *Checks*; the amounts of the various checks are



DEPOSITED WITH
COMMERCIAL NATIONAL BANK
For Paul A. Stone
3131 S. Maple Ave.
KANSAS CITY, MO. Nov. 3, 19-
PLEASE LIST EACH CHECK SEPARATELY

	DOLLARS	CTS.
CURRENCY	10	00
SILVER	8	75
CHECKS AS FOLLOWS:		
15-16	28	40
Gary, Ind.	13	25
Greeley, Colo.	5	80
TOTAL	66	20

Fig. 2. Deposit ticket.



DEPOSITED WITH THE
COMMERCIAL NATIONAL BANK
For Carl Smith
Address 321 Concord St.
KANSAS CITY, MO., Oct. 23, 19

			DOLLARS	CENTS
CURRENCY			15	00
SILVER			6	55
CHECKS				
	19	83		
	11	76		
	9	17	40	76
TOTAL $			62	31

ENDORSE ALL CHECKS AND DRAFTS

Fig. 3. Deposit ticket.

entered in these columns. The amounts of the checks are totaled, and the total is placed in the dollars and cents columns on the right side of the deposit ticket, under the totals of the other items.

The depositor should learn the bank's wishes in regard to the listing of checks on the deposit ticket.

To determine the amount of the deposit, the currency, silver, and check totals are added, and the sum is then placed at the bottom of the deposit ticket.

EXERCISES

A. Oral

1. Read as quickly as possible the total for each of the following lists of deposits:

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)	(k)
$9	$7	$8	$12	$8	$13	$16	$18	$20	$11	$9
6	3	9	10	11	4	5	9	4	5	8
4	6	7	8	9	10	14	3	6	6	13
7	7	7	9	2	8	7	15	14	8	9
8	9	9	9	8	7	9	9	7	23	9

2. Add quickly each of the following lists of deposits:

	(a)	(b)	(c)	(d)	(e)	(f)	
(a)	$8	$7	$5	$4	$6	$3	= ———
(b)	9	6	7	3	5	4	= ———
(c)	6	8	4	9	7	5	= ———
(d)	7	9	3	1	8	6	= ———
(e)	5	4	9	7	4	8	= ———
(f)	4	5	8	6	9	7	= ———

B. Written

3. The following change was taken to the bank: 56 pennies, 24 nickels, 17 dimes, 11 quarters, 13 half-dollars, and 3 dollar bills. What was the total amount deposited?

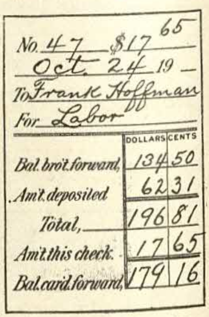
4. Mary deposited $13.75 a week for 47 weeks. What was her total deposit during this time?

5. Tom made the following deposits at his bank: $18.96 in coins, $37.00 in currency, and $59.37 in checks. What was his total deposit?

6. There was a balance of $437.98 in Carl's bank account before deposits of $68.45, $216.95, $75.65, and $168.87 were made. What is the new bank balance?

7. Checks for the following amounts were deposited in the bank: $216.08, $21.19, $185.37, $4.65, $81.43, $538.96, and $67.95. Find the total amount deposited.

8. To a balance of $176.56 in the bank, deposits of $264.39, $47.98, $127.49, and $63.87 are added. What is the new bank balance?


No. 47 $17.65
Oct. 24 19—
To Frank Hoffman
For Labor

	DOLLARS	CENTS
Bal. bro't forward	134	50
Am't deposited	62	31
Total	196	81
Am't this check	17	65
Bal. car'd forward	179	16

Fig. 4. The stub record of the check.

9. Find the total of the following items:

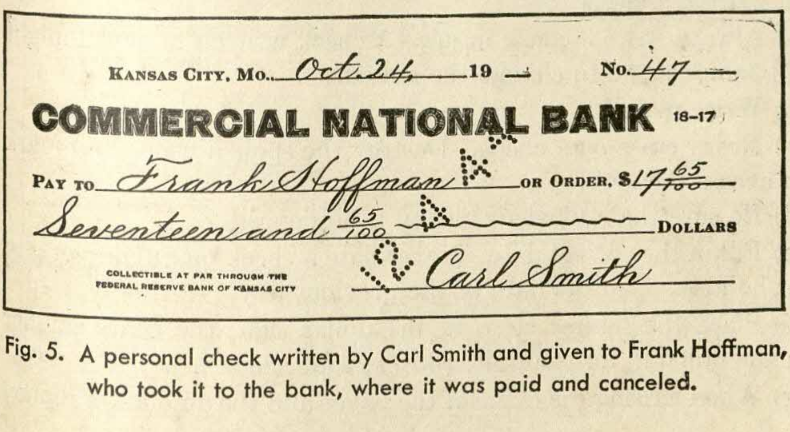
Silver	$65.18
Currency	266.00
Checks:	
Farmers Bank	8.65
Seattle, Wash.	17.88
Chicago, Ill.	323.57

10. Total the following items:

Silver	$56.91
Large Notes	175.00
Small Notes	16.00
Checks:	
10–8	447.62
10–12	9.30
10–15	26.71

93. Checkbook. A *checkbook* is issued to the depositor at the time that he opens his checking account in the bank, and also at any later time that he needs more checks. Each checkbook consists of a certain number of checks and an equal number of check stubs. The checks are perforated so that they can be easily torn out. The stubs remain permanently in the checkbook. The checks and stubs should be numbered consecutively, and the number on each check should correspond with the number on its stub.

94. Checkbook stub. The stub of each check should be filled out carefully, as it is the depositor's record of the number of the check, the date, the amount, to whom it was paid, and for what it was paid. The arrangement of the items on a check stub may vary. Figure 4 shows an arrangement in common use. The check stub should show at any time the exact amount of money in the bank.


KANSAS CITY, MO. Oct. 24, 19 — No. 47

COMMERCIAL NATIONAL BANK 18-17

PAY TO Frank Hoffman OR ORDER. $17 65/100

Seventeen and 65/100 DOLLARS

COLLECTIBLE AT PAR THROUGH THE FEDERAL RESERVE BANK OF KANSAS CITY

Carl Smith

Fig. 5. A personal check written by Carl Smith and given to Frank Hoffman, who took it to the bank, where it was paid and canceled.

The check stub should be filled out before the check is written, for then: (a) the writer of the check is less likely to make a mistake when writing the check, (b) he is less likely to write a check for a sum greater than the amount of money that he has in the bank, and (c) he is assured of a record of each check written.

EXERCISES

1. If the balance after the 10th check is written is \$345.97, what will be the balance carried forward to the 11th stub?

2. If the balance on the stub is \$345.97, and \$399.89 is deposited, what is the total in the bank?

3. If the balance on the stub is \$164.82, and a check is written for \$73.95, what is the new balance?

4. James Brown's checkbook showed a balance of \$350.62. He deposited \$46.62. He then wrote a check for \$209.16. What was the new balance?

5. If the balance on a stub is \$161.96, a deposit of \$74.28 is made, and then a check is written for \$66.76, how much is left in the bank?

6. The balance after the 36th check is written is \$175.20. If a deposit of \$37.89 is entered on the stub of the 37th check and the amount of the 37th check is \$57.37, what is the balance carried forward to the 38th stub?

95. Personal check. Figure 5 illustrates a *personal check.* Carl Smith, the person whose signature appears at the bottom of the check, is the *drawer*; the Commercial National Bank, the bank that is to pay the amount of the check, is the *drawee*; and Frank Hoffman, the one to whom the money is to be paid, is the *payee.*

96. Writing the check. The following cautions should be observed when writing a check.

(a) Always write a check in ink. A check written in pencil might tempt some person to change the amount.

(b) Write plainly.

(c) Never erase on a check. Destroy the spoiled check and mark *Void* across the stub.

(d) Be sure that sufficient money is on deposit.

(e) Fill in the correct date. Never date a check on Sunday or any other holiday. Use the date of the previous day.

(f) Place the figures close to the dollar sign, the cents smaller than the dollars; for example, $\$35\frac{45}{100}$, $\$35^{\underline{\underline{45}}}$, $\$35/^{45}$.

(g) When writing the name of the payee and the amount of money

in words, begin as close as possible to the left edge of the check. Draw in a wavy line to fill any remaining space on the line.

(h) When possible, stamp the amount on the check with one of the patented machines designed for this purpose.

(i) Write the signature exactly as written on the signature card on file in the bank. The bank will probably not cash the check if the signatures are not similar.

(j) Never place the signature on a blank check ahead of the time that the check is to be written.

97. Indorsing the check. Figure 5 is a check which belongs to Frank Hoffman; it was made out to him by Carl Smith. Before Frank Hoffman can collect any money on the check, he must indorse it. His check then becomes the property of the person to whom he gives it. Frank Hoffman may write any one of several forms of indorsement on the check.

Three of the most common forms of indorsement are shown in figure 6: (a) a blank indorsement, (b) a full indorsement, and (c) a restrictive indorsement.

If Frank Hoffman writes only his name on the back of the check, he does not specify to whom payment is to be made. He merely releases his ownership to anyone who may have possession of the check. This is a *blank indorsement*, as illustrated in figure 6A. If Frank Hoffman lost this check with a blank indorsement on it, the finder might try to cash it, since there is no stated owner in the indorsement. Frank Hoffman should immediately report the loss to the drawer of the check, who should ask the bank on which it is written to *Stop Payment*.

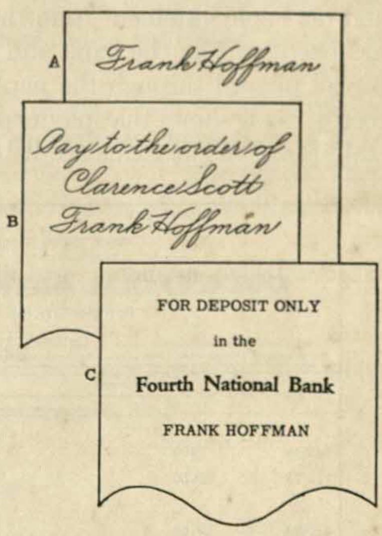


Fig. 6. Three common forms of indorsement.

If Frank Hoffman writes *Pay to the order of* and then writes the name of the party to whom he is giving the check, he thereby specifies to whom payment is to be made. By placing his signature under the name of the party, he releases his ownership of the check.

This is a *full indorsement,* as illustrated in figure 6B. To whom does Frank Hoffman release his ownership in this figure? A full indorsement is safer than a blank indorsement and should always be used when a check is being transferred from one person to another.

If Frank Hoffman wishes to deposit the check, he can protect himself by indorsing it as in figure 6C. Such an indorsement is known as a *restrictive indorsement.* If lost, the check would be of no value to the finder, since the indorsement shows that the check is to be deposited. Hence, it cannot be cashed by anyone.

An indorsement is placed across the back of the check near the stub end. The payee must write his name in the indorsement exactly as it was written on the face of the check by the drawer. Even though his name was misspelled on the check, the payee must copy the spelling and place below this name his signature correctly spelled.

98. Bank statement. The *bank statement* is issued the first day of the month. At that time the depositor should get his canceled checks and his bank statement from the bank. A *canceled check* is one that has been paid by the bank and has the word *Paid* and the date received pierced through the paper. A canceled check makes a good receipt, as it shows that payment has been made. Figure 7 is a bank statement. What items are listed on the bank statement?

Carl Smith
321 Concord Street
Kansas City, Missouri.
IN ACCOUNT WITH

COMMERCIAL NATIONAL BANK

KANSAS CITY, MO.

Statement of your Account to close of business October 31, 19—.

PLEASE EXAMINE AT ONCE. IF NO ERROR IS REPORTED IN TEN DAYS THE ACCOUNT WILL BE CONSIDERED CORRECT.

DATE	CHECKS IN DETAIL		DATE	DEPOSITS	DATE	BALANCE
	BALANCE BROUGHT FORWARD					
			Oct 13	150.00	Oct 13	150.00
Oct 14	5.00				Oct 14	145.00
Oct 17	10.50				Oct 17	134.50
			Oct 23	62.31	Oct 23	196.81
Oct 24	17.65				Oct 24	179.16
Oct 31	15.00	6.00			Oct 31	159.16

USE YOUR PASS BOOK ONLY AS A RECEIPT BOOK WHEN MAKING DEPOSITS

THE LAST AMOUNT IN THIS COLUMN IS YOUR BALANCE.

Fig. 7. Bank statement of Carl Smith's account.

99. Reconciliation. The stub of the checkbook, if properly kept, shows the correct balance of money in the bank. The bank statement also shows the correct balance in the bank. However, many times these two balances do not agree. The question then is, which one is correct? Both may be right or both may be wrong. The process by which the stub balance and the bank statement balance are checked against one another is called *reconciliation.*

To ascertain the correct balance, proceed as follows:

(a) See that the stubs contain every deposit listed on the bank statement. *Add any deposits not already added on stubs.*

(b) Look for penalties, such as fines or fees, that may have been subtracted on the bank statement but not on the stub balance. *Subtract penalties from the stub balance.*

(c) Arrange the canceled checks according to the number of each check, the first number on top. Compare every check with the corresponding stub. If each check and stub agree, place a large *O. K.* on the check and on the stub. There may be checks for which there are no stubs. This may happen, for instance, when a counter check has been written at the bank. *Subtract checks not already subtracted from the stub balance.*

(d) List the numbers and amounts of any stubs for which there are no canceled checks. Those checks have not been turned in to the bank for payment; hence the bank has not subtracted them from the balance. *Subtract these checks outstanding from the bank statement.*

(e) If the two balances still do not agree, prove each addition and subtraction on the stubs.

Here is an example showing how a reconciliation is made. (The letters in parentheses refer to the preceding rules.)

SEPTEMBER 1

A. Stub balance		$417.47
(a) Deposit not added on stubs		25.00
		$442.47
(b) Fines not subtracted on stubs		1.00
		$441.47
(c) Checks not subtracted on stubs	$21.50	
	36.75	
	28.14	
	$86.39	86.39
Correct balance		$355.08

B. Bank statement balance			$531.58
(d) Checks outstanding	#18	$106.50	
	#20	70.00	
		$176.50	176.50
Correct balance			$355.08

Hence the correct balance in the bank is $355.08.

EXERCISES

Make a reconciliation for each of the following:

1. On January 1 Jay Ward's account stood as follows: stub balance, $316.09; bank statement balance, $755.74; deposits not recorded on stubs, $96.69 and $61.78; checks outstanding, #64 for $21.48, #68 for $67.03, #75 for $188.17, and #76 for $4.50.

2. On February 1 John Webb's account stood as follows: bank statement balance, $2,762.36; checks outstanding, #84 for $416.25, #93 for $26.03, #94 for $289.67; fines, $1.00; stub balance, $1,963.27; deposit missing on stub, $114.89; check with no stub, $46.75.

3. Your stub balance is $161.42; outstanding checks, #16 for $18.75 and #20 for $4.82; bank statement balance, $223.79; fine, $1.00; checks not subtracted, #12 for $6.15 and #14 for $10.80; deposit not added on stub, $56.75.

4. On April 1 Joan Parmer's bank account was as follows: stub balance, $318.76; bank statement balance, $320.69; checks not subtracted on stubs, $12.80 and $7.25; outstanding checks, #12 for $10.40, #13 for $4.38, and #15 for $7.20.

5. On May 1 Carl Martin's bank account was: bank statement balance, $769.77; stub balance, $739.26; checks outstanding, #20 for $15.35, #22 for $12.18, and #25 for $20.50; checks not subtracted on stubs, $5.12 and $12.40.

6. Mary Clark's bank account on March 1 was: stub balance, $324.80; bank statement balance, $374.60; deposit not added on stub, $45.12; checks not recorded on stubs, $18.25, $3.78, and $10.24; checks outstanding, #31 for $12.40, #33 for $18.20, and #34 for $6.35.

7. On July 1 Henry Hunter's bank account was: checks outstanding, #42 for $18.75, #43 for $12.92, and #44 for $6.90; deposits not recorded on stubs, $41.20 and $32.45; stub balance, $125.66; bank statement balance, $237.88.

8. Bert Casper's bank account on June 1 was: stub balance, $251.18; bank statement balance, $270.84; deposit not recorded on stub, $18.30; checks not recorded on stubs, $15.80 and $9.36; outstanding checks, $10.00, $8.64, and $7.88.

9. (OPTIONAL) Outstanding checks, #49 for \$17.80, #51 for \$26.00, #53 for \$5.00, and #54 for \$7.75; fines, \$1.00, 25¢, and 25¢; stub balance, \$68.72; bank statement balance, \$138.17; deposit not added on stub, \$14.40.

10. (OPTIONAL) Bank statement balance, \$73.04; check not subtracted on stub, \$50.26; deposits not added, \$25.20, \$17.80, and \$36.19; stub balance, \$44.11.

100. Interest. The sum of money lent or borrowed is the principal (p); the money paid for the use of the principal is the interest (i); the percentage of principal that is to be paid as interest is the rate (r); the period allowed for the repayment of the principal is the time (t); and the sum of the principal and interest is the amount (a).

To find the interest, use the following formula:

$$\text{Interest} = \text{principal} \times \text{rate} \times \text{time (in years)}.$$

The formula is written: $i = p \times r \times t$. The time (t) may be counted 365 days to the year or 360 days to the year. Banks count 360 days when computing simple interest. In either case, the formula applies.

Notice that when counting 360 days to the year, the interest on \$500 at 6% for 60 days is \$5. A short cut in computing interest is the "60-day" or "6%" method. If you are told that the interest on

Examples

Find the interest on \$500 at 6% for 60 days.

(1) 365 days to the year:

$i = p \times r \times t$

$$i = \$\overset{5}{\cancel{500}} \times \frac{6}{\underset{1}{\cancel{100}}} \times \frac{\overset{12}{\cancel{60}}}{\underset{73}{\cancel{365}}} = \frac{360}{73} = \$4.93$$

(2) 360 days to the year:

$i = p \times r \times t$

$$i = \$\overset{5}{\cancel{500}} \times \frac{\overset{1}{\cancel{6}}}{\underset{1}{\cancel{100}}} \times \frac{\overset{1}{\cancel{60}}}{\underset{\underset{1}{\cancel{60}}}{\cancel{360}}} = \$5$$

\$100	for 60 da. at 6% is	\$1.00
\$200	" " " " " "	\$2.00
\$420	" " " " " "	\$4.20
\$1,800	" " " " " "	\$18.00

can you discover the short cut?

Rule. *To find the interest for 60 days at 6 per cent, move the decimal point in the principal 2 places to the left.*

To find the interest for 30 days at 6 per cent, divide the interest for 60 days by 2.

EXERCISES

1. Using the 60-day method, find the interest on each of the following loans at 6%; give the answers at a glance:

(a) $300 for 60 da.	(d) $ 22 for 30 da.	(g) $480 for 15 da.
(b) 460 “ 30 “	(e) 717 “ 60 “	(h) 60 “ 30 “
(c) 655 “ 60 “	(f) 890 “ 30 “	(i) 180 “ 15 “

2. Using the 6% method, find orally the interest at 6% on each of the following:

(a) $600 for 60 da.	(g) $550 for 60 da.	(m) $1500 for 60 da.
(b) 600 “ 30 “	(h) 550 “ 15 “	(n) 1500 “ 15 “
(c) 600 “ 90 “	(i) 550 “ 75 “	(o) 1500 “ 75 “
(d) 300 “ 60 “	(j) 120 “ 60 “	(p) 3600 “ 60 “
(e) 300 “ 10 “	(k) 120 “ 10 “	(q) 3600 “ 10 “
(f) 300 “ 70 “	(l) 120 “ 70 “	(r) 3600 “ 70 “

3. Find orally the interest at 6% on each of the following:

(a) $ 480 for 60 da.	(e) $660 for 10 da.	(i) $ 132 for 10 da.
(b) 480 “ 10 “	(f) 66 “ 10 “	(j) 96 “ 10 “
(c) 7200 “ 10 “	(g) 360 “ 10 “	(k) 108 “ 10 “
(d) 1200 “ 10 “	(h) 240 “ 10 “	(l) 1080 “ 10 “

4. Find orally the interest on each of the following at 6%:

(a) $ 900 for 60 da.	(f) $ 420 for 20 da.	(k) $2100 for 20 da.
(b) 900 “ 20 “	(g) 1800 “ 20 “	(l) 93 “ 20 “
(c) 33 “ 20 “	(h) 240 “ 20 “	(m) 210 “ 20 “
(d) 48 “ 20 “	(i) 18 “ 20 “	(n) 390 “ 20 “
(e) 15,000 “ 20 “	(j) 144 “ 20 “	(o) 21 “ 20 “

5. (OPTIONAL) Find orally the interest for 60 days on each of the following:

(a) $600 at 6%	(e) $240 at 3%	(i) $666 at 1%	(m) $96 at 6%
(b) 600 “ 3%	(f) 240 “ 1%	(j) 36 “ 6%	(n) 96 “ 3%
(c) 600 “ 1%	(g) 666 “ 6%	(k) 36 “ 3%	(o) 96 “ 2%
(d) 240 “ 6%	(h) 666 “ 3%	(l) 36 “ 2%	(p) 96 “ 1%

6. Find the interest on each of the following at 6% for 60 days, 30 days, 10 days, 5 days, and 1 day:

(a) $7,200, (b) $240, (c) $360, (d) $480, (e) $1,080, (f) $120.

7. Find the interest at 6% on (See method on next page.):

(a) $960 for 80 da.	(f) $1600 for 88 da.
(b) 720 “ 75 “	(g) 420 “ 36 “
(c) 425 “ 72 “	(h) 487 “ 120 “
(d) 848 “ 96 “	(i) 3648 “ 150 “
(e) 870 “ 100 “	(j) 480 “ 42 “

8. Find the interest on \$840 at 6% for 88 days.

Method

Interest for 60 da. =	\$8.40	
" " 20 da. =	2.80	($\frac{1}{3}$ of 60-da. int.)
" " 6 da. =	.84	($\frac{1}{10}$ of 60-da. int.)
" " 2 da. =	.28	($\frac{1}{3}$ of 6-da. int.)
Interest for 88 da. =	\$12.32	

If the rate is not 6%, but 5% for instance, first find the interest at 6% and then take $\frac{5}{6}$ of the result.

9. Using the 6% method, find the interest on:

(a)	\$ 420	at 4%	for 90 da.	(f)	\$ 387	at 8%	for 180 da.
(b)	450	" 4%	" 72 "	(g)	456	" 3%	" 240 "
(c)	1200	" 5%	" 90 "	(h)	48.60	" 3%	" 90 "
(d)	3600	" 8%	" 88 "	(i)	672	" 8%	" 190 "
(e)	720	" 7%	" 6 "	(j)	1260	" 4%	" 75 "

101. Simple interest graph. Figure 8 is a graph by which the annual interest at various rates may be determined. Each horizontal

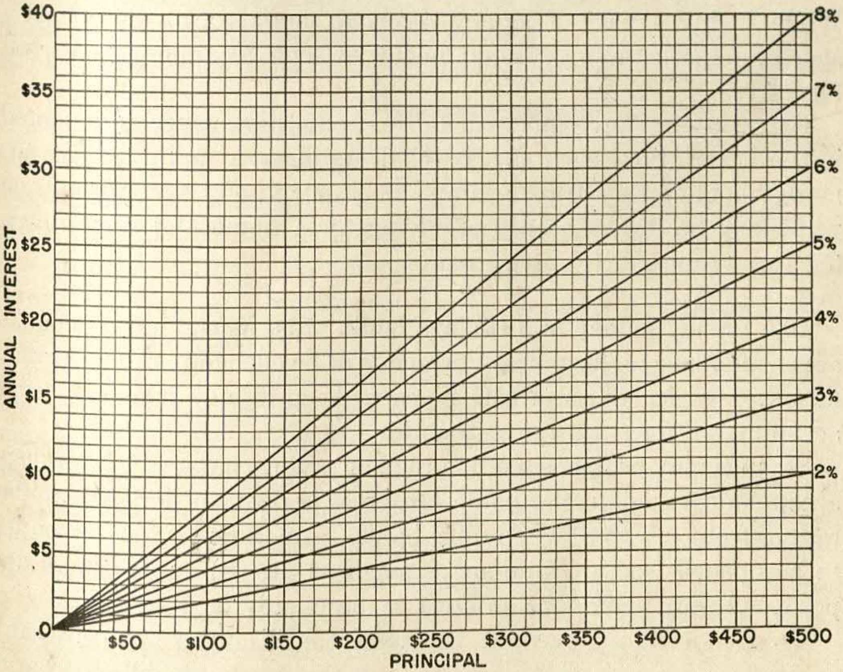


Fig. 8. A simple interest graph.

line is divided into 50 divisions, each division representing $10 principal. Each vertical line is divided into 40 divisions, each division representing $1 interest.

To find the interest on $100 at 2%, follow the 2% rate line to the vertical line representing $100 principal. Since the intersection of these two lines is on the horizontal line representing $2 interest, the annual interest on $100 at 2% is $2. Similarly, the interest on $350 at 8% is found to be $28.

EXERCISES

Using the interest graph, find the annual interest on each of the following:

(a) $150 at 2%	(f) $300 at 3%	(k) $150 at 8%	(p) $280 at 5%
(b) 350 " 4%	(g) 50 " 8%	(l) 450 " 2%	(q) 150 " 6%
(c) 200 " 5%	(h) 150 " 4%	(m) 500 " 8%	(r) 260 " 5%
(d) 450 " 8%	(i) 250 " 6%	(n) 300 " 4%	(s) 100 " 7%
(e) 500 " 7%	(j) 300 " 7%	(o) 300 " 8%	(t) 450 " 4%

102. Saving for a purpose. When should a person start saving? While saving a part of money received should begin at an early age, systematic saving should begin as soon as money is earned regularly.

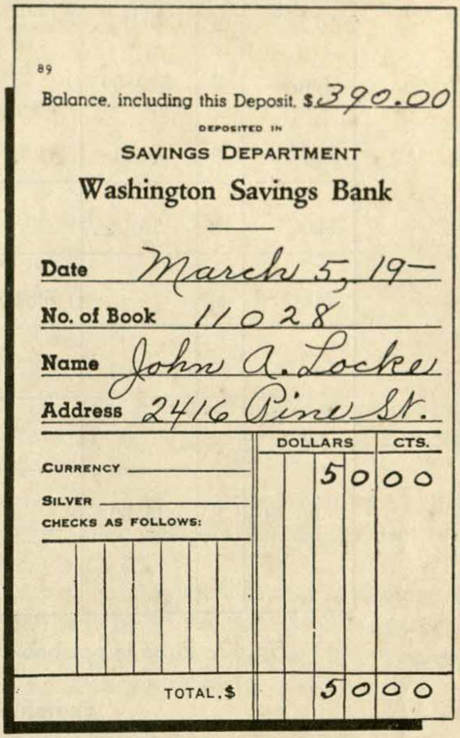
Early savings probably will be for something especially desired in the near future, such as a baseball uniform, a radio, a used car, more schooling, or a trip to camp. Such purposeful savings later on will likely be in the nature of saving for a home and furnishings, for a rainy day, or for investments.

103. Saving money in a savings bank. Just as commercial banks are a safe place to keep money which is to be checked out for current use, savings banks are a safe place to keep money which is being accumulated for a later investment.

Savings may be deposited in a regular savings bank or in a savings account of a commercial bank. Interest is paid on deposits at the rate of from 1% to 2%. Savings banks (a) permit savings accounts to be opened with an amount as small as $1, (b) accept small deposits, (c) pay compound interest semiannually or quarterly.

To open a savings account, the depositor fills out a signature card as he does for a checking account. A savings bank differs from a commercial bank in the manner by which the depositor deposits and

withdraws his money. In filling out a deposit ticket in the savings bank, two additional items are listed—the number of the passbook and the new balance after this deposit is made. See figure 9. The teller enters the amount of the deposit and the new balance in the passbook and returns the passbook to the depositor. When money is to be withdrawn, the depositor fills out a *withdrawal blank* and presents it with his passbook to the teller. The depositor receives the money from the teller, who enters the amount of withdrawal and the new balance in the book and returns the book to the owner. See figure 10 on the following page.


89

Balance, including this Deposit $390.00

DEPOSITED IN

SAVINGS DEPARTMENT

Washington Savings Bank

Date March 5, 19—

No. of Book 11028

Name John A. Locke

Address 2416 Pine St.

	DOLLARS	CTS.
CURRENCY	50	00
SILVER		
CHECKS AS FOLLOWS:		
TOTAL $	50	00

Fig. 9. Deposit ticket for a savings account.

104. Compound interest. Compounding of interest means that as interest is earned, it is added to the principal, making a new principal for the next interest period. The interest-paying periods may be annual (every 12 months), semiannual (every six months), or quarterly (every three months).

When interest is compounded annually, the given rate is used and the interest is added to the principal at the close of each 12 months. This new principal (the old principal plus the interest) is used for the next year, and so it may continue for years.

When interest is compounded semiannually, one-half of the given rate is used and the interest is added to the principal at the close of each six months, making two interest periods each year.

When interest is compounded quarterly, one-fourth of the given rate is used and the interest is added to the principal at the close of each three months, making four interest periods each year.

NO. OF BOOK 11028

Date		Deposit	Withdrawal	Interest	Balance
Jan. 2	B	300.00			300.00
Feb. 5	B	40.00			340.00
Mar. 5	A	50.00			390.00
May 20	B		60.00		330.00
Jun. 3	A	20.00			350.00
Jul. 1				1.47	351.47
Jul. 10	B	15.80			367.27

Fig. 10. Page in passbook for savings account.

Examples

Find the compound interest on \$400 at 2% for 2 years. (Compute the interest on dollars only. Disregard cents.)

Example 1

Compounded annually:
\$400.00 1st Prin.
8.00 1st Int.
\$408.00 2nd Prin.
8.16 2nd Int.
\$416.16 Amt. at close of 2 yr.

Example 2

Compounded semiannually:
\$400.00 1st Prin.
4.00 1st Int.
\$404.00 2nd Prin.
4.04 2nd Int.
\$408.04 3rd Prin.
4.08 3rd Int.
\$412.12 4th Prin.
4.12 4th Int.
\$416.24 Amt. at close of 2 yr.

Example 3

Compounded quarterly:
\$400.00 1st Prin.
2.00 1st Int.
\$402.00 2nd Prin.
2.01 2nd Int.
\$404.01 3rd Prin.
2.02 3rd Int.
\$406.03 4th Prin.
2.03 4th Int.
\$408.06 5th Prin.
2.04 5th Int.
\$410.10 6th Prin.
2.05 6th Int.
\$412.15 7th Prin.
2.06 7th Int.
\$414.21 8th Prin.
2.07 8th Int.
\$416.28 Amt. at close of 2 yr.

EXERCISES

Compute the amount of the following with compound interest:

1. \$500 @ 3% for 4 yr. annually.
2. \$300 @ 4% for 3 yr. annually.
3. \$420 @ 3% for 5 yr. annually.
4. \$300 @ 4% for 2 yr. semiannually.
5. \$240 @ 2% for 3 yr. semiannually.
6. \$1,000 @ 2% for 4 yr. semiannually.
7. \$800 @ 3% for 2 yr. semiannually.
8. \$600 @ 4% for 1 yr. quarterly.
9. \$1,200 @ 2% for 2 yr. quarterly.
10. \$1,600 @ 4% for 3 yr. quarterly.

11. Mr. Jackson deposited \$2,560 on Jan. 1 in a savings account paying $1\frac{1}{2}$% compounded semiannually. How much interest did his money earn by July 1? What was the amount?

12. Mr. Plank deposited \$1,840 on July 1 in a savings account paying 2% compounded semiannually. How much was the interest by the following Jan. 1? The amount?

13. Mrs. McCall deposited \$1,500 in a savings bank paying $1\frac{1}{2}$% interest compounded annually. Calculate the amount at the end of three years.

14. John Baxter put \$800 in a savings account earning 2% interest compounded quarterly. What will be the amount in $1\frac{1}{2}$ years?

105. Compound interest table. Computing compound interest

A COMPOUND INTEREST TABLE

Showing what \$1 will amount to at compound interest

Period	1 per cent	$1\frac{1}{2}$ per cent	2 per cent	3 per cent
1	1.0100 000	1.0150 000	1.0200 0000	1.0300 0000
2	1.0201 000	1.0302 250	1.0404 0000	1.0609 0000
3	1.0303 010	1.0456 784	1.0612 0800	1.0927 2700
4	1.0406 040	1.0613 636	1.0824 3216	1.1255 0881
5	1.0510 100	1.0772 840	1.1040 8080	1.1592 7407
6	1.0615 202	1.0934 433	1.1261 6242	1.1940 5230
7	1.0721 354	1.1098 450	1.1486 8507	1.2298 7387
8	1.0828 567	1.1264 926	1.1716 5928	1.2667 7008
9	1.0936 853	1.1433 900	1.1950 9257	1.3047 7318
10	1.1046 221	1.1605 408	1.2189 9442	1.3439 1638
11	1.1156 683	1.1779 489	1.2433 7431	1.3842 3387
12	1.1268 250	1.1956 182	1.2682 4179	1.4257 6089
13	1.1380 933	1.2135 524	1.2936 0603	1.4685 3371
14	1.1494 742	1.2317 557	1.3194 7876	1.5125 8972
15	1.1609 690	1.2502 321	1.3458 6834	1.5579 6742
16	1.1725 786	1.2689 855	1.3727 8570	1.6047 0644
17	1.1843 044	1.2880 203	1.4002 4142	1.6528 4763
18	1.1961 475	1.3073 406	1.4282 4625	1.7024 3306
19	1.2081 090	1.3269 507	1.4568 1117	1.7535 0605
20	1.2201 900	1.3468 550	1.4859 4740	1.8061 1123

Fig. 11

can become a very slow process if the time extends through many years. Banks and individuals who compute interest usually save time by using a table. See figure 11.

Examples

Using the table, find the amount when the interest is compounded:

(1) Of \$300 annually for 10 years at $1\frac{1}{2}\%$:
Rate $= 1\frac{1}{2}\%$ annually.
Periods $= 10$.
Follow down $1\frac{1}{2}\%$ column to the 10th period.
\$1.1605408 Amt. of \$1 for 10 yr. @ $1\frac{1}{2}\%$
$\times$ 300
\$348.1622400
or
\$348.16 Amt. of \$300 for 10 yr. at $1\frac{1}{2}\%$.

(2) Of \$500 semiannually for 8 years at 2%:
Rate $= 2\%$ annually $= 1\%$ semiannually.
Periods $= 2 \times 8 = 16$.
Follow down 1% column to the 16th period.
\$1.1725786 Amt. of \$1 for 8 yr. @ 2%
$\times$ 500
\$586.2893000
or
\$586.29 Amt. of \$500 for 8 yr. at 2%.

EXERCISES

Using the table, find the amount of compound interest on the following:

1. \$600 for 5 yr. @ 3% annually.

2. \$400 for 16 yr. @ 2% annually.

3. \$800 for 20 yr. @ $1\frac{1}{2}\%$ annually.

4. \$500 for 5 yr. @ 2% semiannually.

5. \$700 for 10 yr. @ 3% semiannually.

6. \$200 for 6 yr. @ 3% semiannually.

7. \$1,000 for 20 yr. @ 1% annually.

8. \$2,000 for 10 yr. @ 2% semiannually.

9. \$300 for 7 yr. @ 3% semiannually.

10. \$800 for 4 yr. @ 2% annually.

11. If on your 8th birthday your father had deposited \$100 for you in a savings account @ 2% compounded semiannually, how much would the deposit be worth at your own birthday this year?

12. If you, at age 20, deposit \$500 in a savings account at 3% compounded semiannually, how much will the account amount to when you are 30 years of age?

106. Saving in a savings and loan association. One systematic way of saving is to buy shares in a savings and loan association. The purchaser agrees to purchase a certain number of shares in the association, for which he is to pay in monthly installments over a number of years.

The payments are usually 50 cents per month for each $100 share. The association pays compound interest of from 2% to 3% on the payments that are made by the purchaser. The share is paid up, or matured, when the payments made plus the interest earned equal the face value of the share being bought. It takes close to 12 years for a $100 share to mature. At maturity the purchaser may receive $100, the face value of the share, or he may leave the $100 with the association and receive dividends on it.

Savings and loan associations, which are members of the Federal Savings and Loan Insurance Corporation, are insured up to $10,000 for each account.

The money placed in savings and investment accounts in a savings and loan association is loaned to purchasers of real estate. The amount of loan obtainable varies from 60 to 80 per cent of the value of the property being purchased. The borrower receives the money at once, the loan of which is secured by a mortgage on the property. He is to pay back the loan in monthly installments which must also include interest on the unpaid balance.

EXERCISES

1. (a) Clark Jones wishes to save $10 per month. How many shares can he buy at 50 cents per month for each $100 share?

(b) What will be the value of his shares when paid up?

(c) If the shares are paid up in 11 yr. 7 mo., how much will Clark have paid for them?

2. (a) Jerry Kemp finds that he can save $8 per month. For this payment how many shares can Jerry buy at 50 cents per month for a $100 share?

(b) How much will the association pay him when his shares mature?

(c) If Jerry's shares mature in 11 yr. 5 mo., how much has been paid in dues for them?

3. (a) Mae Martin is investing $15 per month in shares at 50 cents per month per $100 share. For how many shares has she subscribed?

(b) What will be the value of the shares when they mature?

(c) If in 11 yr. 6 mo. the shares are paid for, how much has Mae paid out in payments for them?

4. Mr. Carter subscribes for 10 shares of par value $100 each. If his payments are 50 cents per month per share, what will be his full payment each month?

5. (a) At 50 cents per month for a $100 share, what will be the monthly payment if 25 such shares are subscribed for by Mr. Slater?

(b) What will the payments total at maturity if paid in 11 yr. 6 mo.?

(c) What will be the total value of the shares at maturity?

6. Mr. Harris wishes to buy property worth $8,000. He has $3,000 for a down payment and subscribes for enough $100 shares to cover a loan to complete the purchase price. (a) For how many shares will he have to subscribe?

(b) Find his monthly dues at 50 cents per share.

(c) What per cent of the cost did he borrow?

7. Property worth $10,000 is bought by Mr. Senior. He pays $4,000 down and finances the remainder of the cost by subscribing for $100 shares at 50 cents per month per share. (a) For how many shares did he subscribe?

(b) Find their value at maturity.

(c) What per cent of the cost did Mr. Senior borrow?

8. Mr. Jacobs buys real estate for $12,000. He pays $4,000 down and borrows the remainder from a building and loan association at 50 cents per month for each $100 share. (a) For how many shares must Mr. Jacobs subscribe?

(b) If the shares are paid up in 11 yr. 7 mo., how much was paid to the association in dues?

(c) What per cent of the cost was the loan?

INVESTMENTS

VOCABULARY

1. investment	4. par value	7. dividend
2. corporation	5. common stock	8. market value
3. capital stock	6. preferred stock	9. brokerage

107. Investments. By saving regularly, Mr. Hill has built up a savings account that is large enough for an investment. Realizing that he is not well informed on investments, Mr. Hill consults his banker, who advises him to consider:

(1) *Safety.* Will the money be safe? While no investment is 100 per cent safe, some investments are practically so while some others are extremely risky. Safety of the principal is of the greatest importance. No investment ranks higher in safety than United States Government bonds.

(2) *Salability.* How quickly can the investment be converted into cash if needed? Some investments have a ready market; others may take months to convert into cash. Bonds and stocks are highly salable. A business or real estate may be slow in selling.

(3) *Income.* What rate of income will the investment yield? It is usually true that a highly safe investment yields a low rate of income while a risky investment offers a high rate. For the small investor, it is wise to consider safety first. Bonds bear a low rate of interest. A business, real estate, and stocks may, or may not, earn a high rate of income for the owner.

108. Bonds. The government and large corporations often need to borrow huge amounts of money. These loans are secured by *bonds* issued by the borrower. The bonds are a type of promissory note in that the borrower promises to pay back the loan with specified interest at a specified time. The time is usually ten or more years later.

109. Corporation bonds. The Eastern Railroad Corporation, wishing to extend its lines and improve its service, needs $5,000,000, a larger sum of money than it is able to borrow from a bank or from any single individual. However, it may borrow the money from a number of different persons by issuing bonds. These bonds may be in denominations of $1,000, $500, $100, or less, payable from 5 years to 50 years after the date they are issued, and with interest at about 5%. Such a bond might be listed in the newspapers as "Eastern 5's, 1980," meaning that the Eastern Railroad bond pays 5% interest and matures in 1980. Corporation bonds are often made safe by a mortgage on the property of the corporation.

The price at which a bond can be bought is its *market value.* The value printed upon the bond is its face value, sometimes called its *par value.* If the market value is quoted at more than the face value, the bond is said to be *above par*; if at less than face value, it is said to be *below par.*

EXERCISES

A. Oral

1. Some $1,000 bonds are sold at the following prices. How much above or below par value is the market value of each bond?

(a) $996	(e) $1,067.46	(i) $982.25	(m) $1,169.85
(b) $1,097	(f) $992.50	(j) $1,043.80	(n) $988.24
(c) $1,016	(g) $987.70	(k) $997.75	(o) $1,014.17
(d) $989	(h) $1,145.45	(l) $1,111.11	(p) $980.28

2. The broker usually charges a brokerage fee of \$2 for buying a \$1,000 bond and \$1 for buying a bond of lower denomination. What is the brokerage for buying the following bonds?

(a) 28 bonds, par value \$1,000	(e) 16 bonds, par value \$100
(b) 36 " " " \$1,000	(f) 29 " " " \$500
(c) 48 " " " \$1,000	(g) 14 " " " \$500
(d) 24 " " " \$1,000	(h) 18 " " " \$100

3. What is the semiannual interest on a \$1,000 bond at 6%? 5%? 4%?

B. Written

4. If a \$1,000 bond is quoted at 95, the market value is 95% of \$1,000, or \$950.

Find the market value and the semiannual interest on the following bonds:

(a) Two \$1,000 Atlas Oil 6's at 97.
(b) Six \$1,000 Missouri Power $5\frac{1}{2}$'s at 109.
(c) Ten \$500 Cent. Elec. $6\frac{1}{2}$'s at 112.
(d) Twenty \$100 Govt. $3\frac{3}{4}$'s at 99.

5. Find the cost of the following bonds:

(a) Five \$100 Liberty $4\frac{1}{4}$'s bought at 104.
(b) Twenty \$1,000 Treasury $3\frac{3}{4}$'s bought at 100.
(c) Four \$500 Balt. & Ohio $4\frac{1}{2}$'s bought at $92\frac{7}{8}$.
(d) Ten \$1,000 Western Union $6\frac{1}{2}$'s bought at 93.

110. United States Government bonds. A common investment for many citizens is United States Government bonds. These bonds represent money loaned to the United States. They are safe, for they are backed by the government's promise to pay.

Government bonds may be bought and sold on the market, their prices being quoted in per cents on the financial page of the daily newspapers.

One issue gave these quotations:

(1) $2\frac{1}{2}$ s	'72–'67	105.8
(2) $2\frac{1}{2}$ s	'67–'62	103.24
(3) $2\frac{1}{4}$ s	'62–'59	101.18
(4) $2\frac{3}{4}$ s	'65–'60	113.22
(5) 3 s	'55	102.18

The number (1) bonds pay $2\frac{1}{2}$% interest, some maturing in 1967 and others in 1972. They are priced at 105.8 per cent of their face value. The cost of a \$100 bond would be \$105.80. A \$1,000 bond would cost \$1,058. These bonds are priced at above their face value.

The popular United States Savings bonds Series E are not transferable but may be redeemed by the government after 60 days from the purchase date. They are 10-year bonds, so, if cashed prior to their maturity date, part of the interest is sacrificed. Series E bonds are registered bonds and at maturity are worth one-third more than their purchase price. This increase in value is equivalent to 2.9% interest a year compounded semiannually if held to maturity.

SERIES E BONDS

COST PRICE	MATURITY VALUE	COST PRICE	MATURITY VALUE
$18.75	$25.00	$150.00	$200.00
37.50	50.00	375.00	500.00
75.00	100.00	750.00	1,000.00

EXERCISES

1. How much is the interest received on a $25 Series E bond at maturity? What does the interest average per year?

2. How much interest is received on a $50 Series E bond at maturity? What is the average interest per year? How does the interest per year compare with that on the $25 Series E bond? Which is the better investment, one $50 bond or two $25 bonds?

3. Mary Hart bought a Series E bond for $150. What will be its value at maturity? How much interest will it earn in 10 years? What will be the average interest per year?

4. Mr. Reed asked his banker to invest $1,125 for him in Series E bonds. If the banker buys the largest denominations possible, how many bonds and what denominations will he buy? What will the bonds be worth in 10 years? How much interest will they earn?

5. At maturity a Series E bond will earn ? per cent interest compounded ? .

6. By saving $1.25 per week, how many weeks would it take to save enough to buy a $25 Series E bond?

7. Find the cost of a $100 government bond at 103.24.

8. Find the cost of five $100 government bonds at 101.18.

9. Find the cost of two $1,000 government bonds at 113.22.

10. Find the cost of ten $1,000 government bonds at 102.18.

11. Compute the annual interest on exercises 7 and 8. The interest is computed on the face value of $100 each, not on the purchase price.

12. Compute the annual interest on the bonds in exercises 9 and 10. (See page 166 for interest rates.)

111. Corporations and their stocks. As an investment, Mr. Payne purchased shares of stock in the A. B. Corporation. This purchase of shares made Mr. Payne a part owner of the corporation. As one of the owners he will receive a portion of the earnings. When the business is prosperous, he will likely receive a good yield on his money. When business is slow, his yield will drop, possibly to nothing. Unless Mr. Payne is well informed about this corporation, he is taking a great risk in buying the shares.

Money received from the sale of shares of stock in a corporation is called its *capital stock*. Buyers of the shares are called *stockholders* or *shareholders*. The original price stated on a share of stock is called the *par value*.

The usual kinds of stock issued by corporations are *preferred stock* and *common stock*. Preferred stockholders receive a fixed rate of profits. "Six per cent preferred" stock means that the holder will receive annually 6% of the total par value of his stock in dividends. The remaining profits, large or small, will be divided among the common stockholders in proportion to the number of shares held by each.

A *dividend* is the amount of the profits paid by a corporation to its stockholders. Dividends are always based on the par value of the stock. For instance, Mr. Black, who owns stock with a par value of $100 per share and paying a 4% dividend, will receive a $4 dividend (4% of $100) on each of his shares, regardless of the purchase price or the present value of the stock.

The price at which stock can be bought is its *market value*. If the market value is quoted at more than the par value, the stock is said to be *above par*; if less than par value, it is said to be *below par*.

Stock is usually sold or bought by an agent called a *stockbroker*. The fee charged for this service is called *brokerage*.

To reduce the risk of investments in common stocks, brokers offer *diversification* in common stock buying. This means that an investor in buying shares of stock under such a plan is putting his money into a long list of carefully selected shares instead of putting all his money into the shares of one corporation. One certificate represents ownership in 25, 30, 50, or whatever number of corporations make up the series. This protection of principal and income is available to the small investor as well as to the investor of large amounts.

EXERCISES

Solve the following exercises. When the number of the exercise is in parentheses, prove the results.

(1) If the capital stock of a corporation is increased from \$750,000 to one million dollars, what is the per cent of increase?

(2) Four men own all the shares of a corporation. If Brown owns 50 shares, Smith 49 shares, Black 87 shares, and Jones 62 shares, and if the shares have a par value of \$50 each, what is the total capital stock of the corporation?

(3) The capital stock of a corporation is \$50,000. If the par value of each share is \$50, how many shares may be issued?

(4) A corporation with a capital stock of \$700,000 issues 20,000 shares; what is the par value of each share?

(5) Mr. Allen buys 25 shares of stock at 75 (\$75 per share). How much does the stock cost him without brokerage?

6. If Mr. Allen had invested the money at 6% interest, what would have been his interest the first year?

7. Using short cuts, give orally the cost of each of the following (without brokerage):

(a) 100 shares at 38.75	(e) 10 shares at $56\frac{1}{2}$	(i) 144 shares at $8\frac{1}{3}$
(b) 25 " " 48	(f) 100 " " $126\frac{1}{4}$	(j) 39 " " $33\frac{1}{3}$
(c) 50 " " 37	(g) 48 " " $12\frac{1}{2}$	(k) 24 " " $66\frac{2}{3}$
(d) 20 " " $84\frac{1}{4}$	(h) 126 " " $16\frac{2}{3}$	(l) 16 " " 75

(8) Find the market value of the following stocks:

(a) 25 shares at 87	(d) 137 shares at $97\frac{1}{2}$	(g) 63 shares at $26\frac{3}{8}$
(b) 65 " " 38	(e) 59 " " 134	(h) 125 " " $218\frac{1}{2}$
(c) 93 " " 45	(f) 147 " " $75\frac{3}{4}$	(i) 348 " " $36\frac{1}{4}$

(9) A dividend of $3\frac{1}{2}$% is declared for the year. How much dividend will be paid on 25 shares of stock, par value \$100 each?

10. A semiannual dividend of $3\frac{1}{2}$% is declared. What per cent is this a year? $3\frac{1}{2}$% quarterly is what per cent a year?

11. A semiannual dividend of $2\frac{3}{4}$% paid on stock with a par value of \$75 is how much dividend per year on 1 share of stock? On 36 shares?

(12) How many shares of Prairie Oil stock at $42\frac{3}{4}$ can be purchased for \$2,864.25?

13. What is the loss on one share of stock purchased at $48\frac{1}{2}$ and sold at $39\frac{3}{4}$? On 52 shares?

(14) Find the rate of dividend on these shares of stock:

CAPITAL STOCK	DIVIDEND	CAPITAL STOCK	DIVIDEND
(a) $300,000	$15,000	(d) $750,000	$40,000
(b) $150,000	$6,000	(e) $60,000	$2,400
(c) $ 20,000	$1,500	(f) $1,800,000	$72,000

15. Find the brokerage on the following at an average brokerage fee of 16 cents per share:

(a) 29 shares at 40	(d) 112 shares at 17	(g) 79 shares at 83
(b) 46 " " 66	(e) 88 " " $9\frac{3}{4}$	(h) 53 " " 148
(c) 63 " " 121	(f) 134 " " 54	(i) 68 " " $3\frac{1}{4}$

16. (OPTIONAL) Find the rate of income on an investment in stocks bought at the prices quoted below and paying the stated rates of dividend. Consider the par value as $100 per share and disregard brokerage:

(a) $50, 4%	(d) $150, 6%	(g) $98, 5%
(b) $200, 4%	(e) $125, 8%	(h) $$82\frac{1}{2}$, 4%
(c) $80, 6%	(f) $180, 10%	(i) $$96\frac{1}{4}$, 7%

17. (OPTIONAL) At what price should 5% stock ($100 par value) be purchased to yield an investment of 6%?

112. Investing in life insurance. In life insurance money is paid by the insured in order to receive money in return from the insurer. The return may be to a beneficiary or it may be to the insured.

The main purpose when payments are made on an endowment policy or on an annuity is that of investment, the return on the investment to be made to the insured as a cash payment, or as a series of cash payments, at some specified time.

How does life insurance rank as an investment?

(1) *Safety.* Life insurance is safe—second to no investment unless it is government bonds.

(2) *Income.* The rate is low, as would be expected with such a high degree of safety. The finances of life insurance are so skillfully managed that, if the policy is in force the expected time, the insured will receive all that he has paid for but no more.

(3) *Salability.* Life insurance policies have a cash-refund and loan value after they have been in force for at least two years. The value is printed on the policy. Money is sacrificed when a policy is cashed prior to its date of maturity. The nearer the policy is to maturity, the greater the cash value. Not only will the insurance company loan money on a policy, but most banks will accept a policy as security on a loan.

Endowment policies and annuities provide a satisfactory method of supplying an income from investment.

Care should be taken not to overstock on any one kind of investment or on investments as a whole. Deep study should be given before making an investment. How will it fit into the budget? Dividing the total payments for the year by 12 will tell whether the monthly budget can take care of the extra expenditure.

SUMMARY QUESTIONS

1. How has the Federal Government made bank deposits safe?
2. List the services of a commercial bank.
3. What business papers are left at banks for collection?
4. What do banks consider as good security?
5. Checking accounts are opened in ? banks.
6. What use is made of the signature card by the bank?
7. "Currency" is ? .
8. "Silver" includes ? .
9. How are checks listed on the deposit ticket?
10. Give three reasons why the check stub should be filled out before the check is written.
11. What care should be taken in writing the amount of a check in figures?
12. Why is it not safe to sign a check ahead of the time that the check is to be written?
13. Distinguish between the drawer and the payee of a check.
14. What should the drawer of a check do if the check is lost?
15. Why is it not safe to put a blank indorsement on a check before taking it to the bank to be cashed?
16. Joe Pratt is the payee of a check received. He wishes to give the check to J. W. Kerns. How should he word a safe indorsement?
17. If Joe Pratt wished to deposit the check in The First National Bank, how would he word a safe indorsement?
18. What is a canceled check?
19. Why should canceled checks be saved?
20. Frank Marrs has received his bank statement. The balance does not agree with the balance on his checkbook stub. (a) What might cause this difference, as far as the bank statement is concerned? (b) As far as the stub of the checkbook is concerned?

21. What two items are to be filled out on a deposit ticket for a savings account that are not required for a checking account?

22. How is money withdrawn from a savings account?

23. What advantage does a savings account have over a checking account?

24. What advantage does a checking account have over a savings account?

25. Highly safe investments usually have a ? rate of yield while risky investments usually have a ? rate of yield.

26. What is the difference between common and preferred stock?

27. What is the meaning of "par value" of stocks?

28. What is "brokerage"?

29. How do bonds differ from stock?

30. Distinguish between *market value* and *par value* of stocks.

31. How does compound interest differ from simple interest?

32. Illustrate that a Series E bond yields $3\frac{1}{3}\%$ simple interest.

33. What is done with the money that is paid into a building and loan association?

34. In what way is a building and loan association a savings institution?

35. To what amount does the Federal Government protect deposits in a building and loan association?

36. The yield of building and loan shares is slightly ? than the yield of a savings account.

37. It has been recommended that a person have funds divided among cash, a fixed-income investment, and a common stock investment. In what financial institutions might a person put his money to secure this?

CHAPTER X

PROBLEMS OF THE CONSUMER

VOCABULARY

1. budget 2. income 3. expenditures

113. Planned spending. Every person, early in life, needs to learn how to handle money. A child, upon being given his first pennies to care for, is at a ripe age to be trained in planning the spending and saving of these pennies.

Only by having a plan for spending and saving can we hope to get that which we want most for our money.

When we grow older and begin to earn our own money, the responsibility of deciding how this money shall be used is mostly ours. We shall want to manage our money so that it will bring us the greatest value and happiness possible. Just how to divide earnings so as to meet needed expenses, enjoy some of the luxuries of life, give to worthy causes, and have a fair savings account requires serious thought. No doubt you have heard more than one person say something to this effect, "My salary is all gone, but for the life of me I don't know how I have spent it." Evidently such persons have no definite plan for their expenditures and savings, but hand out their money in a haphazard manner. Men and women, and even boys and girls, who wish to spend and save wisely must consider very carefully just how to apportion their money.

114. Budget. It is wise to plan expenses carefully and, as far as possible, to spend according to the plan. The habit of recording and classifying expenditures is one of the best aids to wise spending and systematic saving. A statement of your probable expenditures based upon expected income is called a *budget.*

115. Making a weekly budget for a pupil. Suppose that each week Jim Hale, a high school freshman, earns $4.00 and receives

from his father an allowance of $1.00. Jim plans how best to use his $5.00. By so doing week after week Jim will accomplish several things: (a) He will get with his money the things that he wants most. (b) He will learn not to spend extravagantly on certain items. (c) He will learn how to handle his money so as to have some savings. (d) He will have a record showing what he did with his money. Jim makes a simple plan of expenditure as shown in figure 1.

Budget of. . Jim Hale . . . . Week of. Oct. 20-27 . .

Expected Income			Actual Income	
Earnings.	4	00	4	50
Allowance and Gifts	1	00	1	00
Total.	5	00	5	50

Items	Expected Expenses		Actual Expenses		More	Less
Lunch	1	20	1	25	05	
Carfare	1	10	1	00		10
Savings	1	50	1	75	25	
Supplies.		10		30	20	
Recreation		65		65		
Gifts		25		55	30	
Miscellaneous. . . .		20				20
Total	5	00	5	50	80	30

Fig. 1. A weekly budget for a pupil.

Jim first records and totals his *Expected Income* of $4.00 and $1.00 on his budget form. He then carefully considers just how he can best use the $5.00. He decides first on the necessary items, *Lunch* and *Car Fare*. He records the amounts $1.20 and $1.10 for his lunch and car fare. Jim wishes to save $15 for his Christmas shopping, so he plans to save $1.50 each week for 10 weeks. He lists the $1.50 under *Savings*. These first three items total $3.80, which leaves him $1.20 for his other expenses. After careful consideration of each remaining item, he sets aside 10 cents for note paper under *Supplies*. For *Recreation* Jim plans to attend the school mixer (15¢) and a

skating party (50¢). Under *Gifts* he records his offering to Sunday School and a small amount that he is saving each week for his Red Cross donation which comes later in the year. The 20 cents which is unaccounted for he lists as *Miscellaneous* to take care of any expense item not listed.

Jim has now accounted for the use of the $5.00 that he expects to receive. In order to know how well he will live by his plan, he

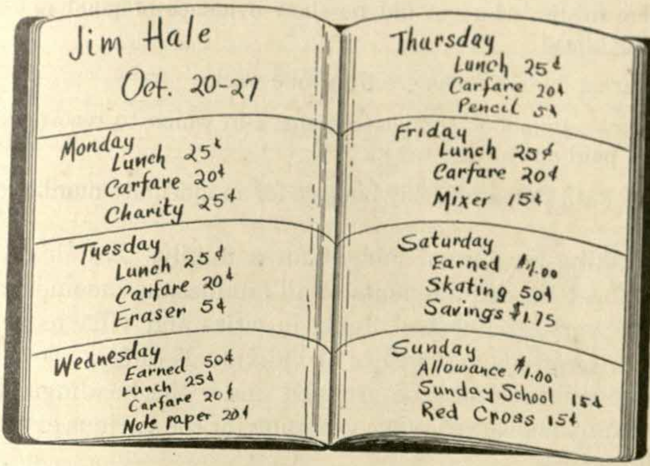


Fig. 2. The notebook kept by Jim Hale in connection with his budget.

keeps a record in a notebook of all money actually received and spent during the week. See figure 2.

At the close of the week Jim totals the items that are alike. By doing some extra work Jim earned $4.50 instead of the expected $4. His *Actual Income* totals how much? His *Actual Expenses* must also total $5.50. As would be expected, there were some items on which Jim spent more than he had planned to spend and other items on which he had spent less. Through practice Jim will be able more nearly to make his expected and actual expenses agree.

Jim finds that he spent $1.25 for *Lunch.* This is 5 cents more than he had planned to spend. Where on the *Budget* did he put the $1.25? The 5 cents? How much did he actually spend for *Car Fare*? Is this more or less than Jim's plan? By how much? Where did he put this amount? On what items did he spend more than he expected to spend? On what items less? On which item did his estimate agree with his actual expense? How much do his *Actual*

Expenses total? Does this amount agree with his *Actual Income*? If so, the budget and record of expenses are correct. The difference between the total of the *More* and *Less* columns (80¢ – 30¢) must equal the difference between the *Expected Expenses* and the *Actual Expenses* ($5.50 – $5.00).

EXERCISES

1. Where in Jim's budget did he show evidence of good management by planning ahead?

2. Prepare a budget for yourself for one week.

3. Prepare a notebook similar to figure 2 in which to record money received and paid out for the week.

4. (OPTIONAL) Prepare weekly budgets for an indefinite number of weeks.

116. Making an annual budget for a family. No single budget plan will meet the requirements of all families, as incomes and sizes of families vary, the cost of living in cities and villages is not the same, and the needs and wishes of individuals differ.

The expenditures may be grouped under the headings that best suit the family concerned. One grouping for the various expenditures is as follows:

Food—All food materials.
Clothing—Ready-made, home-made, laundry.
Housing—Property taxes, repairs, property insurance, interest on mortgage, rent.
House operation—Fuel, light, gas, telephone, household supplies, equipment, and maid service.
Transportation—Upkeep and insurance on automobile; streetcar, bus, and taxi service.
Health—Medical, dental, and optical care.
Personal care—Toilet articles, barber, and beauty shop.
Recreation—Books, magazines, music, radio, entertainment.
Education—Books, magazines, musicals, lectures, school tuition, and supplies.
Taxes—Income and other personal taxes.
Gifts—Religious, charitable, Christmas, birthday, and so on.
Insurance—Life insurance.
Savings—Money deposited in bank, interest on dividends and other investments, Social Security.

In order to plan your personal or family program of income and expenditures for a given period of time, it is necessary: (a) to

approximate your income for the period, (b) to group the various items of necessary living expenses as in the following table, and to place a limit on each group in proportion to the total amount of income, (c) to distribute carefully the remaining income among such items as Education, Recreation, Gifts, Insurance, and Savings.

A SUGGESTED BUDGET FOR FAMILIES CONSISTING OF FATHER, MOTHER, AND TWO CHILDREN WITH INCOMES OF $2,400, $3,600, AND $5,200 PER YEAR

Budget Items	$2,400 per Yr.	$3,600 per Yr.	$5,200 per Yr.
Food	30%— $720	22%— $792	18%— $936
Clothing	15%— 360	12%— 432	10%— 520
Housing	25%— 600	20%— 720	15%— 780
House Operation	8%— 192	10%— 360	12%— 624
Transportation	3%— 72	4%— 144	6%— 312
Health and Personal Care	3%— 72	3%— 108	3%— 156
Recreation and Education	5%— 120	8%— 288	10%— 520
Taxes	3%— 72	6%— 216	8%— 416
Gifts	1%— 24	5%— 180	6%— 312
Life Insurance and Savings ...	7%— 168	10%— 360	12%— 624
Total	100%—$2,400	100%—$3,600	100%—$5,200

Fig. 3

117. Cash record. After a budget has been set up for the year, it should be followed with a monthly *cash record* of all money received and paid out. This record will make it possible to know just how closely the budget has been followed. The amount of expenditures at the close of the year will not exactly equal the amount estimated at the beginning of the year. However, if certain items of the budget have not been reasonably estimated, more careful consideration should be given to these items for the following year. Keeping a cash record gives the satisfaction of knowing just how money was spent. This is valuable information for making out an income tax statement.

For most individuals and families, a simple form of cash record is better than a complicated form. A common form is illustrated in figure 4. It was compiled jointly by Mr. and Mrs. Allen and their two children Sue and Bob. The *Description* column may be omitted; however, it is worth while to keep it for future reference.

Cash Record

Received		Items	Paid Out	
20	00	On hand Oct. 1st.		
425	00	Salary		
15	00	Dividends		
		Food	52	40
		Clothing	42	88
		Housing	55	00
		House Operation	10	50
		Furnishings	14	24
		Automobile	25	00
		Transportation	4	50
		Health	8	00
		Personal Care	5	10
		Recreation	6	40
		Education	7	20
		Taxes	56	80
		Gifts	20	00
		Insurance	50	00
		Savings	82	00
		On hand Oct. 31st.	19	98
460	00		460	00

Fig. 4A. This is the record of money *received* and money paid out by the Allen family.

118. Making the cash record. As money is received and as it is spent, the amounts are recorded by the Allens in a notebook kept for that purpose. At the end of the month the Allens list the items recorded in the notebook with the amounts under the various headings and total each list separately.

They are now ready to record the items and their amounts on the cash record form, recording the *Received* items first. Under

Cash Record
Description
Mrs. A. 18.36 - Mrs. A. 2.40 - Sue 12.24 - Bob 9.88
Payment 45.00 - Fire Insurance 10.00
Gas 3.30 - Light 2.10 - Tele. 3.25 - Water 1.85
Curtains 6.12 - Chair 8.12
Payment 15.00 - Upkeep 10.00
Street car and bus.
Dental fee
Newspaper 1.80 - Music lesson 3.00
School supplies 2.40
Income 48.30 - Personal 8.50
Church 5.00 - Charity 15.00
Life Insurance
Social Security 12.75 - Bond 37.50
Retirement 10.00 Bank 21.75

Fig. 4B. This is a list of items for which the money was paid out.

Received Mr. Allen lists three amounts—the $20 that he had *On hand* from his September earnings, his *Salary* of $425, and the $15 received as *Dividends* on investments. The Allens find that their *Food* costs them $52.40. This amount is placed under the *Paid Out* column. Clothing purchased amounts to $42.88. After recording this amount under the *Paid Out* column, the $42.88 is separated into the smaller amounts that were spent on the various members of the

family. These amounts are listed under the column *Description.* How much was spent for clothes for each member of the family? What is the total cost of these clothes? What was paid for under the title *Housing*? Under *House Operation*? Under *Taxes*? Under *Gifts*?

119. Balancing the cash record. The cash record in figure 4 was balanced on October 31. To balance a cash record, proceed as follows:

(a) Find the total of the Received column.

(b) Find the total of the Paid Out column through Savings.

(c) Find the amount of money on hand at the end of the month by subtracting the total money paid out from the total money received. This amount should equal the amount of money actually on hand.

(d) Enter the words *On hand* and the date under *Items* and the amount under *Paid Out.*

(e) The total of the *Paid Out* column should now equal the total of the *Received* column.

If the two totals agree and the cash actually on hand is the same as calculated in (c), the record is correct.

EXERCISES

1. Answer the following questions on the budgets of figure 3:

(a) As the family income increased, the per cents for food ? while the cost for food ? . Account for your answer.

(b) How is it possible for a family of 4 to be as well fed on $720 per year as one of 4 on $936 per year?

(c) What probably has caused the cost of house operation to be so much higher for the family earning $5,200 per year than for the other two families?

(d) What might be the cause of transportation costing so much more for the family having the highest income?

(e) List the items in which the per cent of income decreased as the income increased. How can you account for this?

2. Which items in a family budget will likely vary a considerable amount from summer to winter?

3. Account for the increase allowed for necessary living expenses (food, clothing, and housing) in recent years.

4. How has the increase in living expenses affected the allowance for savings and life insurance?

5. Account for the rapid increase in taxes as income increases.

6. The greater the earnings, the ? the percentage saved.

7. Explain how a family can get as enjoyable and wholesome recreation by spending $2.00 as by spending $5.00 per week.

8. How would education likely be enriched by an allowance of $2.50 per week instead of $1.00 per week?

9. Which items in the budget are looking forward to the future?

10. Prepare an annual budget for an average family whose income is $3,000 per year.

120. Deriving correct values in the home. Every person has three basic needs: he needs food, clothing, and shelter. Beyond these needs he has desires for other things that will enrich his life. He likely wants such things as an automobile, television set, first-class entertainment, travel, insurance, besides other numerous items. He has just so much money with which to purchase these "wants." As he can spend each dollar of his income

Courtesy National Cash Register Co.

PRICE is probably the largest single factor in the selection of an article.

but once, it behooves him to buy intelligently. He must first satisfy his basic needs. To do this wisely, he must answer many questions for himself. What cut of meat shall I buy? What make of shoes are superior in quality? Shall I buy composition or wood shingles? The "best buy" is not determined by its price, its brand, or its advertising and sales claims. By carefully thinking over at home what and how much of an item is needed and then investigating at the store the various brands as to quality and price, a person can be more certain of having his needs met and of having money left for enrichment of his life.

121. Thrift in buying food. The task of 33,000,000 homemakers trying to feed their families food that is nourishing and enjoyable and bought at a fair price is difficult.

In order to judge the food offered for sale, it is far better to select purchases in person than it is to order them by telephone. Before leaving for the store, a list of food to be bought with the prices quoted in the paper at available stores should be made.

Courtesy National Cash Register Co.

STORES OFTEN LOWER THE PRICE PER ARTICLE in order to encourage quantity buying.

The price of food is controlled by such things as:

(1) *Quality.* Yet a high price does not necessarily mean a superior article, nor does a low price always indicate an inferior product.

(2) *Quantity.* Ordinarily it is cheaper to buy staples in large amounts.

(3) *Season.* Fresh fruits and vegetables bought out of season are expensive.

EXERCISES

1. Mrs. Bly has a grocery list as follows:

Beef roast, 2 lb. (store A, 89¢ per lb.—store B, 77¢)
Shortening (store A, 83¢ for 3 lb.—store B, 73¢)
Sugar (store A, 46¢ for 5 lb.—store B, 41¢)
Berries (store A, 33¢ per qt.—store B, 39¢)

At both stores the beef is stamped *U. S. Good** and is the same cut of

*The Federal grade names which appear in purple markings on beef, lamb, and veal are: *U. S. Choice* (top grade), *U. S. Good* (middle grade)—very acceptable, and *U. S. Commercial* (3rd grade)—less tender, usually a good buy, but requires long, slow cooking. From *Family Fare*, published by the United States Department of Agriculture.

meat. The shortening and sugar are of different brands but both are acceptable. The berries are equally good.

How much will Mrs. Bly save by buying the first three items at store *B* and the berries at store *A*? The saving* is what per cent?

2. Mrs. Gay can buy orangeade at store *X* in a 46-oz. can for 31¢ or at store *Y* the same size and brand for 33¢. Store *Z* has a different brand in a 46-oz. can for 25¢. Mrs. Gay enjoys the first brand but has not tried the other brand. Which can would you advise Mrs. Gay to buy? Why?

Courtesy National Cash Register Co.

CAREFUL PLANNING AND THOUGHTFUL SELECTION aid in wise spending.

3. The same brand of orange juice is advertised at 31¢ for a 46-oz. can and at 10¢ for a 12-oz. can. If the 46-oz. size can be kept until used, what per cent is saved on the ounce by buying it instead of the 12-oz. can?

4. A store advertises a certain brand of tomato juice at 19¢ per can or \$2.15 per case (12 cans). How much is saved on a dozen cans by buying by the case? What per cent is saved?

5. The same brand of corn is offered in one store at 3 cans for 25¢ and in a nearby store at 3 cans for 29¢. How much could be saved on a purchase of one dozen cans? What per cent could be saved?

6. A store offers the same brand of peaches in two different-size cans. It has a #$2\frac{1}{2}$ can ($3\frac{1}{4}$ cups) for 23¢ and a #10 can (12 cups) for 59¢. What is the price per cup (to the nearest mill) for each can? How much could be saved per cup by buying the larger can? What per cent could be saved per cup?

7. Mrs. Kerr, wishing to take advantage of a food sale, bought: 6 cans of peas (12¢ value) at 3 cans for 25¢, 12 cans of pork and beans (18¢ value) at 2 cans for 23¢. How much did she save? What per cent did she save?

* Always base the per cent saved on the higher price.

8. Mrs. Clark in planning for a large group needs 36 cups of canned berries. She can buy #2 cans ($2\frac{1}{4}$ cups) for 20¢ per can or #10 cans (12 cups) for 98¢. How many of the #2 cans would she have to buy? How many #10 cans would she need? What would be the cost of buying the #2 cans? The #10 cans? Which is the cheaper buy? How much cheaper?

9. Find the increase in cost (in per cent) of buying each of the following foods out of season:

Strawberries	(in season) 29¢ per qt.—(out of season) 69¢ per qt.
Asparagus	(in season) 15¢ per lb.—(out of season) 28¢ per lb.
Tomatoes..........	(in season) 10¢ per lb.—(out of season) 25¢ per lb.
Corn	(in season) 30¢ per doz. ears—(out of season) 4 ears for 29¢

10. Arrange in order of price per article, the smallest first: 2 for a quarter; 3 for 50¢; 7 for $1; 13¢ each.

11. Strawberries are priced at 28¢ per quart box. A crate (16 quarts) of berries costs $3.75. How much is a quart box at the crate price?

12. At 3 for 55¢ what will a dozen cantaloupes cost?

13. At 2 for 1¢, how many pieces of candy do you receive for 5¢?

14. At 3 for 5¢, what will 2 doz. peaches cost?

15. If the price of a head of cabbage is reduced from 10¢ to 5¢, what is the per cent reduction?

16. Sugar purchased in a 5-lb. sack is 44¢ per sack, while a 100-lb. sack costs $7.75. What is the difference per pound between the two costs?

17. If the price of a head of lettuce is raised from 5¢ to 10¢, what is the per cent of increase?

18. This recipe for custard ice cream serves 8 people:

1 pkg. gelatin	$\frac{3}{4}$ cup sugar	$\frac{1}{4}$ teaspoonful salt
2 tablespoonfuls cold water	1 pint milk	$1\frac{1}{2}$ teaspoonfuls vanilla
	2 eggs	1 pint cream

Rewrite the recipe so as to serve (a) 4 people, (b) 2 people, (c) 16 people, and (d) 12 people.

19. Thirty cents per quart is what price per gallon? What price per pint?

20. Dinner plates cost $2.70 per dozen. What will 10 plates cost?

21. Potatoes are bought from a farmer at $1.90 per bushel, while the store price is 35¢ per half-peck. How much cheaper is the farmer's price per bushel?

22. A pork shoulder roast should be baked in the oven 12 minutes at 500° Fahrenheit plus 20 minutes per pound at 300°. How long should a 5-lb. roast be baked? A $4\frac{1}{2}$-lb. roast?

23. If one cup of oatmeal is mixed with 3 cups of water and $\frac{1}{2}$ teaspoonful of salt, it serves 2 people. What mixture would serve 3 people? 5 people? 6 people?

24. These measures are common in the kitchen:

3 teaspoonfuls	$= 1$ tablespoonful	$= \frac{1}{2}$ fluid ounce
8 tablespoonfuls	$= \frac{1}{2}$ cupful	$= 4$ fluid ounces
1 cupful	$= \frac{1}{2}$ pint (2 gills)	$= 8$ fluid ounces
4 cupfuls	$= 1$ quart	

(a) Double the measures on the left and find each equivalent measure in the other columns. (b) Take $\frac{1}{2}$ of each of the measures on the left and find their equivalents. (c) Take $\frac{1}{4}$ of each of the measures on the right and find each of the equivalents.

25. Canned goods used in the home come in different sizes, as listed:

Can Size	*Diameter*	*Height*	*Contents*
8 oz.	$2\frac{11}{16}$ in.	$3\frac{1}{4}$ in.	1 cup
Picnic	$2\frac{11}{16}$ in.	4 in.	$1\frac{1}{4}$ cups
Baby foods	$2\frac{1}{8}$ in.	$2\frac{3}{16}$ in.	$\frac{1}{2}$ cup
No. 1	$3\frac{1}{16}$ in.	$4\frac{11}{16}$ in.	2 cups
No. 1F	$3\frac{3}{8}$ in.	$2\frac{7}{32}$ in.	1 cup
No. 2	$3\frac{7}{16}$ in.	$4\frac{1}{2}$ in.	$2\frac{1}{2}$ cups
No. $2\frac{1}{2}$	$4\frac{1}{16}$ in.	$4\frac{11}{16}$ in.	$3\frac{1}{2}$ cups

(a) A No. $2\frac{1}{2}$ can holds how many times as much as a No. 1 can? (b) If a No. 1 can of tomatoes costs 12¢, what does each cup of tomatoes in the can cost? (c) Find the price that should be marked on each of the other cans according to the number of cups of tomatoes contained.

122. Thrift in buying clothing. There are various ways to learn about the quality, value, and style of clothing.

(1) *Advertisements* usually picture the style and state the kind of fabric, the price, the color, and the size of suits, dresses, coats, shoes, and certain other clothing. While personal inspection should follow before a purchase is made, advertisements help to start the shopping at home. Comparisons can be made and a list of "must sees" jotted down. All advertisements cannot be relied upon, but, in general, an established business cannot afford to put out false statements about its merchandise.

(2) *Salesclerks* can be of great help if they are well informed about the merchandise in their charge and if the customer knows how to tell the salesclerks what he wants.

(3) *Labels* are very important, for they present true statements. The labels "100% wool," "51 gauge," "Sanforized," and "pure silk" on clothing are an assurance of quality. However, not all clothing is labeled.

(4) *Testing agencies* put out information about the wearing quality of certain clothing, such as shoes, hose, hats, and of fabrics. By

laboratory methods these agencies can test the wearability of merchandise. Such information is limited but quite reliable.

EXERCISES

1. At a clearance sale a \$35 suit is offered for \$14. How much is the reduction? What per cent?

2. Shoes which originally sold for \$10.95 are on sale for \$7.90. Find the per cent of reduction.

3. Coats originally selling for \$79.95 are on sale for one-third off. What is the sale price?

4. What disadvantage may there be in buying shoes at a July shoe sale?

5. Would you expect a \$3.98 pair of shoes to have the leather, workmanship, style, and comfort of a \$10.95 pair? Why?

6. A woman's coat is marked \$189. On sale, it has a 12% discount. What is the sale price?

7. A reduction from \$8 to \$7 is what per cent reduction?

8. An increase from \$7 to \$8 is what per cent increase?

9. Tablecloths damaged by water were sold at \$6.25. How much was the reduction, in per cent, if the marked price had been \$8.50?

10. What is the reduction in the price of men's suede jackets reduced from \$7.50 to \$5.98? What is the per cent reduction?

11. What are the rates of discount on a sport coat reduced from \$36 to \$30 and a sport coat reduced from \$25 to \$20?

12. Which is the better bargain: an overcoat reduced from \$40 to \$35, or the same overcoat reduced 15%?

13. A pair of shoes cost \$4.88. The sales tax of 2% must be added. What is the actual cost?

14. Mrs. Jones bought the following dress materials: $4\frac{1}{2}$ yd. gingham @ 34¢ per yard; $3\frac{1}{4}$ yd. percale @ 26¢ per yard; $1\frac{1}{2}$ yd. ribbon @ 26¢ per yard; and $2\frac{1}{3}$ doz. buttons @ 42¢ per dozen. What was the total cost of the articles?

15. Margaret buys a fur coat for \$270. She pays \$90 cash and agrees to pay \$30 each month until the coat is paid for. How many months are required to pay for the coat?

16. Find to the nearest cent the cost of $19\frac{1}{2}$ yd. of linen at \$.95 per yard.

17. Mary's graduation dress ready-made costs \$17.50. If Mary can make the dress from 4 yards of material at 78¢ per yard, 10 buttons at 48¢ per dozen, collar material for \$1.12, and a pattern for 50¢, how much does she save by making the dress?

FINANCING PURCHASES

VOCABULARY

1. credit	4. day of maturity	7. proceeds
2. installment	5. collateral	8. equity
3. promissory note	6. bank discount	9. depreciation

123. Credit and installment buying. Buying on credit is popular with the American people. Just a few years ago the total consumer credit in the United States had risen to more than $14,000,000,000. Opening a charge account is simple. The merchant merely wants to know that the prospective purchaser will be able to pay for his purchases and that he *will* pay for them.

In a small community, a person's reputation is usually sufficient guarantee for acceptance or cause for rejection. In a city, most stores have a credit manager who interviews the applicants desiring credit. The credit manager also checks on each applicant's income and past credit record. Whether the applicant is allowed credit or not depends upon the result of the investigation. If accepted, and purchases are made, a statement is rendered to the customer at the end of the month, and payment for all purchases placed on that charge account for the month is due in full, with no extra charge, by the tenth of the next month.

Users of credit should (1) keep within a planned budget, (2) shop around, and (3) buy only as needed.

Installment buying is another form of credit buying. It is more often used for the purchase of expensive durable articles, such as furniture, appliances, automobiles, and television. A *down payment* is usually made at the time of the purchase. The rest of the purchase price is paid in weekly or monthly payments until the article is wholly paid for. The store makes a small fixed charge for this service called a *service charge* or a *carrying charge.* This charge represents interest on the unpaid balance and the cost of the bookkeeping involved.

Installment buying has its dangers and its advantages. The easy-payment plan tempts some people to buy articles which they cannot afford to buy. In installment buying, if it becomes impossible for the buyer to complete the payments on an article so purchased, the seller has the right to take back the property, causing the buyer to lose the article and the payments that he has made on it. Installment

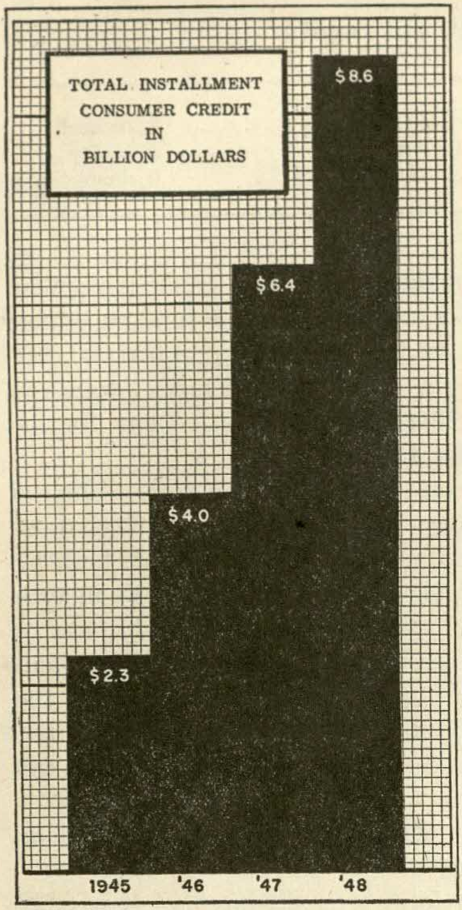


Fig. 4. Installment buying, 1945–48.

buying gives the consumer the advantage of getting use of the article sooner than he would if he had to wait to save the cash before buying. Articles should be bought on time payments *only* after careful consideration of the budget.

Figure 4 shows the trend in installment credit buying in the United States over a period of four years. (a) What was the total amount of installment buying for each of the four years? (b) The amount of credit in 1947 was approximately how many times as much as in 1945? (c) The 1948 amount is about how many times that of 1945? (d) How many dollars more was the credit in 1948 than in 1945? (e) Which year shows the greatest increase over the preceding year? (f) Account for this rise in installment buying.

Example

Mr. Mann buys a \$50 radio on a 10-payment plan. He makes a down payment of \$10. The unpaid balance bears a 6% service charge. What is the amount of each of the ten payments?

Purchase price	\$50.00
Down payment	10.00
Unpaid balance	\$40.00
	.06
Service charge	\$2.40

\$40.00 + \$2.40 = \$42.40

\$42.40 ÷ 10 = \$4.24, the amount of each payment.

EXERCISES

What is the amount of the payment in each of the following?

Purchase Price	*Down Payment*	*Service Charge*	*Number of Payments*
1. $ 75	$ 15	5%	10
2. 800	100	6%	20
3. 50	5	1%	5
4. 1,200	200	6%	25
5. 250	25	2%	10
6. 1,750	250	7%	48
7. 7,000	1,000	6%	72
8. 980	150	½%	20
9. 175	25	$15	17
10. 47.50	None	1%	15

11. A piano that is priced at $750 cash can be purchased in installments of $15 down and $32 each month for 24 months. Find the difference between the cash price and the installment price.

12. The furniture which the Jones family wants to buy will cost $223 cash or $230 if the Joneses pay $25 down and the rest in installments of $20.50 per month.

(a) How many payments must be made on the installment plan?
(b) What is the difference in the cost of the two plans?

13. An electric washing machine is sold for $23 down and $9 per month for one year.

(a) What is the installment price?
(b) If 9% discount from the installment price is allowed for cash, what is the cash price?

14. Mr. Jones decided to trade his old automobile for a new one. The dealer offered him $425 for his old car. The new car cost $2,412. Mr. Jones was to pay $667 cash and the balance in 12 equal monthly payments. What did he pay per month on the balance?

15. Mr. Sams bought a gas stove for $189.50. He paid $14 down and agreed to pay $1.95 per week until the stove was paid for. For how many weeks was he paying for the stove?

16. Mr. White bought a television set for $279.95. His down payment was $27.95. At 42 cents per day, how many days will it take him to complete his payments?

17. A radio is priced at $22 cash or $1 down and $4 per month for six months. What is the installment price of the radio? How much more is the installment price than the cash price? Considering this additional cost as *interest*, what rate of interest is paid for using this installment plan?

Solution

Installment price	$\$4 \times 6 + \$1$	$= \$25$
Cash price		$= \underline{\ 22}$
Interest (additional cost)		$= \$\ 3$

To find the rate of interest, use the formula $r = \dfrac{i}{pt}$.

The principal (p) is the amount owed each month. As \$4 is paid on the principal each month, the principal becomes less month by month. To arrive at a fair principal to use in computing the interest, the total owed for each of the six months is found.

Solve as follows:

\$25 − \$1 down	=	\$24 owed for 1 mo.
24 − 4 payment	=	20 owed for 1 mo.
20 − 4 "	=	16 owed for 1 mo.
16 − 4 "	=	12 owed for 1 mo.
12 − 4 "	=	8 owed for 1 mo.
8 − 4 "	=	4 owed for 1 mo.
		\$84 owed for 1 mo.

\$84, the total of the monthly amounts owed, is the same as owing \$84 for one month. The interest on the \$84 for 1 month is \$3.

As $i = \$3$, $p = \$84$, $t = 1 \text{ mo.} = \frac{1}{12} \text{ yr.}$,

$$r = \frac{3}{84 \times \frac{1}{12}} = \frac{3}{7},$$

$$r = 42\tfrac{6}{7}\%.$$

18. Mr. James can buy a suit of clothes for \$40 cash or for \$10 down and \$6 per month for six months. What is the cost of the suit by the installment plan? How much more is this than by paying cash? What rate of interest is paid if the installment plan is used?

19. A table can be purchased for \$48 cash or for a \$12 down payment and \$5 per month for eight months. Find the installment price. What is the rate of interest paid if the table is bought on the installment plan?

20. A rug is listed at \$70 cash or \$15 down plus \$12 per month for five months. What is the installment price? Find the rate of interest paid if the installment plan is used.

124. Consumer borrowing. George Morse, who wishes to purchase a refrigerator for his home but does not have the money available to pay cash, may buy on the installment plan. By so doing he figures that the additional cost will be equivalent to about 40% interest. To buy on a cash basis, he will need about \$100 more than he has now.

Mr. Morse looks about for a reasonable rate of interest. He goes to the bank where he is known and finds that he can borrow the money at 6%.

George Morse asks the Commercial National Bank to lend him \$100 for 60 days. He makes a written promise to pay back the amount of the loan. This promise (see figure 5) is called a *promissory note*, or merely a *note*. When the face of the note is paid, the banker returns the note. To make sure that George Morse will keep his promise, the bank may require him to give some form of good security, such as a mortgage on some of his property or safe stocks and bonds. Such security is called *collateral*.

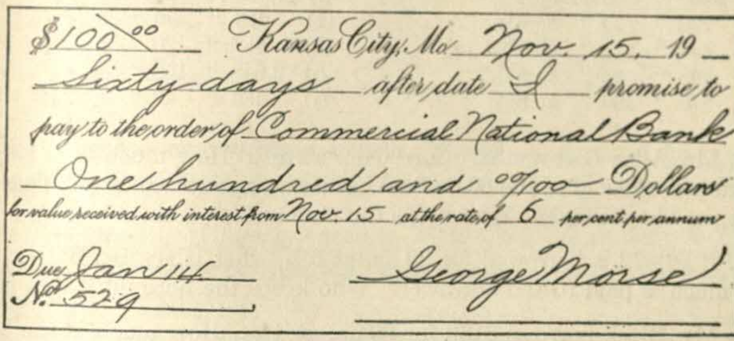

\$100.00 Kansas City, Mo. Nov. 15, 19__
Sixty days after date I promise to
pay to the order of Commercial National Bank
One hundred and 00/100 Dollars
for value received with interest from Nov. 15 at the rate of 6 per cent per annum
Due Jan 14
No. 529
George Morse

Fig. 5. Promissory note

Instead of putting up collateral, the borrower may get one or more responsible persons to guarantee the payment of the note.

Although the amount of the George Morse loan is \$100, the bank will pay Morse at the time of the loan \$100 *minus* the interest from the date of the loan to the day of maturity. Interest is thus deducted in advance. Since the interest on \$100 for 60 days at 6% is \$1, Morse will receive \$99. The collection of interest in advance is called *bank discount*, and the amount that the borrower receives is called the *proceeds*.

Face of Note − Bank Discount = Proceeds
\$100 − \$1 = \$99.

If Mr. Morse needs the money longer than the 60 days, he can have the note renewed.

EXERCISES

1. To find the interest due on his note, Mr. Morse uses the formula: $i = prt$.

$$p = \$100;\ r = 6\%;\ \ t = 60 \text{ da.} = \tfrac{1}{6} \text{ yr.}$$

$$i = \frac{\$100}{1} \times \frac{6}{100} \times \frac{1}{6} = \$1.00.$$

2. Find the interest on a \$450 loan at 6% for 96 days.

3. What is the interest on a \$720 loan at 5% for 80 days?

4. How much interest must be paid on a \$630 loan at 8% for 90 days?

5. Find the interest on:

(a)	\$960 for	80 da.	@ 4%	(f)	\$1,600 for	88 da.	@ $4\frac{1}{2}$%
(b)	720 "	75 "	@ 6%	(g)	420 "	36 "	@ $3\frac{1}{2}$%
(c)	425 "	72 "	@ 3%	(h)	487 "	120 "	@ 3%
(d)	848 "	96 "	@ 5%	(i)	3,648 "	150 "	@ $4\frac{1}{2}$%
(e)	870 "	100 "	@ 6%	(j)	480 "	42 "	@ $2\frac{1}{2}$%

6. Mr. Allen borrows \$275 for 60 da. at 6%. How much does he pay the bank at the end of the 60 da.? What is the bank discount on his note? What are the proceeds? Who keeps the note until it is due?

7. If \$2,472 is borrowed for 30 da. at 6%, what is the bank discount? How much is paid to the borrower? Who keeps the note until it is paid?

8. Mr. Black borrows \$180 for 60 da. at 4%. How much is the bank discount? How much is paid to the borrower?

9. James Stone borrowed \$1,400 for 90 da. at 6%. How much did the bank deduct as interest? How much did James Stone receive?

10. A sum of \$389 is borrowed for 45 da. at 6%. What is the face of the note? How much must be paid on the note? Who has possession of the note until its payment?

11. \$890 is borrowed for 120 da. at 6% interest. How much is the bank discount on this note? What are the proceeds?

12. On Nov. 3, George Long borrows \$860 from the Commercial National Bank for 90 da. at 6%. How much is the bank discount? How much did George Long receive? How much did he owe the bank in 90 days?

13. Mr. Carter needs \$290 to pay emergency expenses. The Citizens Bank loans him the money at 6% for 180 days. How much money does Mr. Carter receive?

If Mr. Carter needs all of the \$290, he must make the face of the note more than \$290. He must make it large enough to cover the \$290 plus the discount. The discount rate is 6% per year, which is equivalent to 3% per $\frac{1}{2}$ yr. \$290 is 97% (100% − 3%) of the amount needed. Hence, the face of the note is \$290 ÷ .97 = \$298.96.

14. Mr. Carter makes the face of the note \$298.96. Prove that he will receive \$290 from the bank.

15. What must be the face of a 180-da. note if the discount is 4% and the proceeds desired are \$490?

16. Mr. Dale must have \$2,646. The discount rate is 8%. How much must he make the face of the 90-da. note? (8% for 1 yr. = ?% for $\frac{1}{4}$ yr.)

17. Determine the face of the note:

PROCEEDS	DISCOUNT	TIME
(a) \$3,136	6%	120 da.
(b) 495	6%	60 da.
(c) 1,728	8%	180 da.
(d) 776	9%	120 da.
(e) 1,440	6%	240 da.
(f) 297	5%	72 da.
(g) 1,455	4%	270 da.
(h) 788	9%	60 da.

125. Renting or owning a home. The three greatest factors influencing the question of whether to own or to rent a home are:

1. *The amount of family income.* If the annual income is \$3,000 or less, the purchase of suitable shelter in most parts of the United States is out of the question.

2. *The source of the annual income.* If the income is seasonal, temporary, or uncertain, then ownership is risky.

3. *The size of the family.* The cost of home ownership is practically the same for one person as for a family of four. Hence, in a small family the cost of home ownership per person may be too high.

To own property means to have taxes to pay, insurance to buy, repairs to make, and, until the property is paid for, payments and interest on mortgage to be met.

Renting a place to live involves paying rent regularly, running the risk of having to move at an inconvenient time, caring for the property of others, and living, to a certain extent, as others would have you live.

Home ownership promotes a feeling of security and pride. It offers a more independent and satisfactory way of living. It is much to be desired in a community.

Because home ownership is a good thing for the individual and the community, the Federal Housing Administration of the United States Government has made it easier for families to get the money

to finance the purchase of a home. It guarantees the payment of property loans through a system of insurance. The loan is made by an approved bank, by a private lender, or by a mortgage, loan, or insurance company. The insured loan costs the borrower a Government fee of $\frac{1}{2}\%$ of the total loan plus $4\frac{1}{2}\%$ interest on the money borrowed. Any buyer who can meet the requirements of the Government should use this type of credit, for it is cheaper than borrowing without the Government's guarantee to the person making the loan. Loan companies usually charge from 6% to 36% interest per year. Such interest rates make the borrowing of money very expensive.

Houses are usually purchased on the monthly installment plan. The buyer of the house pays a sum of money to the seller of the house at the time of the purchase and signs a *mortgage*, which allows the seller of the house to take the house back and, in most cases, to keep all the money the buyer has paid if the buyer does not keep his promise to continue payments on the house. The amount of money that the buyer has paid on the principal is called his *equity*. If he has paid \$2,000 on an \$8,000 house, his equity is \$2,000.

Example

Jones buys a house for \$8,000. He pays \$2,000 down and agrees to pay \$60 per month on the principal, plus interest at 6% on the unpaid balance. What are his payments?

\$8,000	purchase price
− 2,000	down payment
\$6,000	unpaid balance

First Month		*Second Month*	
\$6,000	unpaid balance	\$5,940	unpaid balance
× .005	(6% per yr.)	× .005	(6% per yr.)
\$30.00	interest for first month	\$29.70	interest for second month
+ 60.00	payment on principal	+ 60.00	payment on principal
\$90.00	payment for first month	\$89.70	payment for second month

\$6,000 − \$60 = \$5,940, unpaid balance   \$5,940 − \$60 = \$5,880, unpaid balance

For the third month, continue with \$5,880 as the unpaid balance. Find the amount of interest due; add the interest and the \$60 payment to find the total payment due; then subtract the \$60 payment from the principal (the unpaid balance at the beginning of the month) to find the unpaid balance for the next month.

EXERCISES

1. Complete this table by continuing with total payments and unpaid balances for one year on the sale of the house in the preceding example.

Months	*Principal*	*Payment on Principal*	*Interest at 6%*	*Total Payment*	*Unpaid Balance*
1st	$6,000	$60.00	$30.00	$90.00	$5,940
2nd	5,940	60.00	29.70	89.70	5,880
[Etc.]					

2. Mr. Stone bought a house and lot for $7,500 and paid $4,000 down. He paid the balance like rent. The monthly payment was $50 per month plus interest at 6%. Make a table and calculate his payments for one year. How many years and months will it take to pay for the house?

3. A house rents for $35 per month, and the expenses for the year were: taxes, $65; insurance, $30; water, $5; and repairs, $25. How much of the rent money was left at the end of the year? How much rent was paid the owner for the year?

4. A house was purchased for $12,000. It increased in value 10% in two years. What was the average increase per year for the two years?

5. An owner, after living 10 years in a house that cost him $9,000, sold it for $7,500. (a) How much money did he lose on the house? (b) What was the average loss per year? (c) What was the average loss per month for the 10 years? (d) Could he have rented a house or apartment for this amount per month? (e) Did he make or lose money by owning the house for 10 years?

6. A builder sold a house for $7,200, which was $12\frac{1}{2}$% more than it cost him. What did it cost to build the house?

7. If the expenses on a house were $154.40 for taxes, $32.80 for insurance, and $132.80 for repairs; and if the house rented for $50 per month, how much did the owner have left at the end of the first year?

8. (OPTIONAL) A bungalow cost $6,500. $3,000 was paid down, the interest on the unpaid balance was 6%, and the principal was reduced $40 per month. Make a table showing the total payments for the first six months.

9. (OPTIONAL) On September 4, 1951, Mr. Clark bought a farm costing $8,000. He made a down payment of $5,000. He contracted to pay the balance in $100 installments every month beginning October 4, 1951. The unpaid balance was to bear 6% interest. Find the amounts Mr. Clark will have to pay each month up to and including March 4, 1952.

126. Depreciation. All property *depreciates* with time and use. Jones bought an automobile for $1,945. He kept it for 3 years and then sold it for $1,387. The difference between $1,945 and $1,387 represents what the car lost in value because of wear. The $558 decrease in value is the *depreciation.*

$$\text{Annual Depreciation } (D) = \frac{\text{Value New } (N) - \text{Value Used } (U)}{\text{Time in Years } (T)}$$

Or

$$D = \frac{N - U}{T}$$

Example

What is the annual depreciation on an automobile that cost $3,000 new and was sold for $800 after 5 years' use?

$$D = \frac{N - U}{T}$$

$$D = \frac{\$3{,}000 - \$800}{5}$$

$$D = \frac{\$2{,}200}{5}$$

$$D = \$440$$

The depreciation amounted to $440 per year.

EXERCISES

1. Find the annual depreciation on $450 worth of furniture that was sold for $55 after nine years' use.

2. A 25-year-old house was sold for $1,800. What is the annual depreciation if it cost $3,050 when new?

3. An automobile that cost $2,975 new was used five months and sold for $1,760. What was the monthly depreciation?

4. If the automobile in exercise 3 was driven 4,300 miles before being sold, what was the cost of the car per mile?

5. (a) If $2\frac{1}{2}\%$ is written off the value of a $3,800 house each year, what is the annual depreciation? (b) What is its estimated life?

6. The average life of a large commercial passenger airplane is six years. What is the annual per cent of depreciation?

7. Brown's automobile cost $2,200. Depreciation was calculated at 35% of the cost the first year, 20% of the cost the second year, and 15% of the cost the third year. What was the estimated value at the end of the third year?

8. What is the rate of depreciation on furniture that cost $750 and at the end of 12 years was sold for $150? (*Hint:* $\frac{50}{750}$ = what per cent?)

9. Mr. Jones bought a truck for $1,680. The life of the machine was estimated at six years. Its scrap value at the end of that time is $60. (a) What is the annual rate of depreciation? (b) What is the monthly depreciation?

10. What is the annual depreciation on a piece of machinery that cost $250 if 10% is charged off each year for depreciation? What is the estimated life of the machine? What is its value at the end of 4 years? 9 years? 10 years?

Courtesy National Cash Register Co.

Fig. 6. Making change.

127. Making change. A clerk is expected to give change to a customer in as few coins as possible. The usual method of making change is to add coins to the cost of the purchase until the amount offered in payment is reached. If you should buy a can of soup for 8¢ and give a $1 bill in payment, the clerk would take 2¢ from the cash register, and say, "Ten cents"; then 5¢ and say, "Fifteen cents"; then 10¢ and say, "Twenty-five cents"; then 25¢ and say, "Fifty cents"; then 50¢ and say, "One dollar." He would then count to you in the same order, "8—10—15—25—50—$1."

EXERCISES

Count the change for the following:

1. 2¢ out of a quarter	6. $1.00 out of $10
2. 25¢ " " " half-dollar	7. .63 " " 10
3. 35¢ " " $1	8. 5.40 " " 10
4. $1.40 " " 2	9. .26 " " 5
5. 4.13 " " 5	10. 1.36 " " 5

11. A sweater costing $1.89, a skirt costing $2.98, and a pair of shoes costing $3.87 are purchased with a $20 bill. Count the change to the customer.

12. Frying chickens are advertised at 54 cents per pound. What will a 3-lb. chicken cost? How much change would be received from a $5 bill? Count the change to the customer.

13. John bought a raincoat marked $5.89 at a sale at which he was allowed 25% discount.

(a) What did the raincoat cost him?

(b) Assuming that he gave the clerk a $10 bill, count the change back to him.

14. A sale of 50 yd. of lace at 12½ cents per yard is paid with a $10 bill. How much change is returned to the customer? Count the change.

15. Mary had $25 in currency (paper money). She bought a dozen hens at $1.80 each. Count the change back to her.

SUMMARY QUESTIONS

1. What is the purpose of a budget? How does it help you to get better use from your money?

2. Expenditures for which items should be planned for first when a budget is being made? Why?

3. Expenditures for which items may overlap when education and recreation are being considered?

4. How can keeping a cash record help to make the next budget more nearly accurate in estimates?

5. List a few happenings that might cause the budget to be considerably out of line with actual receipts or expenditures.

6. Under what circumstances would it be wise to make changes in a budget?

7. How might budgeting your time bring better results in your school work?

8. List some things that a high school pupil probably could obtain by budgeting that otherwise might not be obtainable.

9. Why should the children in a family take an active part in planning the family budget?

10. What reasons can you suggest that would justify store A having a higher price than store B has for identical merchandise?

11. Why is the cost of soap powder less per pound when bought in large quantities than when bought in small quantities?

12. Account for the higher cost of melons in December than in August.

13. What advantage is there in purchasing a fur coat at an "August Fur Sale"? What disadvantage?

14. Is there this same disadvantage in buying sheets at a "January White Sale"?

15. What labels have you seen on clothing? On food?

16. Give an example of when one should not "buy the best" in clothing. In food.

17. How can you start your shopping at home?

18. Name some ways of inspecting clothing; food; a radio; a television set; a used automobile.

19. Act out "counting change" with a classmate.

20. A banker once said, "A man always counts his money before leaving the bank window; a woman never does." Should a person count his change, check on scale weights, and look under the first layer of berries when making purchases?

21. What are the three basic needs of every person?

22. Give three reasons why the cost of articles may be reduced.

23. What are two dangers of buying on credit?

24. What is the difference between buying on a charge account and buying on an installment plan?

25. What is the danger of buying an article and giving the seller a mortgage on the article?

26. (a) Give five reasons why a person would want to own his own home. (b) Give five reasons why a person would want to rent a home.

27. (a) What is depreciation? (b) How does depreciation apply to a home? To an automobile?

28. What is meant by *equity*?

29. (OPTIONAL) With one or more classmates plan a debate on the topic, "Resolved, that it is better to pay as you go than to buy on credit."

30. Make a list of articles that are worth borrowing money to obtain. Make a list that would not warrant borrowing to obtain.

ACHIEVEMENT TEST III

Addition—4 minutes

1. 5 7 8 4 6 9	**2.** 45 38 26 57	**3.** 128 277 455 688 329 484	**4.** 31.65 4.87 23.69 .47	**5.** $3.4 + 21.8 + 6$ **6.** $\frac{1}{3} + \frac{5}{6}$ **7.** $\frac{2}{3} + \frac{1}{5} + \frac{3}{5}$

8. $36\frac{5}{7}$ $27\frac{1}{2}$	**9.** $14\frac{3}{4}$ $8\frac{5}{8}$	**10.** 18 lb. 8 oz. 6 lb. 9 oz.

(Achievement Test III—Continued)

Subtraction—4 minutes

11. $\begin{array}{r} 305 \\ \underline{28} \end{array}$

12. $\begin{array}{r} 3030 \\ \underline{286} \end{array}$

13. $\begin{array}{r} 63.14 \\ \underline{17.29} \end{array}$

14. \$1.67 − \$1.39

15. \$5 − 67¢

16. $78\frac{2}{3} - 17.8$

17. $\frac{5}{6} - \frac{2}{3}$

18. $\begin{array}{r} 55\frac{2}{3} \\ \underline{24\frac{8}{9}} \end{array}$

19. $\begin{array}{r} 60\frac{1}{5} \\ \underline{20\frac{2}{3}} \end{array}$

20. $\begin{array}{r} \text{5 ft. 7 in.} \\ \underline{\text{2 ft. 9 in.}} \end{array}$

Multiplication—4 minutes

21. $\begin{array}{r} 87 \\ \underline{49} \end{array}$

22. $\begin{array}{r} 109 \\ \underline{509} \end{array}$

23. $\begin{array}{r} 170 \\ \underline{70} \end{array}$

24. $\begin{array}{r} 8.4 \\ \underline{.07} \end{array}$

25. $\frac{1}{4} \times \frac{1}{4}$

26. $1\frac{3}{4} \times 8$

27. $\frac{1}{6} \times 11\frac{1}{3}$

28. $\frac{5}{8} \times 160$

29. $\begin{array}{r} 57\frac{1}{2} \\ \underline{28\frac{2}{3}} \end{array}$

30. $\begin{array}{r} \text{6 bu. 2 pk.} \\ \underline{3} \end{array}$

Division—4 minutes

31. $9\overline{)219}$

32. $.17\overline{).442}$

33. $45.2 \div 100$

34. $2.8\overline{).420}$

35. $\begin{array}{r} 178 \\ 2.4\overline{)427.2} \end{array}$

36. $\begin{array}{r} 178 \\ .24\overline{)4.272} \end{array}$

37. $\frac{3}{4} \div \frac{3}{5}$

38. $\frac{1}{3} \div \frac{1}{3}$

39. $19\frac{1}{2} \div 3\frac{1}{4}$

40. $2\overline{)\text{7 ft. 5 in.}}$

Per Cent—4 minutes

Write as common fractions:

41. 5% **42.** $62\frac{1}{2}\%$

Write as decimal fractions:

43. 8% **44.** 150%

Write as per cents:

45. $\frac{7}{20}$ **46.** 1.6

47. 20% of 158 = ?

48. 12% of ? = 60

49. ?% of 40 = 18

Arrange in order of size—smallest first:

50. $.087\frac{1}{2}$, $87\frac{1}{2}\%$, $8.7\frac{1}{2}$, $\frac{88}{100}$

Record your scores on the Achievement Chart.

Which of your scores are moving upward? Just one more Achievement Test. Make it a topper.

Review of the Fundamentals of Arithmetic—7

Addition and Subtraction

Add and prove:

1.	2.	3.	4.	5.	6.	7.
5	74	89	256	3284	57.008	$9167.82
8	88	76	342	7158	19.06	8605.75
2	64	35	979	2101	.56	1084.25
1	39	47	508	3764	4.098	5837.75
9	52	58	634	1831	47.52	8755.92
	19	63	826			

8.
12.1645
9.875
.6667
5.4167

9. $3.8 + 4.05 + 6.032 + 3.4 + 26.8 + .3$

10. $.36 + .275 + .308 + .001 + 2.38 + .79$

Add:

11. $\frac{4}{9} + \frac{2}{3}$

12. $\frac{7}{8} + \frac{5}{6}$

13.	14.	15.	16.	17.
$\frac{3}{4}$	$\frac{1}{2}$	$78\frac{3}{4}$	$38\frac{2}{3}$	$26\frac{5}{6}$
$\frac{5}{16}$	$\frac{3}{20}$	$37\frac{1}{4}$	$19\frac{7}{8}$	$14\frac{2}{3}$
$\frac{3}{8}$	$\frac{3}{5}$		$5\frac{1}{4}$	$38\frac{3}{4}$

18. $22\frac{2}{9} + 15\frac{2}{3} + \frac{1}{2} + 19\frac{5}{9}$

19. $17\frac{1}{2} + 5\frac{3}{8} + 14\frac{5}{8} + 33\frac{1}{2}$

20.
7 yd. 2 ft.
6 yd.
8 yd. 1 ft.

Subtract and prove:

21.	22.	23.	24.	25.
19384	68907	6000	32106	1353675
8795	49878	1456	17248	868789

26.	27.	28.
12.3	.07	4.03
7.4	.068	2.8

29. $27.463 - 18.574$

30. $9.73 - 8.7485$

Subtract:

31. $\frac{11}{12} - \frac{1}{3}$

32. $20 - 4\frac{4}{5}$

33.	34.	35.	36.	37.
$38\frac{1}{6}$	77	$486\frac{5}{12}$	$48\frac{5}{8}$	3840
$26\frac{5}{8}$	$49\frac{5}{8}$	$249\frac{2}{3}$	$26\frac{3}{4}$	$2951\frac{7}{16}$

38. $47.06 - 5\frac{1}{3}$

39. $121.3 - 85\frac{2}{3}$

40.
14 yd. 2 ft. 8 in.
8 yd. 1 ft. 10 in.

Review of the Fundamentals of Arithmetic—8

Multiplication and Division

Multiply and prove:

1. 709	2. 604	3. 7096	4. 108	5. 1906
56	97	76	408	69
6. 478	7. 57.9	8. 56.72	9. 974	10. 63.85
96	608	5.76	5.9	3.5

Multiply:

11. $.0468 \times 10$
12. 24.86×10
13. $100 \times .89$
14. $.91 \times 1000$
15. 100×78.1
16. $.68 \times 1000$

17. $\frac{5}{6} \times \frac{2}{15} \times \frac{9}{20}$
18. $5\frac{1}{2} \times 5\frac{1}{2}$
19. $12 \times 8\frac{1}{3}$
20. $72 \times 4\frac{1}{12}$
21. $\frac{1}{3} \times \frac{4}{9} \times \frac{1}{2} \times 6$
22. $40\frac{1}{5} \times 15\frac{3}{8}$
23. $68\frac{1}{2} \times 14\frac{3}{4}$
24. $120\frac{2}{3} \times 240\frac{3}{8}$

25. $18\frac{3}{4} \times 140\frac{5}{6}$

26. 3 hr. 52 min. 45 sec.
    10

Divide and prove:

27. 5091 by 39
28. 4003 by 87
29. 4500 by 189
30. 67,447 ÷ 76
31. 684.5 by .38
32. 458.68 by .97
33. 1792 by .85
34. 34.465 by 148
35. 61 ÷ 10
36. 45.75 ÷ 100
37. 6.75 ÷ 100
38. .4 ÷ 1000

Divide:

39. $\frac{3}{10} \div \frac{2}{5}$
40. $\frac{3}{4} \div \frac{1}{2}$
41. $\frac{5}{6} \div \frac{2}{3}$
42. $2\frac{1}{4} \div 1\frac{1}{8}$
43. $8\frac{1}{2} \div 4\frac{1}{4}$
44. $9\frac{1}{7} \div 4\frac{4}{5}$
45. $6\frac{2}{3} \div 5\frac{1}{3}$
46. $11\frac{7}{9} \div 5\frac{1}{7}$
47. $12\frac{3}{4} \div 5\frac{2}{3}$
48. $10\frac{4}{5} \div 18\frac{3}{4}$
49. $34\frac{1}{2} \div 5\frac{3}{4}$
50. 3)8 bu. 1 pk.

Review of the Fundamentals of Arithmetic—9

Percentages

Express each of the following numbers as (1) a common fraction, (2) a decimal, (3) a percentage:

1. $\frac{1}{3}$
2. $\frac{1}{5}$
3. 50%
4. $\frac{3}{5}$
5. .72
6. $\frac{3}{8}$
7. .01
8. $3\frac{1}{3}$%
9. 28%
10. .875
11. $12\frac{1}{2}$%
12. .35
13. $\frac{1}{25}$
14. .075
15. 175%
16. 1.5
17. $16\frac{2}{3}$%
18. .005
19. 220%
20. $2\frac{3}{8}$

Find the following percentages:

21. 5% of 2,549

22. 150% of .186

23. $14\frac{2}{7}\%$ of 49.84

24. 12% of 12,012

25. 9% of 50,064

26. 120% of 25.5

27. 2% of \$1256

28. $1\frac{3}{4}\%$ of 52

29. 12% of \$88.50

30. $16\frac{2}{3}\%$ of 4728

31. $3\frac{1}{3}\%$ of 93

32. $1\frac{1}{2}\%$ of 169.4

33. 8% of \$47.50

34. $5\frac{1}{2}\%$ of \$250

35. 9% of 67

36. $4\frac{1}{2}\%$ of \$80

37. $12\frac{1}{2}\%$ of \$25.44

38. $\frac{2}{3}\%$ of 480

Fill in the following:

39. 10% of $? = 50$

40. $?\%$ of $40 = 30$

41. $2\frac{3}{4}\%$ of $8 = ?$

42. 2% of $? = 16¢$

43. $?\%$ of $64 = 12$

44. 3% of $? = 22\frac{1}{2}$

45. $?\%$ of $120 = 8$

46. $\frac{1}{2}\%$ of $150 = ?$

47. 72% of $19 = ?$

48. 15% of $? = 13.5$

49. $?\%$ of $\$6 = \$9$

50. $\frac{3}{4}\%$ of $500 = ?$

Solve the following:

51. 4 is what per cent of 50?

52. What per cent of 25 is 75?

53. 6 is what per cent of 45?

54. What per cent of 10 is $1\frac{1}{2}$?

55. 1 is what per cent of 6?

56. What per cent of 15 is 6?

57. 9 is what per cent of 12?

58. 60 is what per cent of 40?

CHAPTER XI

LOCAL, STATE, AND FEDERAL TAXES

VOCABULARY

1. tax
2. levy
3. revenue
4. valuation
5. assessed value
6. tax rate
7. property tax
8. gross income
9. net income
10. exemption
11. deduction
12. per capita
13. internal revenue
14. sales tax
15. luxury tax
16. excise tax
17. inheritance tax
18. tariff
19. license
20. ad valorem duty
21. specific duty

128. Why taxes are necessary. All of us enjoy the comforts and conveniences provided by our local, state, and Federal governments. We have fire and police protection, schools, public libraries, hospitals, streets, highways, parks, mail collection and delivery, and inspected meat, milk, and drugs. We see on every hand where large sums of money have been, or are being, spent for our benefit. If your father were to try to purchase for your family all those conveniences without the aid of the government, the cost would be prohibitive. We look to the government for these services, and in return each of us must bear his share of the total cost. The share each person pays is his *tax.*

129. Kinds of taxes. Both local and state governments are supported largely by taxes on property and real estate. There are special taxes on estates or inheritances of over a certain sum and on the profits of a corporation. There are luxury taxes, sales taxes, amusement taxes, occupation taxes, taxes on telegraph and long-distance telephone messages, and income taxes.

The national government is supported by about 225 different forms of taxes, including income taxes, estate taxes, internal revenue taxes, and taxes on imported articles (customs duties).

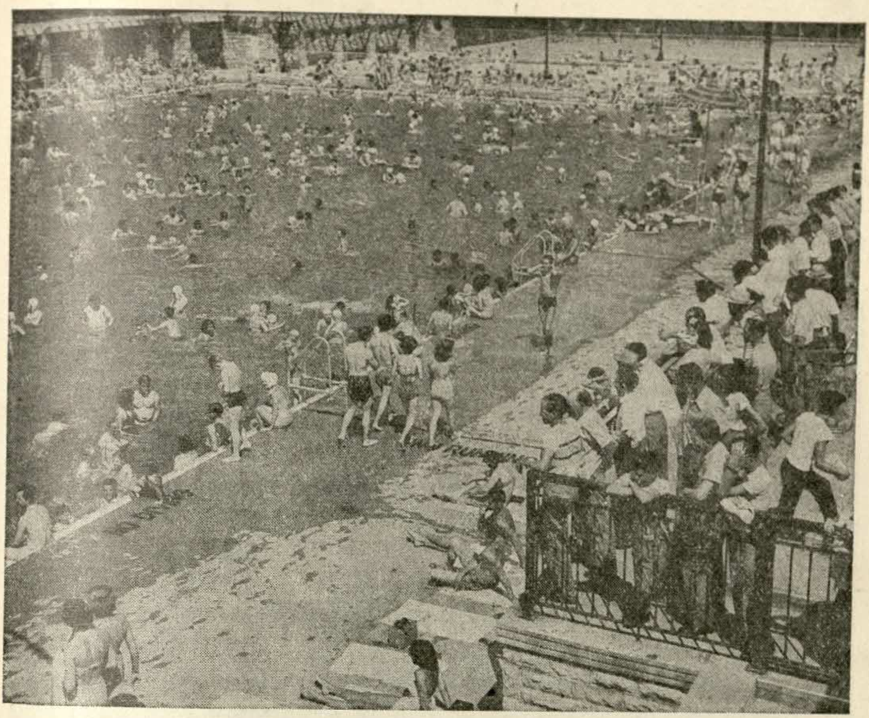


Fig. 1. The expenses of summer playgrounds and free swimming pools are paid from tax money.

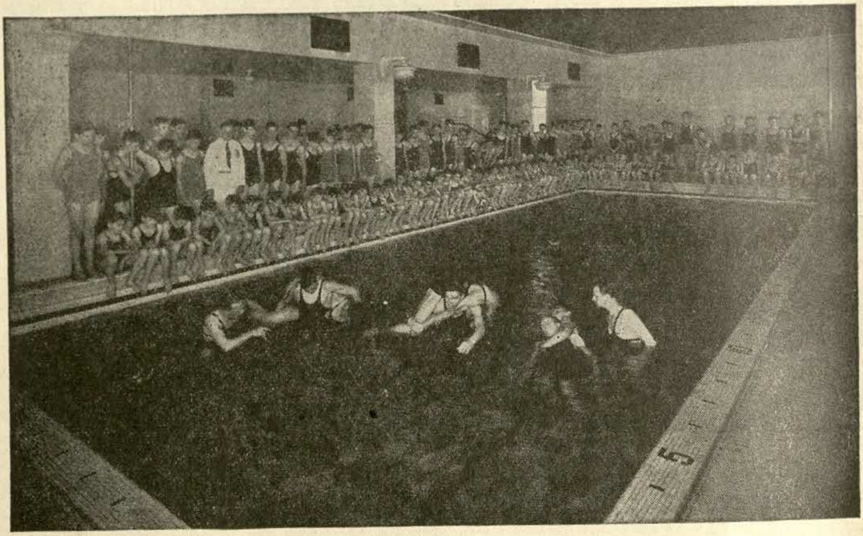
STATE AND LOCAL TAXES

130. General property taxes. States, cities, and counties tax property—real estate and personal property. *Real estate* includes all fixed and immovable property (land, buildings, forests, mines, quarries, and gas and oil wells). Personal property includes property that is easily moved from place to place, such as money, jewelry, stocks, bonds, furniture, livestock, and automobiles. Property taxes are levied in much the same way in all localities. The amount subject to tax is called the *assessed value*. Separate assessments are made for real estate and personal property.

An assessment is made, usually every year, by men called *assessors*. They place a value on every piece of property in their districts. The assessed value is not always the *market value* of the property. In some communities it is customary to assess property at a fractional part of its value—such as one-half or two-thirds of its market value.

The sum of all the assessed values make up the *total assessed value* of the community or state.

The estimated and approved amount of expenditures of a government for the next year is called its *budget*. The local executive body approves the amount to be spent by the local community, the state legislature that for the state, and Congress that for the national government.



OUR SCHOOLS are supported largely by taxes on real and personal property.

131. Fixing the amount of property tax. After the tax rate is determined, the *amount* of the tax is easily found.

Example

What will be the amount of tax on real estate assessed at $4,000 with a tax rate of 1.5%?

$4,000 × .015 = $60, the amount of tax.

To find the amount of tax, multiply the assessed valuation of the property by the rate.

A low assessed valuation on property will necessarily result in a high tax rate in that community; a high assessed valuation will result in a low tax rate. The individuals of a community with a low tax rate and a high valuation may pay a larger tax than those in another community with a higher tax rate and a lower valuation. Why?

EXERCISES

1. Jones owns property assessed at \$6,500 in a city with a tax rate of 23 mills on a dollar. How much is his tax?

2. Find the assessed valuation from each of the following actual values:

Part of Actual Value Assessed	\$1,800	\$2,800	\$3,900	\$12,500	\$37,000
40%	____	____	____	____	____
$\frac{2}{3}$	____	____	____	____	____
$\frac{3}{4}$	____	____	____	____	____
Full value	____	____	____	____	____
50%	____	____	____	____	____
$\frac{4}{5}$	____	____	____	____	____

3. Mr. Franklin owns a house and a vacant lot with market values of \$5,400 and \$900, respectively. Each is assessed at $\frac{2}{3}$ of the market value. What is the tax on each piece of property at the rate of 18 mills on the dollar?

4. Find the tax on Mr. Adams' farm, assessed at \$4,800: (a) if the tax rate is 15 mills on the dollar; (b) if the tax rate is \$2.18 per hundred dollars; (c) if the tax rate is \$15 per thousand dollars.

5. Complete the following table by filling in the correct forms of each rate:

Per Cent Rate	*Mill Rate on the Dollar*	*Dollar Rate on the Hundred*	*Dollar Rate on the Thousand*
2.5%	____	____	____
____	\$.017	____	____
____	____	\$2.30	____
____	____	____	\$19.00

6. In the city of "X" the assessed valuation of all property is \$14,850,-000 and the tax rate is 17 mills. How much is due in taxes?

7. Mr. Fry lives in "X" and owns property assessed at \$5,400. How much is his tax on the property?

8. Mr. Allen owns a farm assessed at \$7,800, 16 cows assessed at \$45 each, 4 horses at \$145 each, and 2 mules at \$190 each. How much is his property tax, if the rate is \$1.75 per hundred dollars?

9. If a city increases the property tax rate from \$1.90 to \$2.50 per \$100

valuation, how much will the taxes of Black & Son be increased on a valuation of $26,500?

10. Find the amount of tax due on each of the following properties:

Market Value	*Per Cent Valuation*	*Assessed Valuation*	*Tax Rate*	*Amount of Tax*
$ 8,000	40%	———	$4.50 per $100	———
$ 5,500	80%	———	3.75%	———
$ 2,500	100%	———	24 mills	———
$ 15,750	66⅔%	———	$36.50 per $1,000	———
$125,000	87½%	———	$48.90 per $1,000	———
$200,000	75%	———	18.9 mills	———
$250,000	50%	———	$3.75 per $100	———

11. Mr. Smith owns property assessed at $5,600 and is subject to the following tax rates per $100:

State tax	$.23
County tax	.39
City tax	1.27
School tax	1.05
Total tax rate	$2.94

(a) How much is each tax? (b) How much is his total tax?

12. The expenditures of a city are $4,008,900. If the population is 232,400, what is the expenditure per capita?

13. (OPTIONAL) In Merryville the budget for school expenditures for one year is as follows:

Salaries	$115,400
Repairs	5,725
Operating expenses	14,250
School bus	1,055
Textbooks	1,290
Library	4,280

What tax rate per $100 is required to meet the budget, if the total assessed value of the real and personal property is $5,680,000?

132. State sales tax. More than half of the states have a tax, known as a *sales tax,* which is levied upon retail sales. The tax varies from 1 to 3 per cent of the purchase price, and, in most cases, the customer pays the per cent to the merchant, who in turn sends the tax to the state.

EXERCISES

1. Mrs. Mills buys merchandise to the amount of $8.60 plus a state sales tax of 2 per cent. What is the total amount of her purchase? Count her change out of a $10 bill.

2. Mr. Foote purchased a $65 suit plus a $2\frac{1}{2}$ per cent sales tax. What was the total cost of his suit?

3. Find the cost, including a sales tax of 3 per cent, for these rugs:

1 Rug, size 7 × 9 ft. @ $15.09 | 2 Rugs, size 2 × 5 ft. @ $9.50

4. Find the cost, including a sales tax of 2 per cent, on the following sales:

2 Wool blankets @ $10.80 | 2 Cotton blankets @ $2.68
3 Cotton blankets @ $2.75

5. What is the cost, including a $2\frac{1}{2}$ per cent sales tax, on the following order?

3 Handkerchiefs @ 39¢ | 1 Pocket Comb @ 15¢
1 Belt @ 75¢ | 4 Spools Thread @ 9¢
1 pair Shoes @ $11.70

6. Mrs. Casey purchased the following order, giving the clerk $20 in currency. (a) Find the total of the bill, including a 2 per cent sales tax. (b) Count Mrs. Casey's change back to her.

6 Ties @ $1.39 | 5 pr. Wool Hose @ $1.15 | 2 pr. Gloves @ $1.65

7. Figure the following order, including a 1 per cent sales tax:

5 loaves Bread @ 19¢ | 2 lb. Coffee @ 89¢
2 lb. Butter @ 87¢ | 2 doz. Eggs @ 62¢
12 lb. Sugar @ 9¢

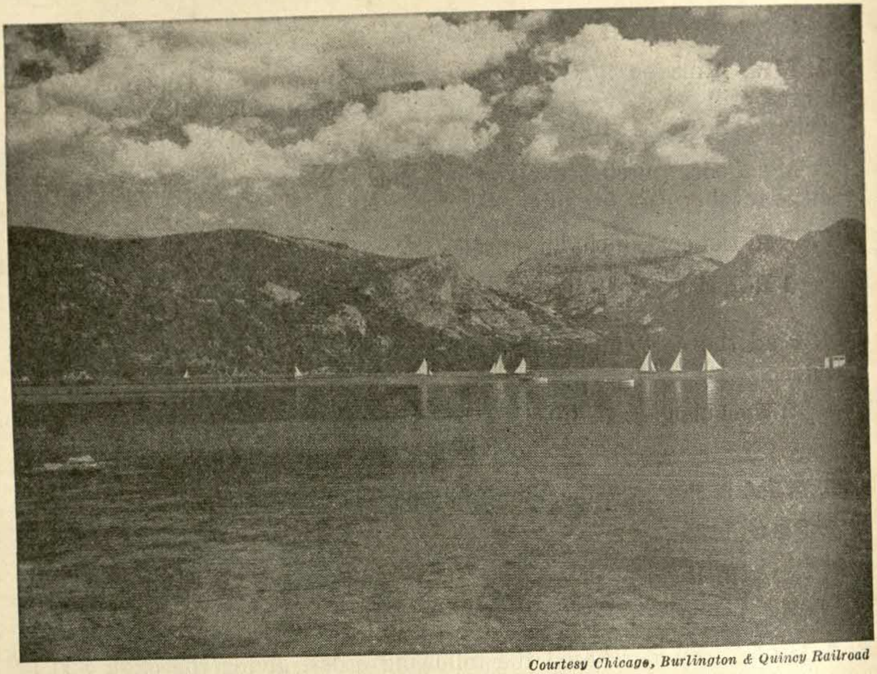
8. What is the total cost of the following order if there is a $2\frac{1}{2}$ per cent sales tax?

6 gross Pins @ $1.65 | 38 yd. Linen @ $2.18
13 yd. Tape @ 27¢ | 19 yd. Silk @ $4.39
27 yd. Ribbon @ 38¢

9. A sales tax of 2 per cent is paid on merchandise purchased in certain states. How much sales tax—in cents and mills—would be paid on groceries costing $4.50? On books costing $5.20? On clothing costing $18.75?

FEDERAL TAXES

133. Federal income tax. The income tax was first adopted by the Federal Government during the War between the States, but was repealed in 1870 and declared unconstitutional. The adoption



Courtesy Chicago, Burlington & Quincy Railroad

Fig. 2. OUR BEAUTIFUL NATIONAL PARKS are purchased and maintained by the Federal Government for our enjoyment.

of the Sixteenth Amendment in 1913 gave Congress power "to lay and collect taxes on incomes from whatever source derived." The income tax was re-adopted in 1913 and is now a source of much revenue. Income taxes are also levied on corporations.

The income tax is a *direct tax*; that is, it is paid directly by the person on whom it is levied.

The Federal income tax return with declaration of the amount of money earned and declarations of deductions and credits must be filed by March 15 of each year. In 1943 a plan was adopted by which a part of salaries and wages are withheld, so that income taxes may be paid *as the income is earned* during the year.

The information* to be given on the Federal income tax return is divided into four groups: (a) that having to do with your salary plus any other income, (b) that which deals with your allowable deductions, (c) that which allows your exemptions, and (d) that which has to do with the computation of your tax.

*The exemptions and rates of Federal income taxes are subject to change by Congress and may vary from year to year.

Income varies with individuals because some persons have only a salary to report while others, who may or may not have a salary, may have dividends from one or more shares of stocks, interest from money, rents from property owned, or profits from business deals. With few exceptions, all sources of income must be totaled. This total is the *gross income.*

Deduction rules may change from year to year, but the purpose of deductions remains the same; that is, to prevent the taxpayer from being taxed unfairly. The government expects you to pay your correct tax—no more and no less. People who have given money to religious, scientific, educational, or charitable institutions, or who have suffered losses from calamities for which they received no insurance, or from debts that are not collectible, are allowed to subtract a figured amount from their income. The income tax law assumes that a person who has had such drains on his income should not have to pay a tax on the full amount of his income. The deductions are subtracted from the gross income. The remainder is the *net income.*

Exemptions are amounts allowed by the government for personal expenses of the taxpayer and his dependents. At present $600 is allowable for the taxpayer himself, his wife, unless she is earning and filing a separate return, and any dependents who are not earning $600 or more. Exemptions are subtracted from the net income. The remainder is the amount upon which the tax is calculated.

While the computation of the amount of tax varies from time to time, there has always been a personal exemption allowable for the head of a family, a smaller exemption for a single wage earner, and an allowance for each dependent. A person who is blind or who is 65 years of age or older is allowed two exemptions for himself. If a person is both blind and 65 years of age or older, three exemptions are allowed. An income tax return requires careful preparation and a knowledge of income and expenditures. A great help in computing income taxes is an accurately kept record of income and expenditures.

Example

Melvin Rhodes has a wife and one dependent child. He has an annual salary of $3,600 and a $400 income from interest and dividends. His deductions amount to $240. They are: contributions to church and charity, $85; interest payment on house, $75; state, county, and city taxes, $80

(Federal taxes are not deductible). His exemptions amount to $1,800 (three persons at $600 each).

(*A*) *Income*

1. Salaries and other compensation for personal services	$3,600	
2. Dividends	150	
3. Interest on bank deposits, notes, corporation bonds	150	
4. Interest on Government obligations	100	
5. Rents and royalties		
6. Net profit from business or profession		
7. Other income		
8. Gross income		$4,000

(*B*) *Deductions*

9. Contributions paid	$85	
10. Interest on debts	75	
11. Taxes	80	
12. Losses from fire, theft, etc.		
13. Medical, dental, etc., expenses		
14. Bad debts		
15. Total deductions		$ 240
16. Net income (item 8 minus item 15)		$3,760

(*C*) *Exemptions*

17. Exemptions	$1,800
18. Amount subject to tax	$1,960

The $1,960 is the amount of income upon which Mr. Rhodes will pay tax. He follows the directions which come with his tax form to determine the amount of his income tax.

Usually it is cheaper for married couples to file a joint return rather than for each one to file a separate return.

EXERCISES

1. Mr. Blair's income consists of a salary of $4,000, dividends of $80, interest of $175, and rents of $480. Find his total income.

2. Mr. Blair's deductions are: contributions, $125; taxes, $85; and bad debts, $100. Find his total deductions.

3. How much is Mr. Blair's net income?

4. Mr. Black's income consists of a salary of $12,000; dividends amounting to $40, $65, and $15; interest from various sources, $25, $38,

and $76; and an income of $780 from a patented article. What is his gross income?

5. Mr. Black had these deductions: contributions to church, $250; to charity, $180; to scientific research, $25; to an educational fund, $16; to the American Red Cross, $8; and to the Boy Scouts of America, $6. He also deducted: taxes, $140 and $28; bad debts, $72; other deductions, $87. Find the total amount of his deductions.

6. What is Mr. Black's net income?

7. Miss Case's gross income is $2,500, and she has paid out a doctor's bill for $80, a hospital bill for $100, a dentist's bill for $60; for medicine, $18; and for health insurance, $30. She may not deduct these sums from her gross income, but she is allowed a deduction of the difference between 5% of her gross income and the total of the above expenses. Find the amount of the deduction.

8. Mr. Barker owns 46 shares of stock with a par value of $100 per share. If he has an annual dividend of 4½ per cent from them, how much is his income increased by this dividend?

9. If Carl Cross has a gross income of $2,760.30, a personal exemption of $600, and a 10 per cent deduction of his gross income, find his balance subject to tax.

10. If Mr. Singer has a gross income of $5,240.25 and a personal deduction of $600, credit for two bad debts of $500 each, and a 10 per cent gross income deduction, what is his balance subject to tax?

11. Mr. Carl Jackson has a salary of $4,800 per year. He has interest on Government bonds of $20, interest on savings account of $17.50, dividends on stock of $25, and rent on garage of $30. He has paid $82.50 to church and charity. His taxes, not including Federal taxes, amount to $125.18. He has a bad debt of $50.

Mr. Jackson's wife is not employed. They have two dependent children. On what amount of income will Mr. Jackson pay a Federal income tax?

12. (OPTIONAL) Obtain a Tax Rate Schedule and calculate the income tax of Mr. Rhodes (in the Example), Mr. Singer (exercise 10) and Mr. Jackson (exercise 11).

134. Import duties. In the United States the taxes or duties on imported articles yield about $400,000,000 income a year. Duties are collected at customs houses at the ports of entry into the United States. The list of articles and the duty on each are fixed by Congress and are called the *tariff*. Articles on which no tax is levied are said to be *on the free list* or to be *duty free*.

Duties are of two kinds, *ad valorem* and *specific*. The ad valorem duty is a fixed per cent of the value of the goods in the foreign

country. The specific duty is a fixed tax per article, yard, pound, dozen, or other unit of measure without regard to value. Both of these duties must be paid on some articles. (See the table of duties.)

A tariff is levied not only for the revenue but also for the protection of our industries and agriculture against foreign competition. If the Federal Government did not provide this protection, many products of our own people would be crowded out of our own markets by goods produced in countries where the standard of living is low and costs of production are kept down by low wages.

A PARTIAL LIST OF IMPORT DUTIES ON ARTICLES BROUGHT INTO THE UNITED STATES

Imported Article	*Import Duty*	
	Ad Valorem	*Specific*
Butter		7¢ lb.
Cameras	20%	
Cosmetics and perfume	20%	
Diamonds (uncut)	10%	
Knives	$17\frac{1}{2}$%	8¢ ea.
Linen	$22\frac{1}{2}$%	
Olive oil		$4\frac{3}{4}$¢ pt.
Oriental rugs	30%	
Playing cards	5%	5¢ pack
Safety razors	$12\frac{1}{2}$%	
Sugar		.47¢ lb.
Toys	35%	

Example

Find the duty on 10 doz. knives purchased in a foreign country for 13¢ each.

The table shows that the ad valorem duty is $17\frac{1}{2}$% and the specific duty 8¢ each.

Ad valorem duty:	120 × \$.13 = \$15.60 cost. . . $17\frac{1}{2}$% of \$16. . .	\$2.80*
Specific duty:	120 × \$.08	9.60
		\$12.40

*Ad valorem duties are calculated on whole dollars only; fifty cents or more counts as a whole dollar.

EXERCISES

1. Find the duty on the following imported articles:

(a) 784 pt. olive oil.
(b) 150 lb. butter.
(c) $7,550 worth of toys.
(d) 15 Oriental rugs valued at $375 each.
(e) 10 tons sugar.
(f) Toys valued at $728.

2. At a cost of $18\frac{3}{4}$ cents per pound, butter is purchased in Switzerland. How much duty is charged for 7,952 lb. imported? What is the total cost in the United States?

3. If 1,470 lb. of cosmetics at 48 cents per pound are imported from France, what is the duty?

4. If the duty on sugar amounts to about $89,100,000 per year, how many pounds are imported?

5. What is the duty on 5,000 safety razors that cost 65 cents each?

6. What is the import duty on 52 yd. of lace 12 in. wide at 28 cents duty per yard? If the cost in Ireland was $1.75 per yard, what is the total cost?

7. (OPTIONAL) Find the tax on 480 lb. of goods valued at $1,260, imported with duties of 19 cents per pound and at 18 per cent ad valorem.

8. (OPTIONAL) Scott & Company, importers of furs, purchased 640 furs from Canada for $5,280. The import duty is 10 per cent. (a) How much duty is paid? (b) What is the total cost? (c) What is the average cost of each fur in Canada? (d) What is the average cost of each fur to Scott & Company?

135. Excise taxes. Taxes on the manufacture, sale, and consumption of certain goods are called *excise taxes.* These taxes bring in about 14 per cent of the government's revenue. The articles that bring in most of this revenue are tobacco, alcoholic liquors, and gasoline.

Excise taxes are paid by the manufacturers or by the retailers. In most cases the amount of the tax is included in the price paid by the customers who use the goods or service. These "hidden" taxes are called *indirect taxes.* It is difficult to know just how much is paid in excise taxes because they are included in the selling prices of most of the goods.

136. Special taxes. Other national and state taxes in the form of licenses (automobile, occupation, hunting, and fishing) are collected. States also levy a tax on property (estates) left by deceased persons.

The national government also collects an *estate tax* on estates of more than \$60,000 valuation.

EXERCISES

1. In a certain city, gasoline retails for 12.5 cents per gallon, the state tax is 4 cents per gallon, and the Federal tax is 1 cent per gallon. What is the total cost of 14 gallons?

2. In a certain state, one cent of its 4 cents state gasoline tax is spent for education and the remainder goes into the road fund. If the income for one year from the gasoline tax is \$13,500,000, how much of it is spent for education? How much is left for the road fund?

3. Find the income received by a city from these special licenses:

15 plumber's licenses at \$15 each
115 restaurant licenses at \$13 each
78 grocer's licenses at \$13 each
9,647 automobile licenses at \$3 each
9,947 dog licenses at \$.50 each
165 hunting and fishing licenses at \$2 each

4. One state and Alaska now levy a tax of 2¢ on every gallon of gasoline sold; 4 states, a 3¢ tax; 18 states and District of Columbia, a 4¢ tax; 2 states, $4\frac{1}{2}$¢; 8 states, 5¢; 1 state, $5\frac{1}{2}$¢; 9 states, 6¢; 1 state, $6\frac{1}{2}$¢; 3 states, 7¢; and 1 state, 9¢.* What is the average gasoline tax per gallon in the United States (leaving out Alaska)?

5. The District of Columbia had receipts of \$5,098,000 in one year from a 4-cent gasoline tax. How many gallons of gasoline did this represent?

6. New York in one year had, with 2,126,400,000 gallons, a larger consumption of gasoline than any other state. If the tax is 4 cents per gallon, what was the gross revenue?

7. If a state spends annually 38 per cent of its sales tax for charity, 26 per cent for old-age pensions, 15 per cent for schools, and the rest for prisons, how much is spent for each, if the revenue from the tax is \$6,495,850?

8. If the \$2,197,209 receipts from the gasoline tax in the District of Columbia in one year represented a gain of 8.6 per cent, what were the receipts the previous year?

9. Make a bar graph showing that in the 1950 Federal Budget the receipts amounted to approximately \$37,800,000,000 and the expenditures to \$43,300,000,000.†

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*1950 World Almanac.
†Charts relating to The President's Budget Message for 1951.

SUMMARY QUESTIONS

1. What is a tax?

2. List five improvements in your community made possible by taxes. Justify taxation for these improvements.

3. How many of the following does your community provide for you?

(a) firemen	(e) boulevards	(i) schools
(b) parks	(f) street lights	(j) courts
(c) libraries	(g) playgrounds	(k) sidewalks
(d) highways	(h) water works	(l) sewers

4. What is the difference between assessed value and market value? When are they the same?

5. List as real or personal property: (a) farm, (b) radio, (c) garage, (d) motorboat, (e) rock quarry, (f) jewelry, (g) bonds, (h) automobile, (i) cows, (j) coal mine, (k) money, (l) gas well.

6. How is the amount of taxation for personal property determined?

7. Explain a tax rate of 18 mills.

8. How does the United States Government support its defense organizations?

9. List ten sources of national revenue.

10. What is the value of the government budget?

11. What is a sales tax?

12. What is a luxury tax? Name three.

13. Give two purposes of customs duties.

14. Explain the difference between specific and ad valorem duties.

15. Name five deductions allowed in the Federal income tax.

16. Explain the difference between exemptions and deductions as used in the income tax.

17. List as direct or indirect taxes: (a) income tax, (b) personal property tax, (c) internal revenue tax, (d) real estate tax, (e) dog tax, (f) automobile driver's license, (g) inheritance tax, (h) luxury tax, (i) excise tax on tobacco, (j) school tax.

18. (OPTIONAL) Is it right to tax improvements on real estate? Why?

19. (OPTIONAL) Explain the sales tax in your state.

20. (OPTIONAL) What is the "gift tax"?

21. (OPTIONAL) Copy and explain the Sixteenth Amendment to the Constitution.

22. (OPTIONAL) Why are schools, hospitals, charitable institutions, and churches not taxed?

23. (OPTIONAL) Paying taxes is a co-operative matter. Explain.

CHAPTER XII

POSITION AND FORM OF OBJECTS

VOCABULARY

1. position	4. slanting	7. meridian
2. horizontal	5. parallel	8. longitude
3. vertical	6. perpendicular	9. latitude

137. Horizontal, vertical, and slanting lines. Straight lines that are in the same relative position as the horizon are said to be *horizontal*. Such lines as the ones on your tablet paper, street car tracks on level ground, boards in a floor, crosspieces on telephone poles, and the line formed by the meeting of a wall and ceiling in a room are horizontal lines. Locate ten more horizontal lines to be found in the classroom. Write ten that you recall seeing outside the classroom.

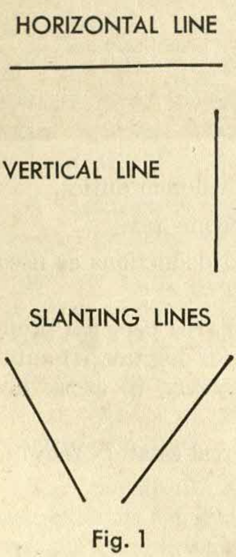


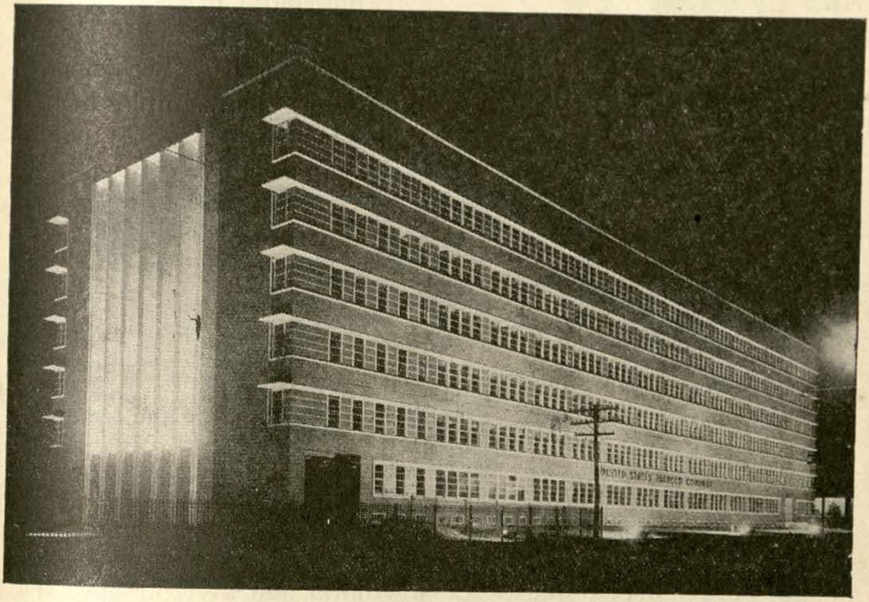
Fig. 1

Lines that are at right angles to the horizon are *vertical*. A vertical line points to the center of the earth. Such lines as telephone poles, fence posts, and tree trunks are vertical. The line formed by the joining of the front wall with a side wall is a vertical line. The up and down edges of a door are also vertical lines. Locate ten other vertical lines in the classroom. Write ten that are found out-of-doors.

Any straight line that is not horizontal or vertical is a *slanting* line (figure 1). Trolley poles on street cars and electric busses, and most limbs of trees form slanting lines. The hands of a clock are slanting except when they point to 3, 6, 9, or 12. At quarter to and quarter past 3 and 9 they are horizontal; at 6 and at 12 they are vertical.

1025—Prentice-Hall: Maths.

The edge of a church steeple is a slanting line. The path of falling raindrops is usually slanting. List five other examples of slanting lines. Hold a pencil in a horizontal position, in a vertical position, in a slanting position.



HORIZONTAL AND VERTICAL PARALLEL LINES IN A BUILDING

138. Parallel lines. Parallel lines may be arranged in a horizontal, vertical, or slanting position. These positions of parallel lines are illustrated in figure 2. By holding two pencils, illustrate each of these three positions in parallel lines. Parallel lines may be extended indefinitely, and yet they will never meet. They will always remain

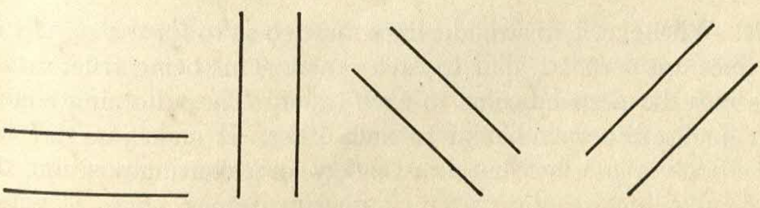
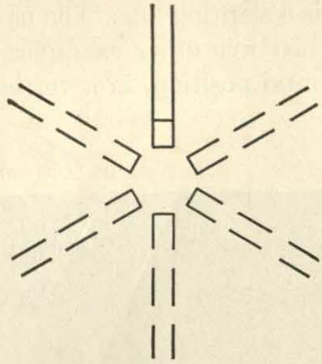


Fig. 2

Fig. 3



the same distance apart throughout their entire length. The edges of a pencil are parallel. The pencil may be placed in vertical, horizontal, or slanting positions and its sides will remain parallel. (See figure 3.) Name ten examples of parallel lines.

EXERCISES

Find out:

1. How parallel lines can be drawn on paper with only a ruler and pencil.

2. How horizontal parallel lines are draw with a T-square.

3. How vertical parallel lines are drawn with a T-square and triangle.

4. How a carpenter draws parallel lines.

5. How one can make the pickets of a fence parallel without using a ruler to measure each picket.

139. Perpendicular lines. All horizontal and vertical lines that meet are perpendicular to each other. (See figure 4.) The angle formed by the meeting of a horizontal and a vertical line is a *right*

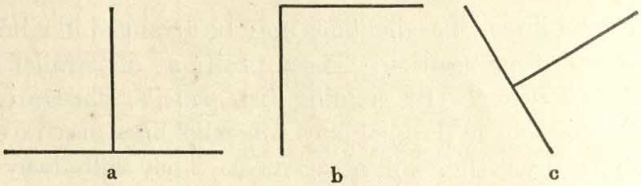


Fig. 4. Perpendicular lines.

angle. Whenever two straight lines meet so as to form a right angle, the lines are perpendicular to each other. This being true, slanting lines may be perpendicular to each other. The adjoining edges of your book are perpendicular to each other. It makes no difference if the book is in a horizontal, a vertical, or a slanting position. Perpendicular lines are illustrated by picture frames, edges of window panes, and so forth. Name ten illustrations of perpendicular lines.

EXERCISES

Find out:

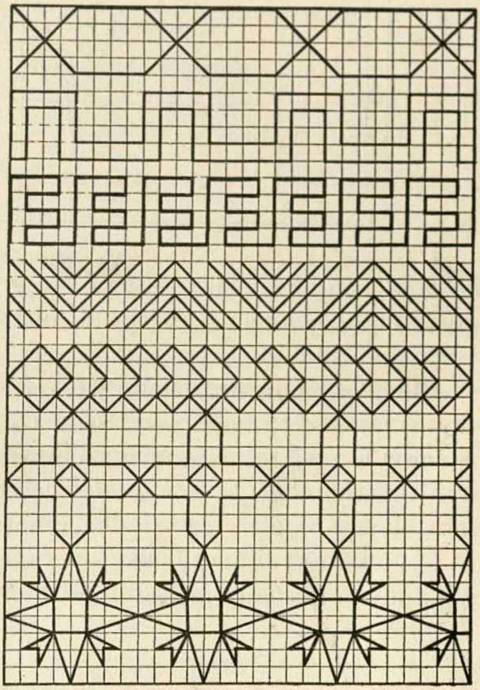
1. How perpendicular lines are drawn with a T-square and triangle.

2. How a perpendicular can be drawn from a point to a line with ruler and pencil.

3. How a carpenter can tell when a wall is plumb (perpendicular).

4. How a nail on a string can be used to tell when a post is vertical.

140. Straight-line designs. Designs are much used for ornamentation. Straight-line designs are often found in towel ends, handkerchief borders, dress patterns, and tiling.



EXERCISES

1. On squared paper, draw the straight-line designs illustrated. Keep clear-cut corners. Use a sharp pencil and a good ruler.

2. Copy straight-line designs found elsewhere.

3. (OPTIONAL) Make original straight-line designs suitable for the border of a handkerchief, for wall paper, and for a rug. The designs may be colored or shaded.

141. Position of places on a map. Places on the surface of the earth are located by means of imaginary lines running north and south, east and west. (See figure 5.) The lines running north and south are called *meridians.* They extend from pole to pole. The meridian which is used as the guide line is the one passing through Greenwich, near London, England. Any place that is east of this meridian (the *prime meridian*) is said to be so many degrees *east*

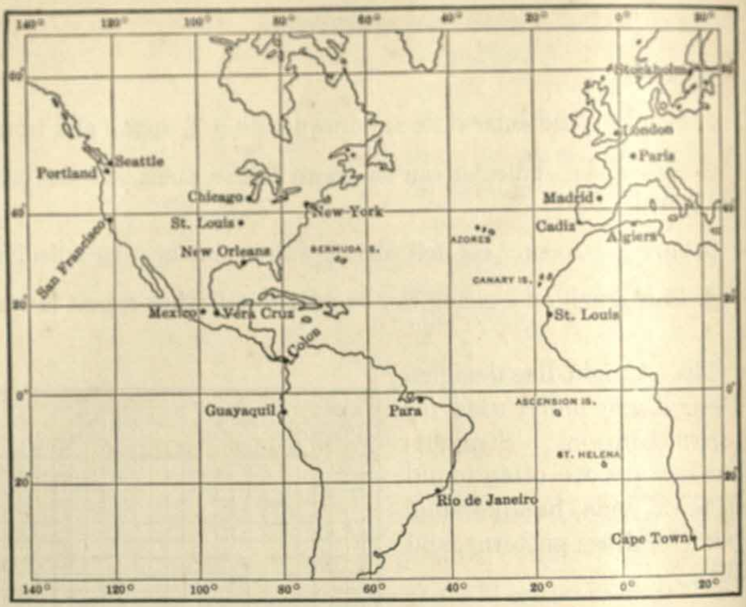


Fig. 5

longitude, while a place west of the prime meridian is so many degrees *west longitude.* Degrees of longitude are read at the top and bottom of a map.

The lines running east and west are called *parallels.* The parallel that is used as the guide line is the equator. Any place north of the equator is said to be so many degrees *north latitude,* while a place south of the equator is so many degrees *south latitude.* Degrees of latitude are read at the sides of a map.

Any number of meridians or parallels may be drawn on the map, but they are usually placed 5°, 10°, or 20° apart.

EXERCISES

Use figure 5 in solving the following exercises:

1. Give the approximate location of each of the following places:

Place	*Longitude*	*Latitude*
Vera Cruz	95° W.	18° N.
San Francisco	____	____
Stockholm	____	____
Guayaquil	____	____
Azores	____	____

Place	Longitude	Latitude
New Orleans	______	______
New York	______	______
Colón	______	______
Paris	______	______
Chicago	______	______

2. What city is approximately 5°W. longitude and 42°N. latitude?

3. What city is near 90°W. longitude and 38°N. latitude?

4. Name an island near 5°W. longitude and 18°S. latitude.

5. Name the city not far from 18°E. longitude and 35°S. latitude.

THE FORM OF PLANE FIGURES

VOCABULARY

1. plane figure	7. trapezoid	13. circle
2. quadrilateral	8. hexagon	14. circumference
3. parallelogram	9. octagon	15. diameter
4. rectangle	10. pentagon	16. radius
5. square	11. decagon	17. radii
6. triangle	12. regular figure	18. arc

142. Plane figures. Shadows on a wall are plane figures because they have only two dimensions, those of length and width. Shadows have no thickness. Any figure cut from paper or printed on cloth may be considered a plane figure, since it has only two dimensions, length and width. Most paper or cloth is so thin that its thickness may be disregarded.

To make cloth attractive to the eye, decorative designs are often woven into it. Or, if the cloth is woven plain, a design may be printed or stamped on. This practice has grown until today we find shops filled with lovely prints, many of which have designs based on geometric figures. Consider a few common geometric designs as found in cloth.

Figure 6 illustrates a plane figure bounded by four straight lines. Any such figure is called a *quadrilateral*. There are several kinds of quadrilaterals.

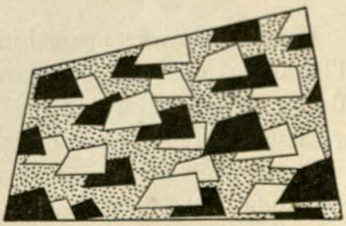
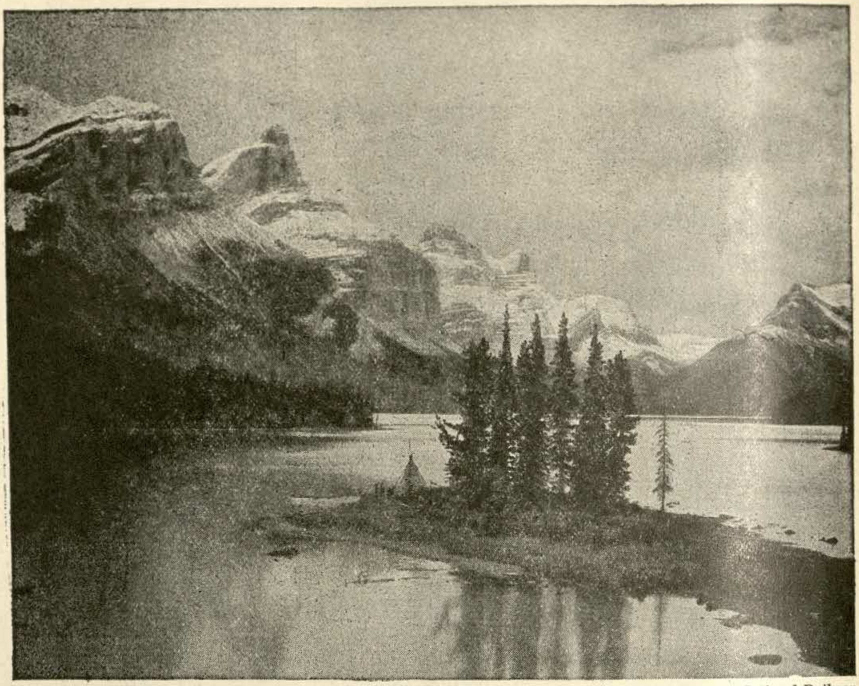


Fig. 6

Figure 7 illustrates a special kind of quadrilateral, called a



Courtesy Canadian National Railway

The surface of quiet water is a plane.

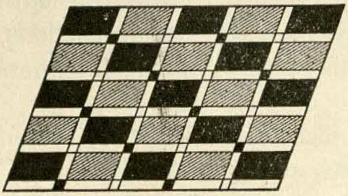


Fig. 7

parallelogram. In this figure both pairs of opposite sides are parallel and equal. Why are the quadrilaterals in figure 6 not parallelograms? All parallelograms are quadrilaterals, but not all quadrilaterals are parallelograms. Explain why this statement is true.

Figure 8 shows a special kind of parallelogram, called a *rectangle.* The angles of a rectangle are all right angles. Why is a rectangle also a parallelogram and a quadrilateral? What is true about a rectangle that need not be true about a parallelogram?

Figure 9 is a cloth pattern composed of *squares.* A square is a rectangle with all the sides equal in length. What kind of angles has a square?

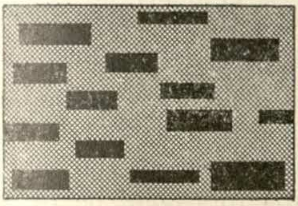


Fig. 8

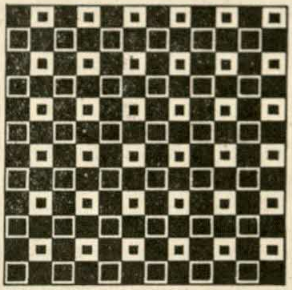


Fig. 9

The *trapezoid*, figure 10, has only one pair of parallel sides. How does a trapezoid differ from a parallelogram? Why is a trapezoid also a quadrilateral? Are the parallel sides equal in length?

In what way are figures 6, 7, 8, 9, and 10 all alike? They are classed as what kind of a plane figure? Which of the figures named is the most general? Which is the most restricted?

The design of figure 11 is composed of *triangles*. "Tri" means "three." A triangle is a plane figure containing three angles. How many sides has a triangle?

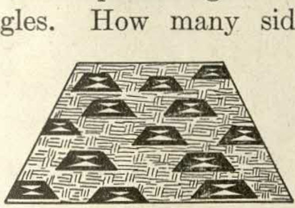


Fig. 10

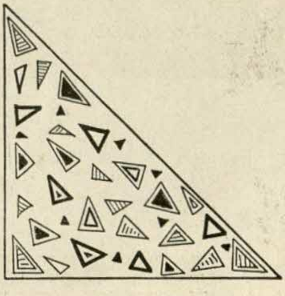


Fig. 11

Figure 12 illustrates a *hexagon*, a six-sided plane figure. A plane figure of eight sides is called an *octagon*; a *pentagon* has five sides; and a *decagon* has ten sides. In design work these figures usually

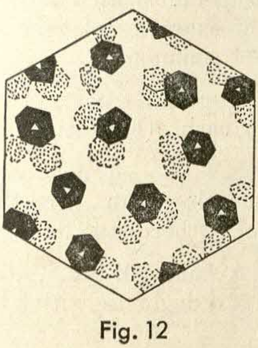


Fig. 12

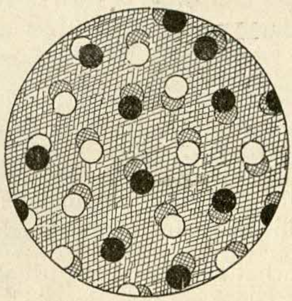


Fig. 13

have equal sides. Such figures are called *regular* hexagons, *regular* octagons, *regular* pentagons, and *regular* decagons.

The *circle*, figure 13, forms the basis of many an interesting design.

The circle is a curved line all points in which are equidistant from a fixed point within, called the *center*. The length of the curved line is called the *circumference*. A *diameter* is a line that passes through the center of the circle and whose ends are on the circle. A *radius* is a line drawn from the center to a point on the circle. *Radii* is the plural of *radius*.

NAVAJO INDIANS spinning wool into yarn to be used in making rugs. Notice the straight-line designs.

EXERCISES

1. Make a drawing of each of the plane figures illustrated and write the correct name under each. List a number of examples of each that are found in your home. Which shape is the most common?

2. A door is the shape of a rectangle. List ten other parts of buildings and name the geometric shape represented by each. (Find as many different shapes as possible.)

3. Point out some rectangles found in the classroom. Are there any squares in the room? Any triangles? Circles? Other plane figures?

4. In what shape is a rug usually made? A saucepan lid? A towel? A napkin? Tiling in the bathroom? The front of a doghouse with a hip roof? A handkerchief? The end of a flower box?

5. What geometric forms do the outlines of these forms of nature suggest?—Snowflakes, top view of cells of honeycomb and wasp's nests, starfish, clover leaf, spruce trees, wildrose, pansy, and trillium.

6. The symbol used by the Y-Teens is a triangle. List other insignia, emblems, or trade-marks that are geometric forms.

143. Plane-figure designs. Borders and other decorations often contain attractive plane-figure designs.

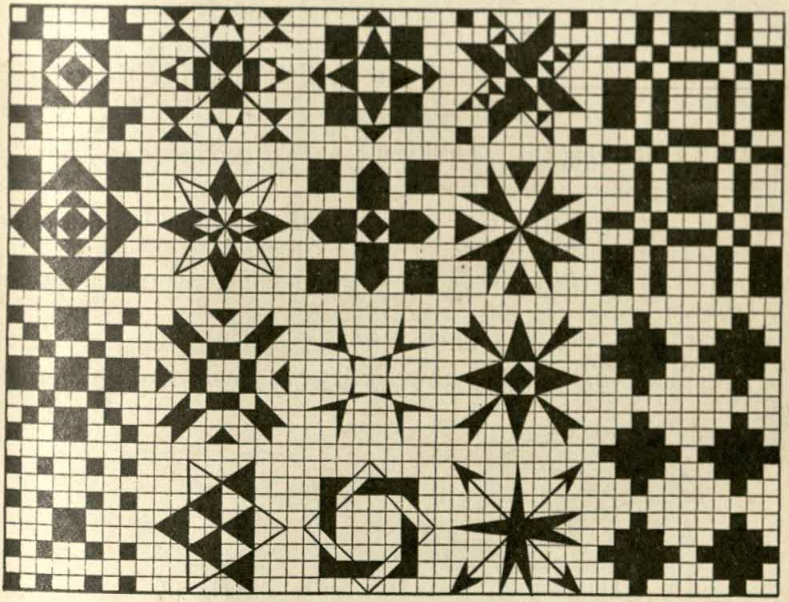


Fig. 14. Plane-figure designs.

EXERCISES

1. Make a copy of the designs shown in figure 14.

2. (OPTIONAL) Make some original designs. The designs may be colored or shaded.

144. How to draw a circle, using the compasses. Learn to adjust your *compasses* to different distances between the pencil point and the steel point. For best results when drawing a circle with compasses, follow this order of procedure:

(a) Adjust the distance between the points of the compasses so

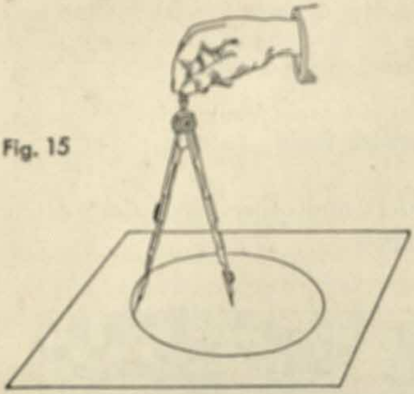


Fig. 15

that it will equal the length of the radius of the circle that is to be drawn.

(b) Hold the compasses at the top, between the thumb and the forefinger.

(c) Stick the steel point into the paper, bearing down lightly.

(d) Starting with the pencil point far to the left, swing it clockwise around the point as an axis, thus drawing the circle.

EXERCISES

1. Practice drawing circles until you can make good ones easily.

2. Draw a circle whose radius is 1 in.

3. Using the same center that was used in the above circle, draw circles whose radii are $1\frac{1}{2}$ in., 2 in., and $2\frac{1}{2}$ in. Circles drawn so as to have the same center are *concentric circles*. Give some examples of concentric circles that you have seen.

4. What are the diameters of the circles in exercise 3?

5. On a line 6 in. long, draw three designs similar to the one shown in figure 16.

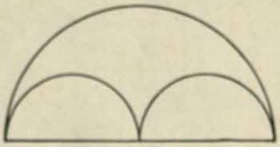


Fig. 16

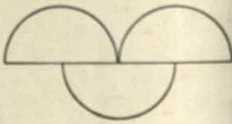


Fig. 17

6. Draw a continuous design similar to the one shown in figure 17.

7. Draw a regular hexagon in the following manner:

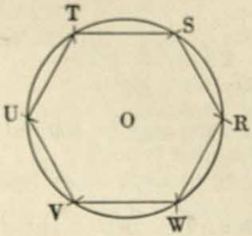


Fig. 18

(a) Draw a circle.

(b) Keep the points of the compasses to a distance equal to the radius of the circle.

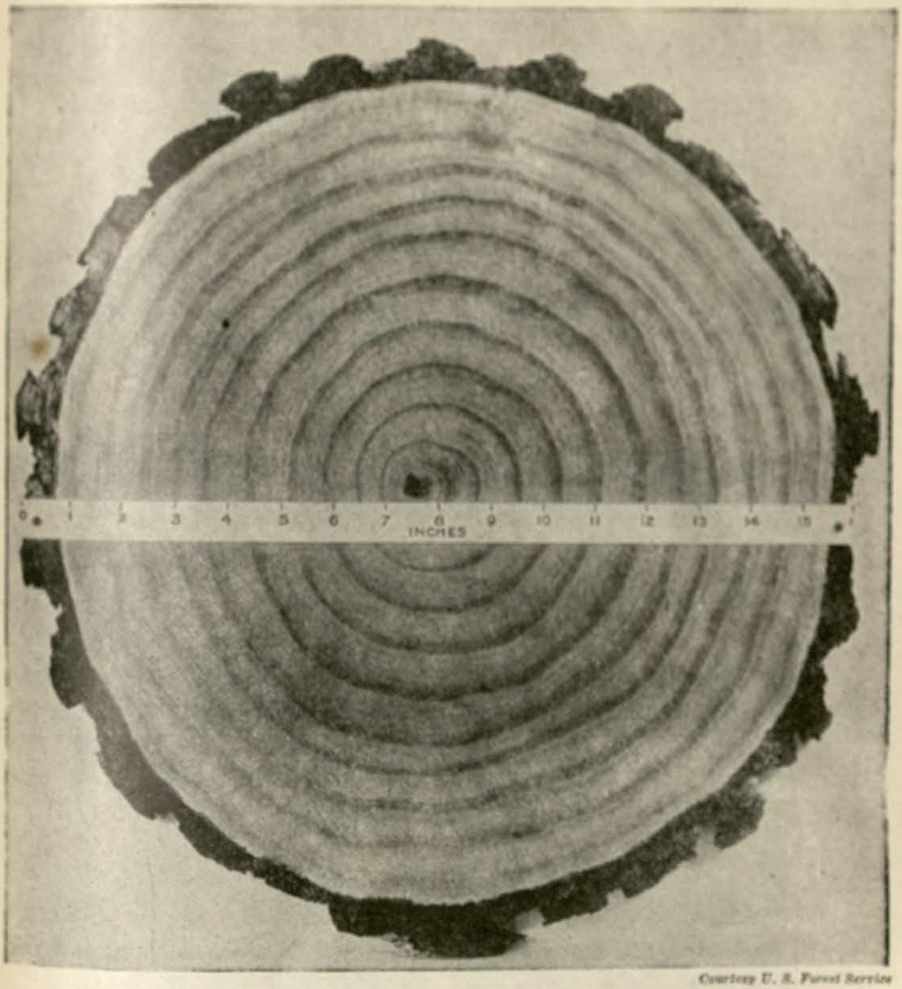
(c) Place the steel point of the compasses at any point upon the circle.

(d) Mark off, using the compasses and starting from this point, six successive points on the circle.

(e) Draw a line connecting the successive points of division.

The figure thus formed is a regular hexagon, as shown in figure 18.

8. Draw an equilateral triangle. Use the preceding method, joining every other point on the circle.



Courtesy U. S. Forest Service

CROSS SECTION OF A TREE TRUNK SHOWING YEARLY RINGS OF GROWTH. Circles with the same center are called "Concentric" circles.

9. Study figure 19. Construct it.

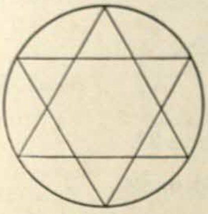


Fig. 19

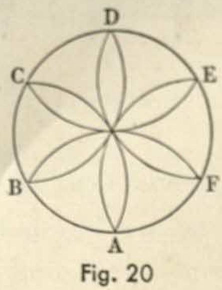


Fig. 20

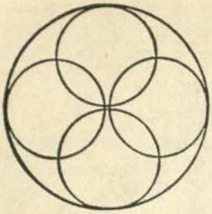


Fig. 21

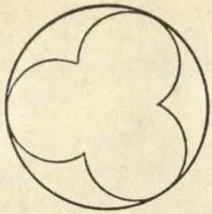


Fig. 22

10. Draw figure 20. *Hint:* From any point on the circle—A— and using the same radius as the radius of the circle, draw the arc BF. (An *arc* is a part of a circle, such as the curved line AE.) From B draw the arc CA. Continue in this manner.

11. Copy the designs in figures 21 and 22. *Hint:* In the first design, the centers of the small circles are on perpendicular diameters. In the second design, divide the large circle into three equal arcs. Draw radii to these points.

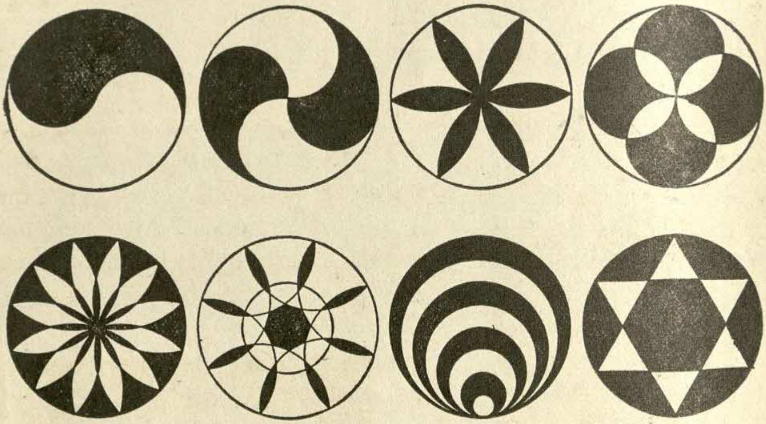


Fig. 23. Circle designs.

Fig. 24. Border designs.

12. (OPTIONAL) Copy the circle designs in figure 23.

13. (OPTIONAL) Copy the border designs in figure 24.

14. (OPTIONAL) Construct original designs based on the circle.

FORMS OF SOLID FIGURES

VOCABULARY

1. solid figure	5. cylinder	9. sphere
2. rectangular prism	6. cone	10. hemisphere
3. cube	7. pyramid	11. thickness
4. triangular prism	8. frustum	12. height

145. Solid figures. Figures that have three dimensions are called *solid figures*, or merely *solids*. The three dimensions are commonly designated length, width, and thickness, as of a book; length, width, and height, as of a room; or length, width, and depth, as of an excavation.

146. Observing solid geometric forms. Geometric solids are found all about us in nature and in the works of man. Let us observe a few illustrations that appear in the construction of buildings.

Figure 25 shows the *rectangular prism* so commonly seen in buildings. It has six faces, all rectangular in shape. A rectangular prism with square faces is called a *cube*. What is a good example of a cube? Why is a cube also a rectangular prism? Why is not every rectangular prism a cube?

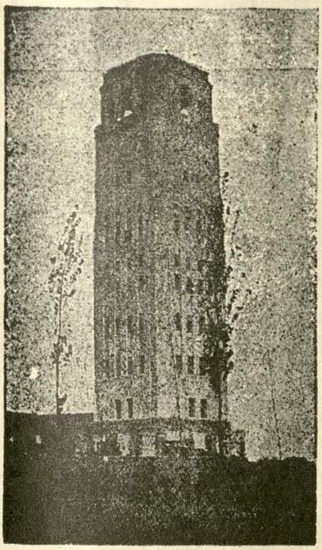
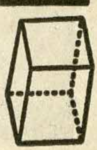


Fig. 25.



RECTANGULAR PRISM

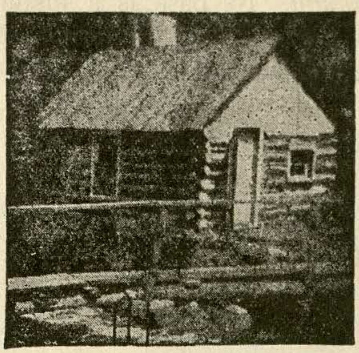
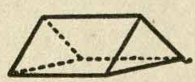


Fig. 26.



TRIANGULAR PRISM

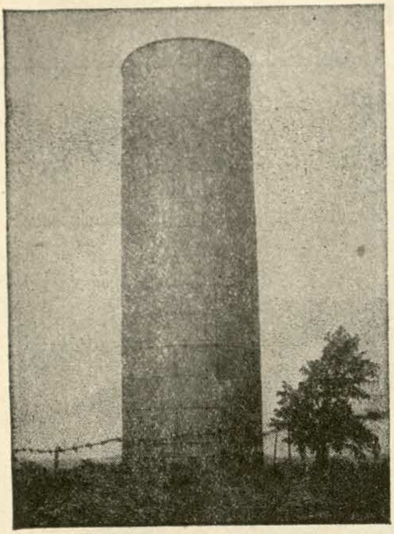
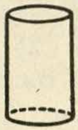


Fig. 27.

CYLINDER



If the top of a house like the one shown in figure 26 were sliced off above the logs, a solid with three rectangles for faces and a triangle for each base would be formed. Such a figure is a *triangular prism.* What other triangular prisms can you name?

Figure 27 illustrates a *cylinder*. It has a curved surface with circular bases.

On top of the cylinder in figure 28 is a *cone.* How many bases has a cone? What is the shape of the base? Name some cone-shaped objects. If the top of the cone were cut off parallel to the base, the lower portion would be called the *frustum* of the cone. Many lamp shades have this shape.

A solid having triangles for its faces, like the upper part of figure 29, is called a *pyramid*. Its base is a plane figure having any number of straight lines for sides. If the top of the pyramid were

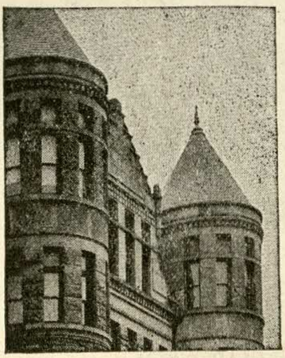
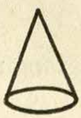


Fig. 28

CONE



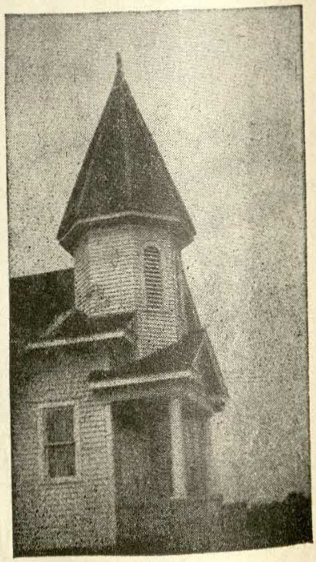
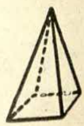


Fig. 29.

PYRAMID



cut off parallel to its base, the lower portion would be called the *frustum* of the pyramid.

The *sphere* (figure 30) is frequently used as an ornament on the tops of buildings, posts, and pedestals. Very often one half of a sphere, a *hemisphere*, is used as a dome on a building.

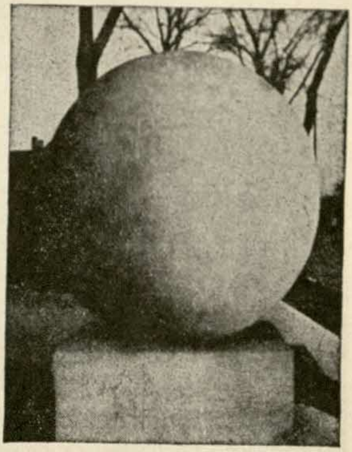


Fig. 30.

SPHERE

EXERCISES

1. Make a drawing of each of the preceding solids and write the correct name under each. List under each drawing as many examples of that solid as you can recall that are found in your home. Which shape is by far the most common?

2. Illustrate a cylinder and a cone, each by a paper model. (Leave bases open. For the cone draw a circle and cut out a portion between two radii.)

3. (OPTIONAL) Make a set of closed cardboard models. (*Hint:* Use the patterns in figure 31 enlarged.)

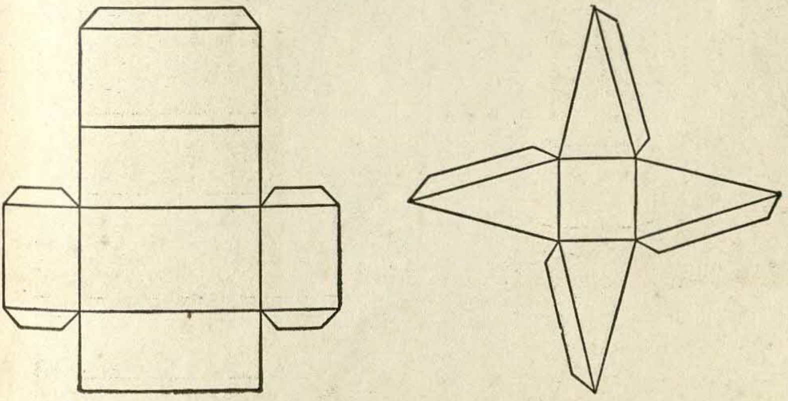
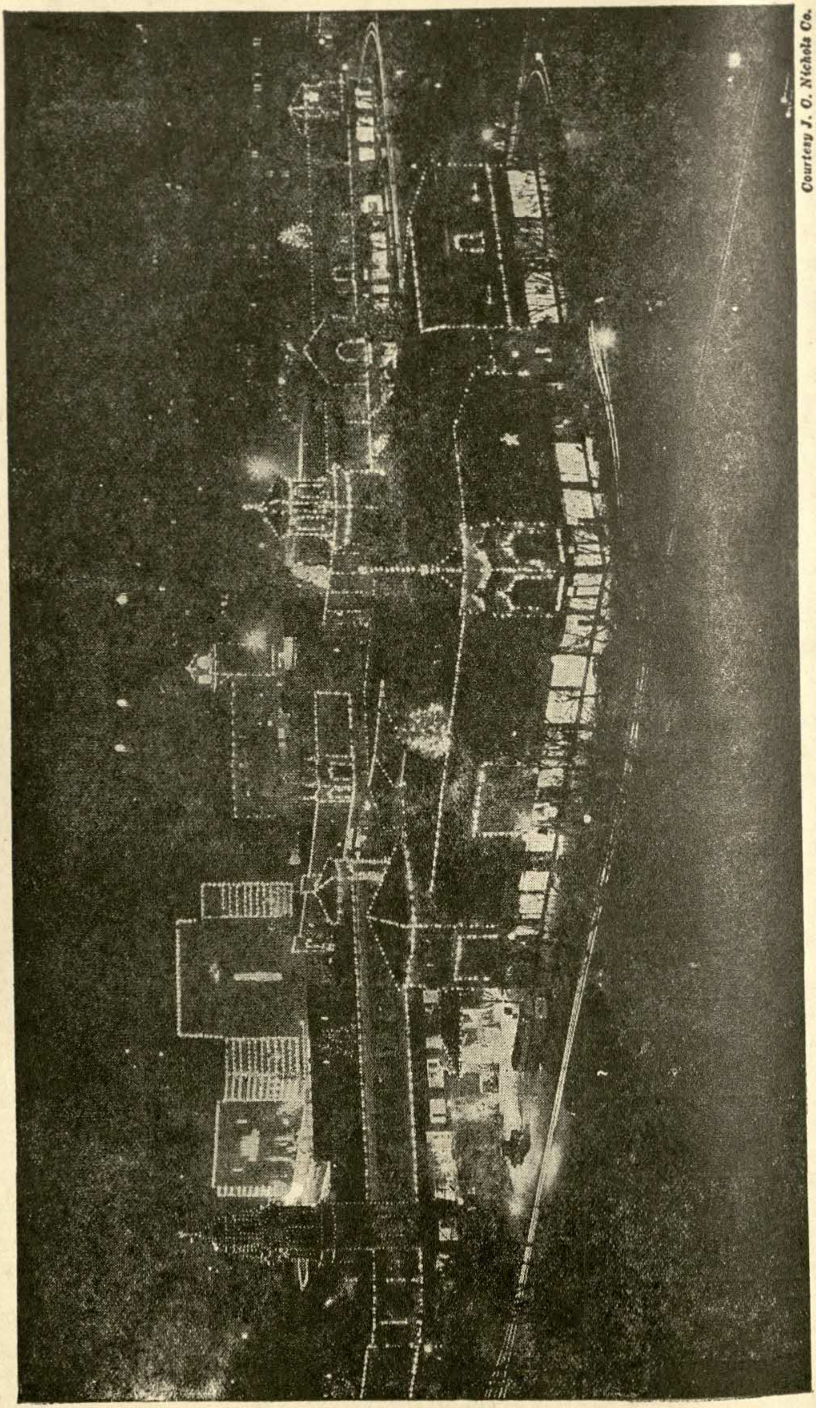


Fig. 31



Courtesy J. C. Nichols Co.

Fig. 32. Country Club Plaza in Kansas City, Missouri.

4. An icicle has the shape of a cone. List ten other objects in nature (trees, fruits, flowers, vegetables, and so forth), naming the geometric shape of each.

5. Make a paper model of a rectangular prism. (Bases need not be made. Fold a rectangular piece of paper into halves, then fold each half into halves again.)

6. Make a paper model of a triangular prism. (Leave ends open. There will be three rectangles for sides.)

7. Make a paper model of a cube.

8. (OPTIONAL) Make other models of the prism and pyramid, using bases of different shapes.

9. (OPTIONAL) Interesting posters can be made by cutting out, mounting, and labeling pictures that illustrate the various geometric forms we have studied.

10. In figure 32 list: (a) the different plane figures to be found; (b) the solid figures.

CHAPTER XIII

THE SIZE OF PLANE AND SOLID FIGURES

VOCABULARY

1. unit of area
2. square unit
3. area
4. perimeter
5. length
6. width
7. altitude
8. base
9. hypotenuse
10. lateral face
11. volume
12. diagonal
13. lateral area
14. formula
15. total area
16. perfect square
17. height
18. π

AREA AND PERIMETER OF PLANE FIGURES

147. The meaning of "area." In the rectangle $WXYZ$, figure 1, imagine that the sides of each small square measure one inch. The length of the rectangle will measure how many inches? The width how many inches? Each of the small squares is a *square inch*. The square inch is the *unit of area* in this case. The number of these squares (units of area) contained in the rectangle represents its *area*. Since there are 5 rows of squares and 8 squares to the row, there are 5×8 squares, or 40 squares. The area, then, is 40 times the unit of area, or 40 square inches.

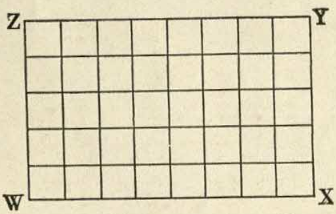


Fig. 1

Now think of each small square as measuring 1 ft. on a side, and the length and width of the rectangle as being 8 ft. and 5 ft., respectively. Each square is a *square foot*. The square foot is the unit of area in this case. Using this unit of area, find the area of the rectangle. The square inch and the square foot are common units of area. Others frequently used are the square yard, the square rod, and the square mile.

The *area* of any surface is the number of square units of a given kind that it contains. The surface $WXYZ$, when we thought of

the squares as being one inch wide, was said to have an area of 40 square inches; but when it was thought of as containing 40 of the square-foot units, its area was said to be 40 square feet. Area is always expressed in square units, such as square inches, square feet, and so on.

148. Perimeter of a rectangle. To find the *perimeter* of $WXYZ$, or the distance around it, add the lengths of the lines that bound it: $WX + XY + YZ + ZW$. How long is the perimeter if the unit of length is 1 in.? The length of a perimeter is always expressed in linear units, such as inches, feet, yards, and so forth.

EXERCISES

1. Using squared paper, show that a rectangle that is 6 units long and 4 units wide has an area of 6×4 square units, or 24 square units.

2. Illustrate the area of a rectangle 7 units long and 5 units wide.

3. In the rectangle $ABCD$, figure 2, let each side of the small squares represent 1 in.

(a) What is the unit of area?
(b) How many of these units of area are contained in the rectangle?
(c) What, then, is the area of the rectangle?
(d) How do you find the perimeter?
(e) What is the unit of length for the perimeter?
(f) Find the perimeter.

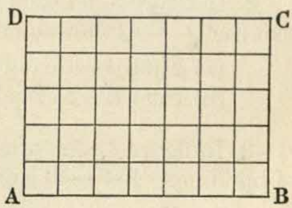


Fig. 2

4. Now let each side of the small squares in rectangle $ABCD$ represent 1 ft.

(a) What is the unit of area?
(b) What is the area of the rectangle?
(c) How long is the perimeter?

5. Using squared paper, let one side of each square represent 1 in. Then draw a square proving that 1 sq. ft. contains 144 sq. in.

6. Using squared paper, let one side of each square represent 1 ft. Draw a square proving that 1 sq. yd. contains 9 sq. ft.

7. On squared paper, let the squares represent 1 sq. in. Then:

(a) Draw a rectangle 6 in. long and 4 in. wide. Find its area. Find its perimeter.
(b) Draw a rectangle 11 in. long and 9 in. wide. Find its area. Find its perimeter.

(c) Draw three rectangles of different dimensions, each having an area of 36 sq. in. Compare the perimeters of the rectangles. The perimeter of which rectangle is the longest? Which the shortest? Do all rectangles of the same area have the same perimeter?

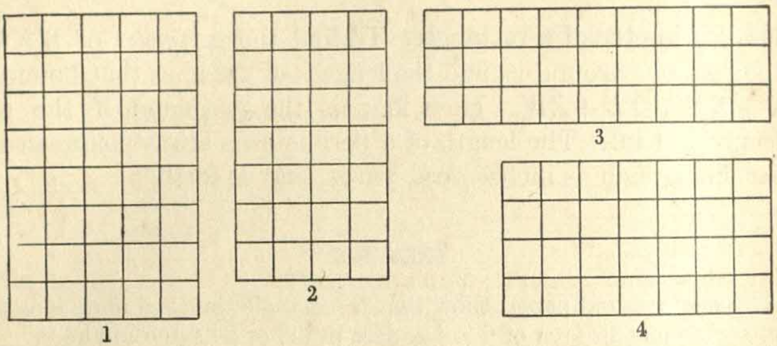


Fig. 3

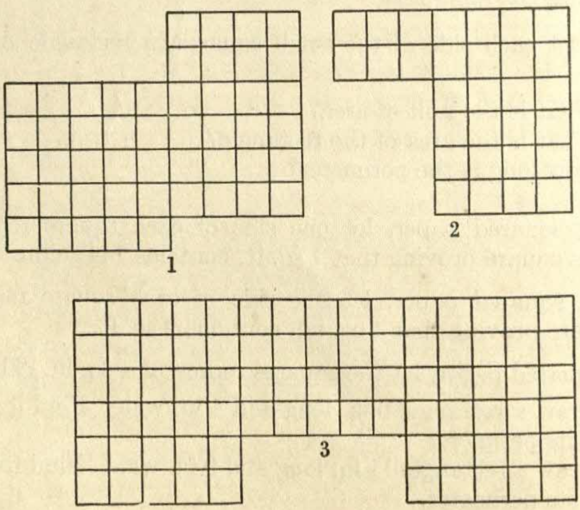
8. In figure 3, let each side of the small squares of the rectangles represent each of the following, in turn: (1) 1 yard, (2) 1 rod, (3) 1 inch, (4) 1 foot.

(a) Find the area of each rectangle.
(b) Find the perimeter of each rectangle.

9. In figure 4, the forms illustrate those often formed by the floor plans of buildings. Let each small square represent a square yard.

(a) How many square yards are there in each figure?
(b) What is the perimeter of each figure?

Fig. 4



10. A picture frame is to be made $18\frac{3}{4}$ in. by $16\frac{1}{2}$ in. Allowing $3\frac{1}{4}$ in. for waste, how many inches of molding are required?

11. On squared paper, let the side of each small square represent 1 ft. Using this scale, draw five rectangles, the perimeter of each being 20 ft. (*Hint:* Use 1 ft., 2 ft., 3 ft., 4 ft., and 5 ft. for the respective widths.) Find and compare the areas of the rectangles. Do all rectangles having the same perimeter have the same area?

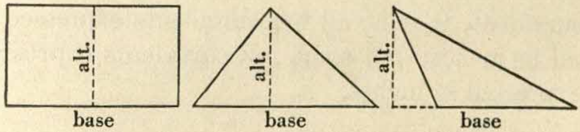
The preceding exercise illustrates the following truth:

For a perimeter of any given length, the square will inclose an area greater than any other rectangle of the same perimeter.

149. Base and altitude of plane figures. The base of a plane figure is the line upon which the figure rests. Any one of the bounding lines may be used for the base if the position of the figure is shifted.

The *altitude* (height or depth) of a figure is the perpendicular distance from the highest point in the figure to its base. For illustrations of base and altitude, see figure 5.

Fig. 5



Which line represents the base of the blackboard? The altitude? Imagine the blackboard to be shifted so that it stands up on its shorter edge. Which line then would represent the base? The altitude? Find illustrations of the base and altitude of other plane figures in the classroom.

150. The area of a rectangle. You have learned that the area of a rectangle is the product of the length multiplied by the width. This rule can be expressed in a much shorter way if initial letters are used for the words *area, length,* and *width* and signs for the words *equals* and *times.* Let A represent *area,* and let l and w denote *length* and *width,* respectively. The rule then becomes: $A = l \times w$. A rule written in this way is called a *formula.* The times sign is usually omitted in formulas to avoid confusion with the letter x.

Letters written side by side in formulas are to be multiplied. The formula for the area of a rectangle may be written:

$$A = lw.$$

Since the length of a figure is usually called its *base* and the width its *altitude*, we can let b stand for *base* and h for *altitude*. The formula $A = lw$ then becomes:

$$A = bh.$$

Example

To find the area of a rectangle whose base is 10 ft. and altitude 8 ft., solve as follows:

$$A = bh \qquad b = 10 \text{ ft.}$$
$$A = 10 \times 8 \qquad h = 8 \text{ ft.}$$
$$A = 80$$

Hence, the area of the rectangle is 80 sq. ft.

Note: In substituting numbers for letters in formulas, it is important to remember two things:

(a) To write merely the number, omitting the letter that it replaces.

(b) To be sure that the numbers are expressed in the same unit of measurement, that is, all measurements expressed in feet, or all expressed in inches; not some measurements expressed in feet and others expressed in inches.

EXERCISES

A. Oral

State the area of each of the following rectangles, using short cuts when possible:

1. $b = 6'$, $h = 12'$

2. $b = 10''$, $h = 7.5''$

3. $b = 20'$, $h = 24''$

4. $l = 24.5'$, $w = 100'$

5. $b = \frac{1}{4}'$, $h = \frac{2}{5}'$

6. $b = \frac{3}{8}'$, $h = \frac{2}{3}'$

7. $l = 24$ rd., $w = 12\frac{1}{2}$ rd.

8. $l = 66$ cm., $w = 16\frac{2}{3}$ cm.

9. $b = 50$ cm., $h = 80$ cm.

10. $b = 7.2'$, $h = .33\frac{1}{3}'$

B. Written

Find the area of each of the following rectangles:

11. $b = .32'$, $h = 14'$

12. $b = 100'$, $h = 88.8''$

13. $l = 2.86'$, $w = 7.2''$

14. $b = 35$ cm., $h = 3.8$ cm.

15. $l = 10\frac{1}{5}'$, $w = 1\frac{2}{3}'$

16. $b = 12\frac{1}{2}''$, $h = 88''$

17. How many square feet are in a lot 180 ft. long and 60 ft. wide? How many square yards are in the lot?

18. Find the cost of covering a lot 40 ft. by 120 ft. with sod at $12\frac{1}{2}$ cents a square yard.

19. How many acres are there in a rectangular field 86 rd. long and 75 rd. wide?

20. A farm in the shape of a rectangle is 80 rd. long and 56 rd. wide. How many acres does it contain?

21. What is the value of the preceding farm at $76 per acre?

22. How many rods of fencing will be needed to inclose it? What will be the total cost of the fence at $.75 per rod?

23. A railway box car is to be painted gray on the sides and black on the ends. How many square feet are painted in each color, if the sides are $40\frac{1}{2}$ ft. long by 10 ft. high and the ends are $9\frac{1}{2}$ ft. wide and 10 ft. high?

24. If a floor contains 288 sq. ft. and is 18 ft. long, how wide is it? $(w = A \div l)$

25. It has been suggested that on a playground 30 sq. ft. of space should be allowed for each pupil. At this rate, how many pupils can safely play at one time on a ground 180 ft. by 120 ft.?

26. A house plan has the shape pictured in figure 6. Find the number of square feet of floor space it contains.

27. Two boys out camping built a fire which, through their carelessness, spread to the forest. It burned an area 120 rd. long and 80 rd. wide. How much was the loss if the timber was worth $500 per acre?

28. Mr. Sands has a rectangular surface 140 ft. by 30 ft. to paint. Which is more economical and how much more: to use paint at $5.80 per gallon that will cover 300 sq. ft. of surface, or paint at $5.15 per gallon that will cover 210 sq. ft. of surface?

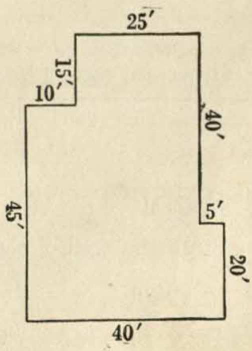


Fig. 6

29. (OPTIONAL) Determine the number of square feet of painted surface in your classroom. Which will be cheaper and how much cheaper: to buy paint for this surface at $2.96 per half-gallon that will cover 150 sq. ft., or to buy paint at $2.45 per half-gallon that will cover 100 sq. ft.?

30. (OPTIONAL) A plot of ground extending 120 rd. along a highway and 20 rd. deep was divided into building lots of 90-ft. frontage by 165 ft. deep. How many lots were thus formed? If the plot of ground cost $600 per acre and each lot sold for $28 a foot frontage, what was the profit on the ground?

31. (OPTIONAL) How many square meters are there in the floor of the classroom?

32. (OPTIONAL) Find the number of square centimeters in the front cover of this textbook.

151. The area of a square. Since the lengths of all four sides of a square are equal, you can find its area by multiplying the length of one side by itself. If the side of a square is 6 units, the square has an area of 6×6 square units, or 36 square units. Let s represent the length of one side of the square; then $A = s \times s$.

6×6 may be written: 6^2 (*six squared*); and the formula $A = s \times s$ may be written:

$$A = s^2.$$

It is read, "A equals s squared."

Example

To find the area of a square whose side is 10 in. long, solve by the following method:

$$A = s^2 \qquad s = 10''$$
$$A = 10^2$$
$$A = 10 \times 10$$
$$A = 100$$

Hence, the area of the square is 100 sq. in.

EXERCISES

A. Oral

State the area of each of the following squares:

1. $s = 40$ in. **4.** $s = 4\frac{1}{2}$ cm. **7.** $s = .02$ ft.

2. $s = 25$ ft. **5.** $s = 100$ mm. **8.** $s = 1.1$ ft.

3. $s = .3$ in. **6.** $s = .4$ rd. **9.** $s = 1.5$ in.

B. Written

Find the area of each of the following squares:

10. $s = 18$ in. **14.** $s = 6.3$ in. **18.** $s = 3.5$ rd. **22.** $s = 4.25$ in.

11. $s = 24$ ft. **15.** $s = 8.25$ ft. **19.** $s = 2.8$ yd. **23.** $s = 4\frac{5}{6}$ ft.

12. $s = 9\frac{1}{2}$ yd. **16.** $s = 14\frac{1}{2}$ cm. **20.** $s = 19\frac{1}{2}$ in. **24.** $s = 9' \, 3''$

13. $s = 4' \, 8''$ **17.** $s = 8.8$ yd. **21.** $s = 27\frac{1}{3}$ yd. **25.** $s = 8' \, 9''$

26. Some boys were given a square lot for a garden. It measured 72 ft. on a side. How many square feet were there in the lot? If they divided it off into plots 6 ft. x 8 ft., how many plots of ground were there?

27. One of the large pyramids of Egypt has a square base 764 ft. long. How many square feet of ground does it cover?

28. A square has an area of 10,000 sq. ft. What is the length of each side?

29. What is the difference between a 6-ft. square and 6 sq. ft.?

30. A building 48 ft. x 60 ft. was built on a lot 90 ft. square. How many square feet were left for the lawn?

31. A plot of ground 1 mile square is divided into four equal square farms. How many acres has each farm? How many pounds of fertilizer are required for each farm if 20 lb. will fertilize 1 acre?

32. (OPTIONAL) How many pounds of fertilizer (20 lb. to the acre) will be required for: (a) A field 80 rd. square? (b) A garden 120 ft. square? (c) A piece of ground containing 2,880 sq. ft.?

33. (OPTIONAL) Measure the back of your mathematics book to the nearest one-half inch. How many square inches does it contain? What is its perimeter?

152. The lateral area of a rectangular prism. A rectangular prism is made up of how many rectangles? What are the dimensions of the base rectangle in figure 7? What is the area of one base? Of both bases together?

The rectangles forming the sides between the bases are called *lateral faces.* The sum of the areas of the lateral faces is called the *lateral area.*

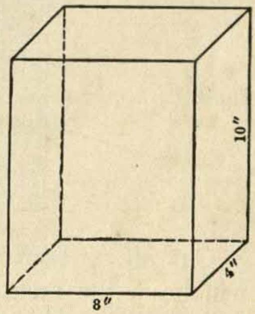


Fig. 7

In figure 7, what is the area of each lateral face? What is the lateral area of the rectangular prism?

What is the perimeter of the base? Multiply the perimeter of the base by the height. How does this result compare with the lateral area? They should agree.

The formula for finding the lateral area of a rectangular prism by the shorter method is:

$$S = Ph,$$

S standing for *lateral area,* P for *perimeter,* and h for *height.*

The *total area* of a rectangular prism is the sum of the area of the bases and the lateral area.

What is the total area of figure 7?

EXERCISES

1. Find the area of each lateral face of the classroom. Find the lateral area of the room. Find the total area of the room.

2. Henry made a tool box 38 in. long, 20 in. wide, and 16 in. high. How many square inches of lumber were there in the box, including the top?

3. How much will it cost at 8 cents per square foot to paint a chest 6 ft. long, $2\frac{1}{2}$ ft. wide, and 18 in. high? (The bottom base is to be left unpainted.)

4. Make a paper carton 2 in. long, $1\frac{1}{2}$ in. wide, and 3 in. high. What is its lateral area? Total area?

5. Draw the pattern for a carton 4 in. long, $1\frac{1}{2}$ in. wide, and 2 in. high. Find the lateral area and the total area.

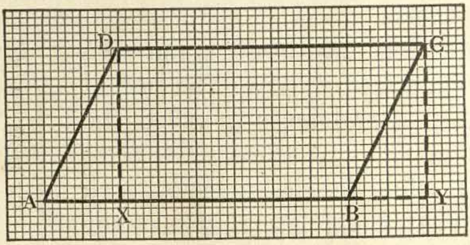
6. Find the cost of covering with paper all sides of ten rectangular boxes each 15 in. long, $8\frac{1}{2}$ in. wide, and 6 in. deep, at 5 cents per square foot.

7. Find the number of square feet in the total surface of a cubical box 5 ft. on each side.

8. Find the cost of plastering a room 16 ft. x 12 ft. x 9 ft. at 15 cents per square foot. (Make no allowance for openings.)

153. The area of a parallelogram. On a piece of squared paper draw a parallelogram 8 units long and 4 units high, as shown in figure 8. The height (altitude) is the perpendicular distance to the base, as DX.

Fig. 8



Cut off $\triangle ADX$ and place it so that AD falls on BC, and you will have the rectangle $XYCD$ of the same base and altitude as the parallelogram $ABCD$.

Why are the bases AB and XY equal? What is the area of the rectangle? Since the area of the rectangle is found by multiplying its base by its altitude, the area of the parallelogram, which is the same, will be found in the same way. What is the area of the parallelogram?

The area of a parallelogram is the product of its base and altitude.

Expressed as a formula, this rule becomes:

$$A = bh.$$

EXERCISES

A. Oral

State the area of each of the following parallelograms:

1. $b = 20'$, $h = 16'$

2. $b = 100'$, $h = 10'$

3. $b = 84''$, $h = 25''$

4. $b = 646$ yd., $h = 50$ yd.

5. $b = 33'$, $h = 66\frac{2}{3}'$

6. $b = 37\frac{1}{2}''$, $h = 88''$

B. Written

Using the formula, find the area of each of the following parallelograms:

7. $b = 26'$, $h = 18'$

8. $b = 3.8''$, $h = .8''$

9. $b = 7\frac{1}{3}'$, $h = 5\frac{1}{4}'$

10. $b = .42$ mm., $h = .8$ mm.

11. $b = 1.8$ cm., $h = 2.6$ cm.

12. $b = 1\frac{1}{2}$ yd., $h = 4\frac{2}{3}$ yd.

In each of the following drawings, indicate the altitude by a heavy dotted line:

13. On squared paper draw a parallelogram whose base is 5 cm. and altitude 2 cm. Let 1 square = 1 sq. cm. Find the area and check the result by counting the square centimeters.

14. On squared paper draw a parallelogram with base of 54 mm. and altitude 22 mm. What is the area of the parallelogram?

15. On paper ruled in square centimeters, draw to a convenient scale a parallelogram with base of 80 ft. and altitude of 16 ft. Indicate the scale used. Find the area. Count squares to verify your result.

16. Draw to a convenient scale on squared paper a parallelogram 120 yd. long and 60 yd. wide. Indicate the scale used. Find the area. Verify by counting the squares.

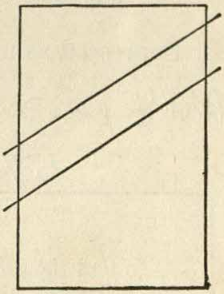


Fig. 9

17. A state road cut across some Boy Scouts' camping ground, as illustrated in figure 9. The portion of the road crossing the camp ground was 840 ft. long and 18 ft. wide. How many square feet of land did the boys lose?

18. How many acres are there in a field in the shape of a parallelogram with a base 80 rd. long and an altitude of 15 rd.?

154. The area of a triangle. On squared paper draw a rectangle $ABCD$ with a base of 10 units and an altitude of 6 units. (See

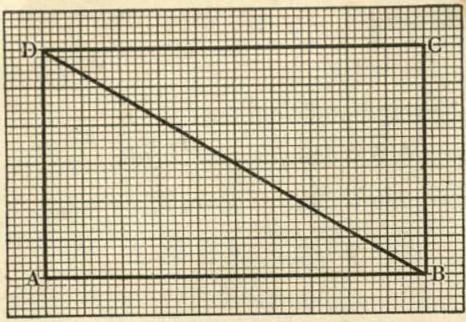


Fig. 10

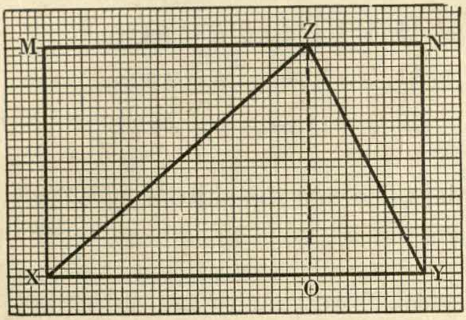


Fig. 11

figure 10.) Draw a diagonal, as BD, forming two triangles each with a base and altitude equal to the base and altitude of the rectangle. Cut out the triangles and fit one on the other. Since they fit exactly, the two triangles are equal. The area of one triangle is just one-half the area of the rectangle.

Draw another rectangle 10 x 6 units. (See figure 11.) Draw the lines XZ and YZ, forming $\triangle XYZ$ with the base 10 units long and the altitude ZO 6 units high. Cut out $\triangle XYZ$ and fit the remaining $\triangle YNZ$ on OZY and $\triangle XZM$ on XZO. Since they fit exactly, the area of $\triangle XYZ$ is just one-half the area of the rectangle of the same base and altitude.

What is the area of the rectangle? Of the triangle?

The area of a triangle is one-half the product of its base and altitude.

Expressed as a formula, this rule becomes:

$$A = \tfrac{1}{2}\,bh$$

LARRY & AND SHARON KEENER
57-58

Example

To find the area of a triangle whose base is 16 in. and whose altitude is 13 in., solve as follows:

$$A = \tfrac{1}{2}\,bh \qquad b = 16 \text{ in.}$$

$$A = \frac{1}{\cancel{2}} \times \overset{8}{\cancel{16}} \times 13 \qquad h = 13 \text{ in.}$$

$$A = 104.$$

Hence, the area of the triangle is 104 sq. in.

EXERCISES

A. Oral

State the area of each of the following triangles:

1. $b = 14'$, $h = 8'$

2. $b = 8''$, $h = 4.2''$

3. $b = \frac{1}{2}'$, $h = 2.8'$

4. $b = \frac{2}{3}''$, $h = 1''$

5. $b = 10$ rd., $h = 40$ rd.

6. $b = 16'$, $h = 9'$

7. $b = 8$ yd., $h = .3$ yd.

8. $b = .2$ cm., $h = .9$ cm.

B. Written

Using the formula, find the area of each of the following triangles:

9. $b = 14'$, $h = 28'$

10. $b = 26''$, $h = 9''$

11. $b = 8\frac{1}{4}''$, $h = 5\frac{1}{3}''$

12. $b = 18$ cm., $h = 14.2$ cm.

13. $b = 12$ mm., $h = 8.5$ mm.

14. $b = 7.2'$, $h = .9'$

15. A triangular sail with a base of 9′ 6″ and an altitude of 11′ 4″ exposes how many square feet of surface when spread out to the wind?

16. The gable end of a house has a base of 18′ 8″ and a height of 8′ 6″. How many square feet does it contain?

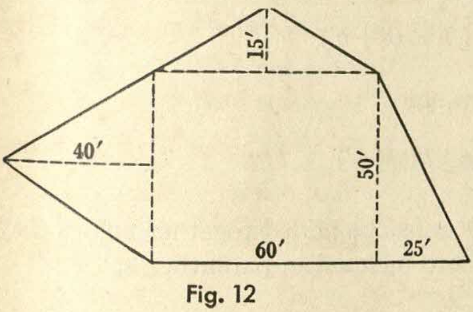


Fig. 12

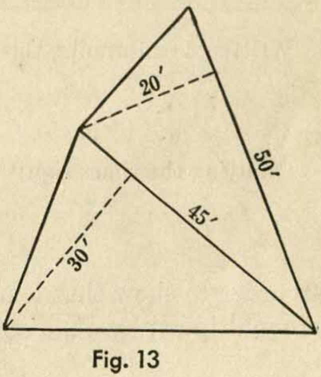


Fig. 13

17. What is the area of figure 12?

18. What is the area of figure 13?

155. (Optional) The area of a trapezoid. Figure 14 is a *trapezoid.* The two opposite sides b and b' (b' is read b *prime*) are parallel and are called the *lower* and *upper* bases, respectively. The non-parallel sides may or may not be

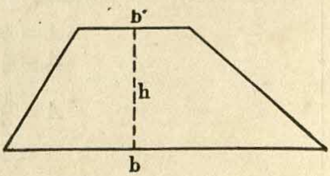
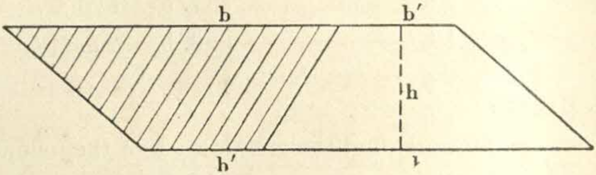


Fig. 14

the same length. The altitude is the perpendicular distance, h, between the two bases.

Draw and cut out two trapezoids of the same size and shape. Inverting one of the trapezoids, place the two together as in figure 15. The figure thus formed is a parallelogram with the same altitude

Fig. 15



as the trapezoid and with base equal to the sum of the upper and lower bases of the trapezoid. The area of the parallelogram is $h \times (b + b')$. Why? Since each trapezoid is just one-half of the parallelogram:

The area of a trapezoid equals one-half the product of its altitude and the sum of its bases.

Written as a formula, this rule becomes:

$$A = \tfrac{1}{2}\, h \times (b + b')$$

Omitting the times sign:

$$A = \tfrac{1}{2}\, h(b + b')$$

In order to show that b and b' must be added together before they are multiplied by $\frac{1}{2}h$, b and b' are inclosed in parentheses.

Example

To find the area of a trapezoid with lower and upper bases 6 in. and 8 in., respectively, and altitude 4 in., solve as follows:

$$A = \tfrac{1}{2}\, h(b + b') \qquad h = 4 \text{ in.}$$
$$A = \tfrac{1}{2} \times 4(8 + 6) \qquad b = 8 \text{ in.}$$
$$b' = 6 \text{ in.}$$
$$A = \frac{1}{\cancel{2}} \times \overset{2}{\cancel{4}} \times 14$$
$$A = 28.$$

Hence, the area of the trapezoid is 28 sq. in.

EXERCISES

A. Oral

State the area of each of the following trapezoids:

1. $h = 4''$, $b = 20''$, $b' = 10''$
2. $h = 2'$, $b = 18'$, $b' = 8'$
3. $h = 10''$, $b = 12''$, $b' = 8''$
4. $h = 5$ yd., $b = 6$ yd., $b' = 4$ yd.
5. $h = 3$ rd., $b = 40$ rd., $b' = 30$ rd.
6. $h = 8$ cm., $b = 10$ cm., $b' = 8$ cm.
7. $h = 1'$, $b = 9'$, $b' = 7'$
8. $h = 20''$, $b = 25''$, $b' = 15''$

B. Written

Using the formula, find the area of each of the following trapezoids:

9. $h = 10''$, $b = 17''$, $b' = 13''$
10. $h = 9'$, $b = 28'$, $b' = 20'$
11. $h = 5$ cm., $b = 16$ cm., $b' = 10$ cm.
12. $h = 3\frac{1}{2}''$, $b = 8\frac{1}{2}''$, $b' = 7\frac{1}{2}''$
13. $h = 4'$, $b = 10'\ 6''$, $b' = 8'\ 4''$
14. $h = 3'\ 4''$, $b = 16'\ 8''$, $b' = 12'\ 4''$
15. $h = 2'\ 8''$, $b = 15'\ 9''$, $b' = 13'$
16. $h = 5$ cm., $b = 9.2$ cm., $b' = 6.4$ cm.

17. The top of a wall seat in a bay window is the shape of a trapezoid with the parallel sides 9 ft. and 7 ft., respectively, and the depth 1 ft. 6 in. How many square feet are there in the top of the seat?

18. A 33-ft. road was cut across one corner of a rectangular field, as shown in figure 16. The owner was offered \$7.50 per square rod for the portion of his land that was used. If one side of the road on his field measured 80 rd. and the other side 72 rd., how much did the owner receive for his land?

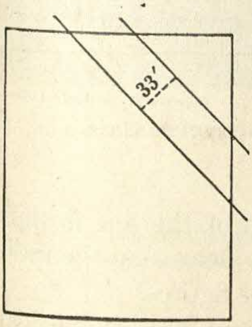


Fig. 16

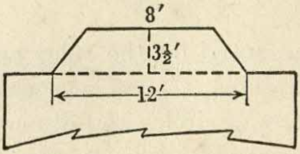
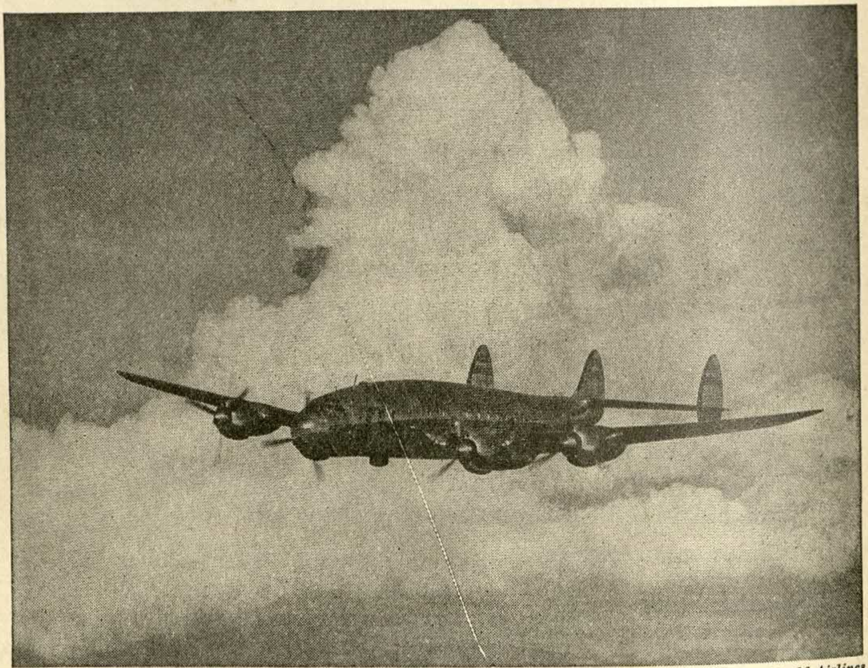


Fig. 17

19. In order to enlarge a room, a bay window with the dimensions shown in figure 17 was added. How many square feet of floor space were added?

20. Betty is making a lamp shade of parchment. Each of the four sides is the shape of a trapezoid, the bases of which are 15 in. and 12 in., with an altitude of 8 in. Find the area of each side.

156. The circumference and diameter of a circle. Short distances are often more conveniently measured with compasses or dividers than by direct application of the ruler. The sharp points of either instrument are adjusted so as to touch both ends of the line to be measured. Then the distances between the points of the instrument



Courtesy Trans-World Airlines

Fig. 18. A whirling propeller moves in an almost complete circle.

used are measured on the ruler and the length of the line is thus determined. This method is especially good for measuring the radii and diameters of circles and dimensions of other figures.

A good way to measure the circumference of any cylindrical object is as follows:

(a) Place a mark at some point on the circumference of the object to be measured.

(b) Place the one-inch mark of a tapeline on this mark.

(c) Wrap the tapeline around the object until it again touches the marked place on the circumference. The distance between these two points on the tapeline will represent the circumference of the object.

Measure carefully the circumference of several cylindrical objects, such as a pie pan, table top, bucket, and wheel. Also measure the diameter of each. Find the ratio of the circumference to the diameter, expressing the results to the nearest hundredth. Arrange the results as follows:

Object	*Circumference*	*Diameter*	*Ratio*
Plate	29.75″	9.5″	3.13 +

Find the average of the Ratio column. It should be about 3.14. This shows us that:

The circumference of a circle is about 3.14 times the length of its diameter.

For more accurate results 3.1416 is used; however, even this ratio is only approximate.* For ordinary purposes the ratio $3\frac{1}{7}$ ($\frac{22}{7}$) is used.

Letting C stand for *circumference*, d for *diameter*, and the Greek letter π (pronounced *pī*) for *ratio*, the formula for the circumference is:

$$C = \pi d$$

$$\text{or, } C = 2\,\pi r. \quad \text{(Why?)}$$

Example

To find the circumference of a circle whose diameter is 35 in., solve as follows:

$$C = \pi d \qquad \pi = 3\tfrac{1}{7}$$

$$d = 35 \text{ in.}$$

$$C = \frac{22}{\cancel{7}_{1}} \times \overset{5}{\cancel{35}}$$

$$C = 110.$$

Hence, the circumference of the circle is 110 in.

---

*The ratio of the circumference to the diameter will never divide out evenly. It divides like this: 3.14159265358979323846264338 32, and on and on.

EXERCISES

Using the ratio $3\frac{1}{7}$, find the circumference of each of the following circles:

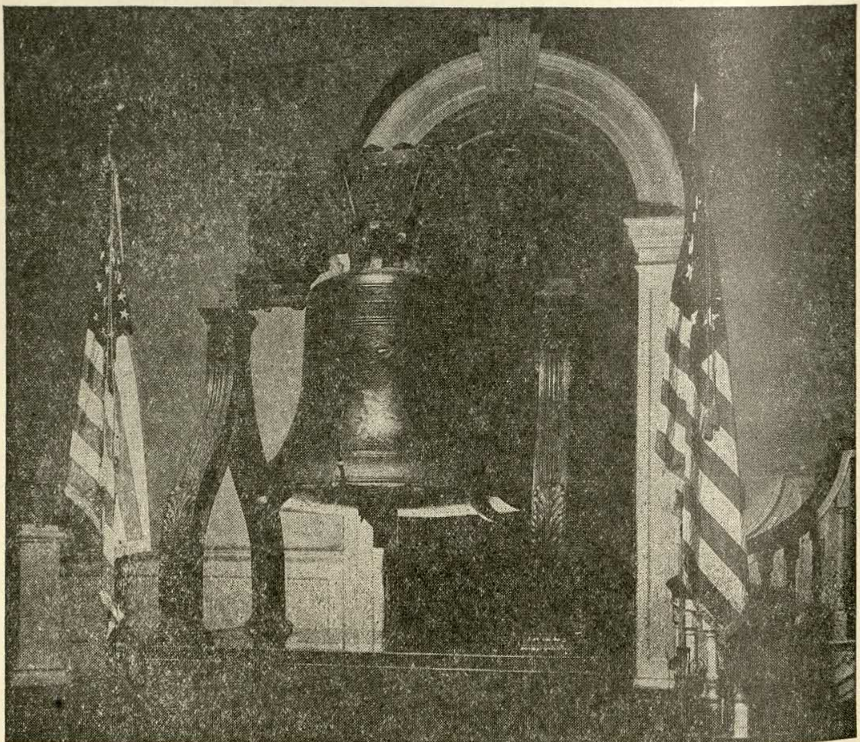
1. $d = 14$ in.	**4.** $d = 3\frac{1}{2}$ ft.	**7.** $r = 4$ yd.
2. $d = 21$ yd.	**5.** $d = 7$ in.	**8.** $d = 5.6$ ft.
3. $d = 49$ cm.	**6.** $r = 28$ in.	**9.** $r = 2.1$ in.

Using the ratio 3.14, find the circumference of each of the following circles:

10. $d = 10$ in.	**12.** $r = 20$ ft.	**14.** $d = 35$ in.
11. $d = 100$ ft.	**13.** $r = 140$ yd.	**15.** $d = 70$ in.

16. How do the circumferences of the circles in exercises 14 and 15 compare? How do you account for this relation?

17. The diameter of a circular boiler is 105 in. What is its circumference?



Courtesy Trans-World Airlines

Fig. 19. The Liberty Bell as seen in Independence Hall in Philadelphia.

18. A garden is in the form of a circle 26 ft. across. How many feet of fencing are required to inclose it?

19. An automobile wheel is $23\frac{1}{2}$ in. in diameter. What distance does it cover in making one complete revolution?

20. Draw a circle with a radius of 3 in. Draw a diameter and construct another diameter perpendicular to it. Cut out $\frac{1}{4}$ of the circle and form the remaining $\frac{3}{4}$ into a funnel. Find the circumference of the base of the funnel by using the formula. Check your result by actual measurement.

21. If the circumference of a circular piece of oilcloth is to be 132 in., what must be its diameter? $(d = C \div \pi)$

22. The circumference of a tree is 88 in. Find its diameter.

23. The circumference around the lip of the *Liberty Bell* is 12 ft. 6 in. What is the diameter at the lip? (See figure 19.)

24. A flywheel in a power plant is 14.14 in. in diameter. What is the circumference?

25. The cross-section of the tree on page 229 has an outside diameter of 16 in. Find the circumference.

26. (OPTIONAL) The diameter of a steel boiler is 57.4 in. What is its circumference?

27. (OPTIONAL) What is the length of a paper wrapper that fits around a 2-in. can, if $\frac{1}{2}$ in. extra is allowed for pasting?

28. (OPTIONAL) The flywheel of a steam engine is 16.8 in. in diameter. How many feet does a point on the circumference travel in one revolution?

To find the *velocity* (sometimes called the *surface speed*) of pulleys and wheels, multiply the circumference in feet by the number of revolutions per minute (r.p.m.).

$$V = \pi \times d \times \text{r.p.m.}$$

Example

Find the velocity in feet per minute of an emery wheel 9 in. in diameter turning at 500 r.p.m.

$$V = \pi \times d \times \text{r.p.m.} \qquad d = 9 \text{ in.}$$

$$\text{r.p.m.} = 500$$

$$V = 3.14 \times \frac{\overset{3}{\cancel{9}}}{\underset{4}{\cancel{12}}} \times \overset{125}{\cancel{500}} = 1177.5$$

Hence, the velocity is 1,177.5 feet per minute.

Solve for velocity in feet per minute:

	Diameter	*r.p.m.*
29.	6′	400
30.	18″	1,100
31.	2′	750
32.	6″	1,800
33.	$7\frac{1}{2}$′	350

34. What is the diameter of a boiler whose circumference is 121 in.?

35. (OPTIONAL) The diameter of each wheel over which a band saw runs is 3.5 ft. The distance between the centers of the two wheels placed one above the other is 3.9 ft. How long is the band saw?

36. (OPTIONAL) A cylindrical lamp stand 8 in. in diameter is cut down in a lathe in the manual-training shop to a diameter of 6 in. What is the new circumference?

37. (OPTIONAL) The circumference of a cylindrical tank is 66 in. What is the diameter?

157. The area of a circle. An interesting method of discovering the rule for the area of a circle is illustrated in figure 20. This circle has a radius of 10 units. The lower right-hand fourth has a square drawn on the radius. As the square is 10 units long, the area of the square on the radius, r^2, is 100 square units.

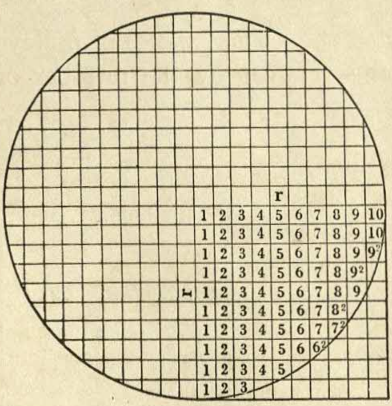


Fig. 20

The square units in the circle are more difficult to count, because of the many fractions of a square unit. In the marked fourth, the number of squares in each row is estimated and listed as follows: 10, 10, $9\frac{1}{2}$ (written as 9^2), $9\frac{1}{2}$, 9, $8\frac{1}{2}$, $7\frac{1}{2}$, $6\frac{1}{2}$, 5, 3—making a total of approximately $78\frac{1}{2}$ square units in each one-fourth of the circle, or approximately 314 square units in the whole circle.

When the approximate area of the circle is compared with the area of the square on the radius, it is found that the former is about 3.14 times as large as the latter ($314 \div 100 = 3.14$).

This leads to the rule:

The area of a circle is about 3.14 (or $3\frac{1}{7}$) times the square of the radius.

$$A = 3.14 \times r^2$$

Using π for 3.14, the formula becomes:

$$A = \pi r^2$$

Example

To find the area of a circle whose radius is $3\frac{1}{2}$ in., solve as follows:

$$A = \pi r^2 \qquad \pi = 3\tfrac{1}{7}\ (\tfrac{22}{7})$$
$$r = 3\tfrac{1}{2} \text{ in.}$$
$$A = \frac{\overset{11}{\cancel{22}}}{\cancel{7}} \times \frac{7}{\cancel{2}} \times \frac{7}{2}$$
$$A = \tfrac{77}{2}, \text{ or } 38\tfrac{1}{2}.$$

Hence, the area of the circle is $38\frac{1}{2}$ sq. in.

EXERCISES

Using the ratio $3\frac{1}{7}$, find the area of each of the following circles:

1. $r = 70$ ft.
2. $r = 140$ in.
3. $r = 2.1$ ft.
4. $r = 2\frac{1}{3}$ yd.
5. $d = 5.6$ rd.
6. $d = 8$ cm.
7. $d = 6$ in.
8. $r = 7$ ft.
9. $r = 14$ ft.

10. In exercises 8 and 9, how do the radii compare in length? How do the areas compare in size?

Using the ratio 3.14, find the area of each of the following circles:

11. $r = 100$ rd.
12. $r = 80$ yd.
13. $d = 8.6$ cm.
14. $d = 7.8$ mm.
15. $r = 10$ ft.
16. $r = 20$ ft.

17. How do the radii in exercises 15 and 16 compare in length? How do the areas compare in size?

18. A wigwam has a diameter on the ground of 13 ft. How many square feet of floor space does it cover?

19. A circular flower bed has a radius of $4\frac{1}{5}$ ft. How many square feet are there in the flower bed?

20. Morning Glory Pool in Yellowstone Park is about 23 ft. across. Approximately how many square feet does the surface of the pool cover? What is the distance in feet around the pool?

21. Find the perimeter and area of figure 21.

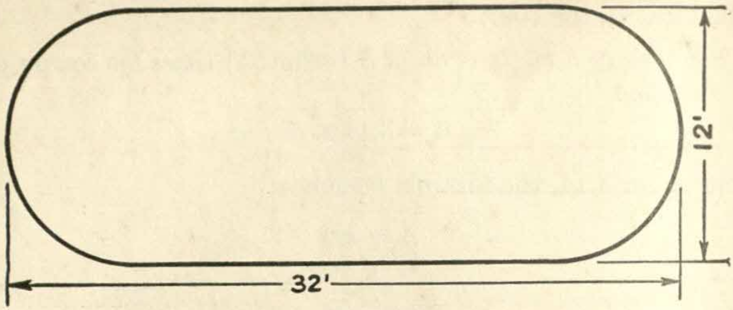


Fig. 21

22. The diameter of a circle is $\frac{1}{2}$ in. Find its area to the nearest thousandth of a square inch; to the nearest ten-thousandth of a square inch. (Use $\pi = 3.1416$.)

23. A steel rod 3.125 in. in diameter must be turned down to 2.287 in. How deep must the cut be?

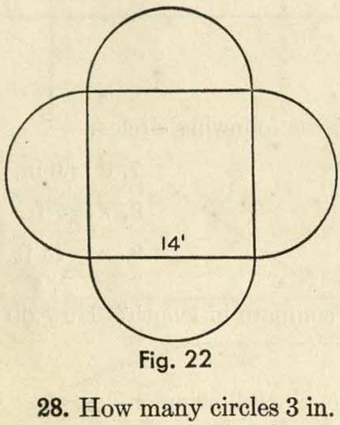


Fig. 22

24. Which has the larger surface, and how much larger: a circular pool with a diameter of 70 ft., or a square pool 70 ft. on each side?

25. If a city with a radius of $2\frac{1}{2}$ mi. doubles its radius to 5 mi., how much has it increased its area?

26. Find the area of figure 22.

27. What is the area of the largest circle that can be cut from a rectangle 6 in. x 9 in.? Make either a full-size or a scale drawing to show how the circle would be cut. Label the dimensions.

28. How many circles 3 in. in diameter can be cut from a strip of decorative paper 9 in. x 60 in.?

29. The outside diameter of a pipe is 3.475 in. and the thickness of the wall is .388 in. Find the inside diameter.

30. The top of a steam table is a metal plate 2′ 3″ by 5′ 4″. Six holes each 9 in. in diameter are cut in the plate. What is the area of the plate remaining?

31. A girl cuts two doilies each 18 in. in diameter, 4 doilies each 9 in. in diameter, and 4 doilies each 7 in. in diameter from a square yard of lace. How many square inches of lace are in the doilies? How many square inches are left?

32. Using the correct formula in each case, find the area of the forms in figure 23 after making the necessary measurements.

33. (OPTIONAL) There is a walk 3 ft. wide around a circular fountain whose diameter is 21 ft. (a) What is the area of the fountain base? (b) What

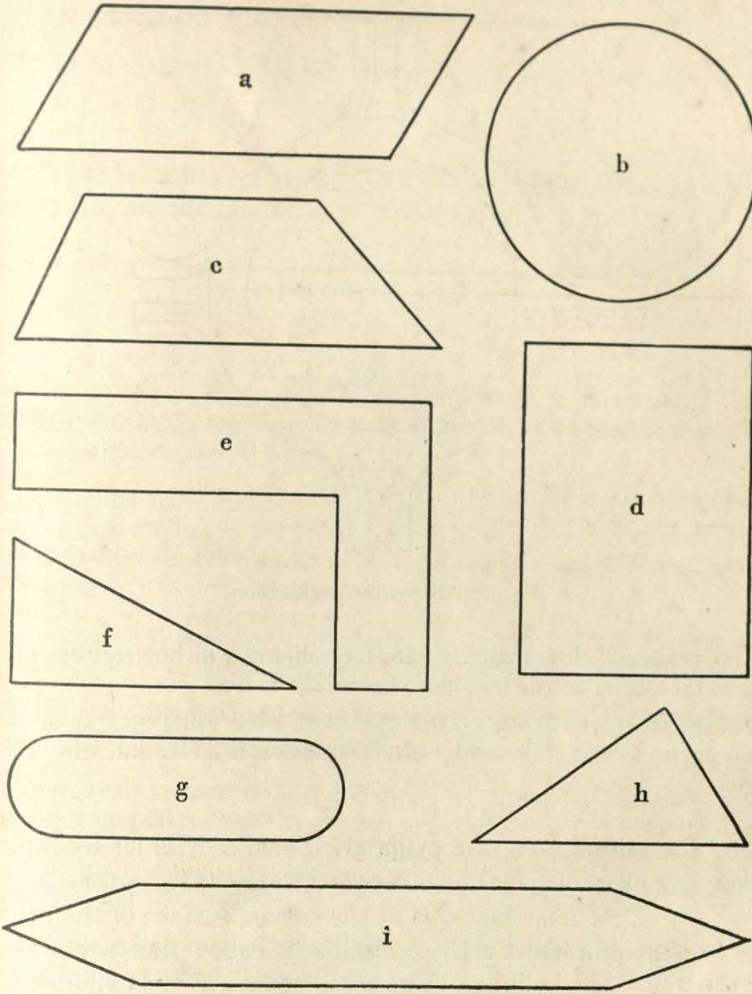


Fig. 23

is the area of the fountain and the walk together? (c) Now that you know these two areas, how can you find the area of the walk? (d) Find the area of the walk.

34. (OPTIONAL) The outside diameter of a circular plate is 22 in. A circular hole in the center is 7 in. across. What is the area of the metal ring? (Use $\pi = 3\frac{1}{7}$.) Area of ring $= \pi \times (R^2 - r^2)$. ($R =$ radius of plate; $r =$ radius of hole.)

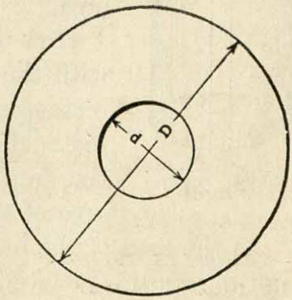


Fig. 24

35. (OPTIONAL) Find the area of figure 24, if $D = 1\frac{1}{2}$ in. and $d = \frac{1}{2}$ in. (Use $\pi = 3\frac{1}{7}$.)

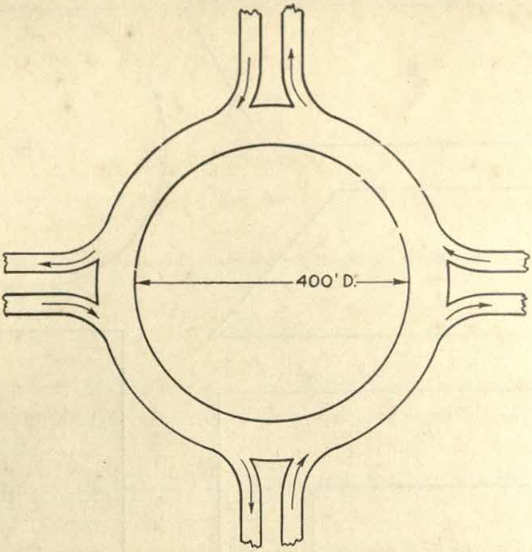


Fig. 25. Highway intersection.

36. (OPTIONAL) How many square rods of sod will be required for the circle in the center of the highway, figure 25?

37. (OPTIONAL) At 45 cents per square yard, what will a contractor charge for paving a circular walk whose inside diameter is 35 ft. and whose width is 7 ft.?

158. The lateral area of a cylinder. Form a cylinder by rolling a rectangular piece of paper as in figure 26. Let the shorter edges just touch. The area of the curved surface of the cylinder formed by the rectangle is called the *lateral area* of the cylinder. The total surface of the cylinder is the sum of the areas of the two bases and the lateral area.

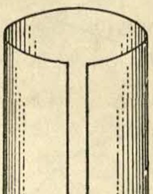


Fig. 26

How does the width of the rectangle compare with the altitude of the cylinder? The length of the rectangle becomes what of the cylinder? Since the rectangle forms the curved surface of the cylinder, the area of the rectangle is the same as the lateral area of the cylinder.

As the area of the rectangle is equal to the product of the length and the width, the lateral area of the cylinder is equal to the product of the circumference (length of rectangle) and the altitude (width of rectangle).

This leads to the rule:

The lateral area of a cylinder is the product of its circumference and altitude.

If S stands for lateral area, C for circumference, and h for altitude, the formula for the lateral area of the cylinder is:

$$S = Ch.$$

EXERCISES

1. What is the lateral area of a cylinder whose circumference is 24 ft. and whose altitude is 8 ft. 6 in.?

2. A cylinder has a radius of $3\frac{1}{2}$ in. and an altitude of 4 in. What is the circumference? What is the lateral area? Roll a piece of paper to make a cylinder with the above dimensions. Measure the circumference to verify your answer.

3. A cylindrical tank has a radius of 7 ft. and an altitude of 20 ft. Find the area of the two bases. Find the lateral area. The total area.

4. At 3 cents per square foot, what would it cost to paint a cylindrical smokestack 40 ft. high and $3\frac{1}{2}$ ft. in diameter?

5. How many square feet of lumber are there in a water tower 21 ft. in diameter and 20 ft. high?

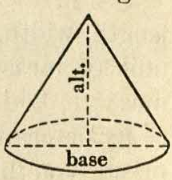
6. How many square feet of metal are there in a hot water tank $1\frac{1}{4}$ ft. in diameter and 6 ft. high?

7. A large roller is a cylinder 6 ft. long and 32 in. in diameter. What area of ground is rolled in one revolution?

8. How many square feet of surface exposed to the air has a steam-heating pipe 18 ft. long and 3 in. in diameter?

VOLUME OF SOLID FIGURES

159. Base and altitude of solid figures. The *base* of a solid figure is the side or face upon which it stands. Any one of the sides may be used for the base if the position of the figure is shifted. The *altitude* of a solid figure is the perpendicular distance from the highest point in the figure to its base.



Place your book on your desk. Which side is its base? What is its altitude? Change its position so that it will have a different base. Now which side is the base? What is the altitude? Find illustrations of the base and altitude of other solid figures in your classroom.

160. The volume of a solid figure. The *volume* of a solid figure is determined by the number of cubic units that it contains. The unit of measure may be a cubic inch, a cubic foot, or a cubic yard, depending upon the size of the solid to be measured. A *cubic inch* is a cube one inch long, one inch wide, and one inch high, like figure 27. What are the dimensions of a cubic foot? Of a cubic yard?

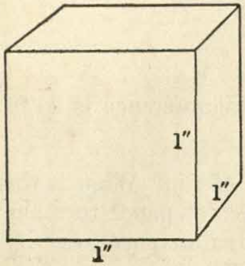


Fig. 27

What unit of measure would be convenient to use in finding the volume of a shoe box? Of a coal bin? Of a room?

161. The volume of a rectangular prism. In figure 28, imagine each small cube to measure 1 inch on an edge. The length of the rectangular prism measures how many inches? The width measures how many inches? The height is how many inches? Each of the small cubes is called a cubic inch. The cubic inch is the unit of volume in this case. The number of these cubes contained in a rectangular prism represents its volume.

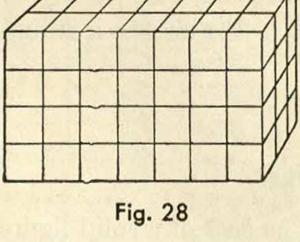


Fig. 28

Since each layer of cubes is made up of 3 rows of cubes with 7 cubes in each row, there are 3×7 cubes, or 21 cubes, in one layer. Four such layers will contain 4×21 cubes, or 84 cubes. The volume, then, is 84 times the unit of volume, or 84 cubic inches.

Now think of each small cube in figure 28 as measuring 1 ft. on an edge, and the length, width, and height as 7 ft., 3 ft., and 4 ft., respectively. The unit of measure now becomes a cubic foot. Using this unit of measure, find the volume of the rectangular prism.

By determining the total number of cubic units in any rectangular prism, you find its volume.

EXERCISES

1. In figure 29, let the edge of each cube represent 1 in.
   (a) What is the unit of volume?
   (b) How many of these units of volume are contained in the rectangular prism?
   (c) What, then, is its volume?

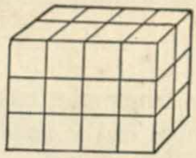


Fig. 29

2. In the same figure, let the edge of each cube represent 1 ft.
   (a) What is the unit of volume?
   (b) What is the volume of the rectangular prism?

3. In the rectangular prisms of figure 30, let one edge of each small cube be represented as follows:

No. 1 = 1 in. No. 2 = 1 ft. No. 3 = 1 yd.

Find the volume of each rectangular prism.

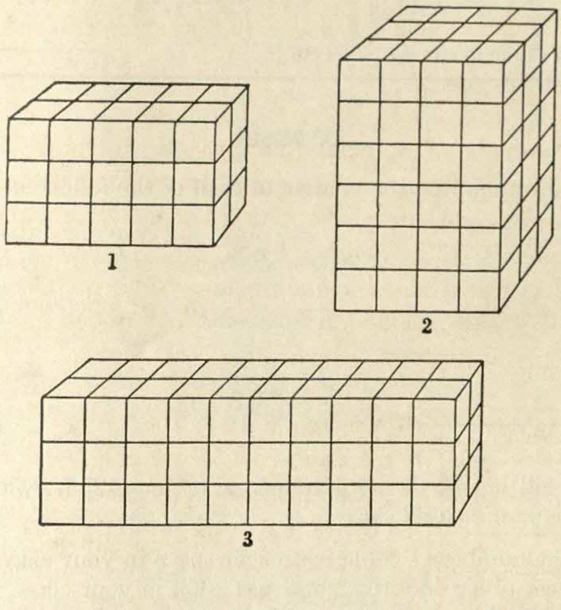


Fig. 30

4. Draw a rectangular solid containing 24 cu. in., letting the length be 3 in.

The volume of a rectangular prism can always be found by the following rule:

The volume of a rectangular prism is the product of its three dimensions.

If V stands for *volume*, l for *length*, w for *width*, and h for *height* or *altitude*, the formula for this rule will then become as follows:

$$V = lwh.$$

Since the base is a rectangle with the dimensions l and w, the area of the base is lw.

Letting B represent the area of the base, and substituting B for lw in the above formula, we have:

$$V = Bh.$$

Example

To find the volume of a box 4 ft. long, 3 ft. wide, and $2\frac{1}{2}$ ft. high, solve as follows:

$$V = lwh \qquad l = 4 \text{ ft.}$$

$$V = \overset{2}{\cancel{4}} \times 3 \times \frac{5}{\cancel{2}} \qquad w = 3 \text{ ft.}$$

$$V = 30 \qquad h = 2\tfrac{1}{2} \text{ ft.}$$

Hence, the volume of the box is 30 cu. ft.

EXERCISES

Using the formula, find the volume of each of the following rectangular prisms:

1. $l = 12$ ft.
   $w = 16$ ft.
   $h = 18$ ft.

2. $l = 3$ ft. 6 in.
   $w = 4$ ft.
   $h = 3$ ft. 4 in.

3. $l = 7.8$ cm.
   $w = 4.1$ cm.
   $h = 2.5$ cm.

4. $l = .6$ ft.
   $w = 1.9$ ft.
   $h = .7$ ft.

5. $l = 8$ mm.
   $w = 2$ mm.
   $h = 2.8$ mm.

6. $l = 8$ in.
   $w = 2$ in.
   $h = 9$ in.

7. What will it cost to dig a trench 40 ft. long, $2\frac{1}{2}$ ft. wide, and 6 ft. deep, at 65 cents per cubic yard?

8. Find the number of cubic feet of air space in your classroom. How many cubic feet of air does this allow per pupil in your class? How many cubic feet would there be per pupil if the room were: (a) twice as long; (b) twice as wide; (c) two times as high; (d) twice as long and twice as wide; (e) with twice as many pupils; (f) with one-half as many pupils; (g) twice as long and with twice as many pupils; (h) twice as long and with one-half as many pupils?

9. A coal bin is 15 ft. long and 12 ft. wide; how many pounds of coal will it hold when filled to a depth of 5 ft., if there are 63 lb. in 1 cu. ft. of coal?

10. How many bars of soap measuring 4 in. x 2 in. x 1 in. are there in a stack of soap 2 ft. x 1 ft. x 8 in.?

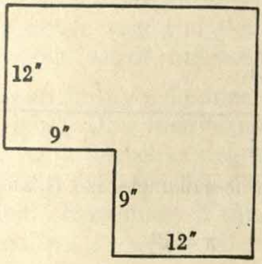
11. A box 8.8 cm. long, 5 cm. wide, and 4.2 cm. deep contains how many cubic centimeters?

12. A freight car is 38 ft. long and 8 ft. wide and is filled with sand to a depth of 3 ft. 9 in. How many cubic feet of sand are in the car? How many cubic yards?

13. A coal bin 14 ft. long, 9 ft. wide, and 6 ft. 4 in. high is half full of coal. How many cubic feet of coal are in the bin? How many tons at 32 cu. ft. per ton?

14. A porch pillar 27 ft. high has for its base the shape illustrated in figure 31. How many cubic feet are there in the pillar?

Fig. 31



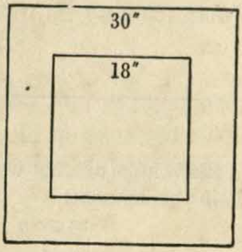


Fig. 32

15. A square chimney 8 ft. high has a base as pictured in figure 32. Find the number of cubic feet of material in the chimney.

16. Carl is making a box 16 in. long and 9 in. wide. He wants it to hold 720 cu. in. How high must he make it?

17. Mr. Cooper wishes to make an icehouse that will hold 6,000 cu. ft. of ice. He has decided to make the ground plan 20 ft. by 20 ft. How high must he make the house?

18. The swimming pool of a certain school is 50 ft. long and 30 ft. wide, and is filled to an average depth of 3 ft. 6 in. How many gallons of water are in the pool? (One cubic foot = $7\frac{1}{2}$ gal., approximately.) If the water costs $1.25 per one thousand cubic feet, what does it cost to fill the pool? If the tank is filled once a month with fresh water for the nine months, find the cost of the water for the school year.

19. (OPTIONAL) If possible, measure the aquarium in your school or home. Find how many cubic inches there are in the aquarium. How many gallons? Check by filling or emptying the tank.

20. (OPTIONAL) Get the necessary measurements of your school pool and find the number of gallons of water required to fill it. Find the cost of each filling at your local rates.

162. The volume of a cylinder. The cylinder in figure 33 is 20 in. high and its base contains 154 sq. in. ($\pi r^2 = \frac{22}{7} \times 7 \times 7 = 154$.)

At a distance 1 in. high on the cylinder there are 154 cu. in. Then at a distance 20 in. high there will be 154 cu. in. $\times$ 20, or 3,080 cu. in.

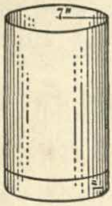


Fig. 33

From the above we derive the following rule:

The volume of a cylinder is equal to the product of the area of the base and the altitude.

As a formula, the rule may be written:

$$V = Bh.$$

Since in the cylinder B represents the area of the circle, $B = \pi r^2$. Then the formula may be written:

$$V = \pi r^2 h.$$

Example

To find the volume of a hot water tank whose diameter is 1 ft. and altitude 5 ft., solve in this manner:

$$V = \pi r^2 h \qquad \pi = 3\tfrac{1}{7}$$

$$V = \frac{\overset{11}{\cancel{22}}}{7} \times \frac{1}{\cancel{2}} \times \frac{1}{2} \times \frac{5}{1} \qquad r = \tfrac{1}{2}\text{ ft.}$$

$$V = \tfrac{55}{14} \qquad h = 5\text{ ft.}$$

$$V = 3\tfrac{13}{14}.$$

Hence, the volume of the tank is $3\frac{13}{14}$ cu. ft.

EXERCISES

Using the formula, find the volume of each of the following cylinders:

1. $r = 5$ in.; $h = 14$ in.
2. $r = 8$ yd.; $h = 3\frac{1}{2}$ yd.
3. $r = 4$ cm.; $h = 14$ cm.
4. $r = \frac{1}{2}$ in.; $h = 14$ in.
5. $r = 7$ ft.; $h = \frac{1}{2}$ ft.
6. $r = \frac{1}{4}$ ft.; $h = \frac{2}{3}$ ft.
7. $r = 2$ in.; $h = 7$ in.
8. $r = 4$ in.; $h = 7$ in.

9. In exercises 7 and 8, how was the volume of the cylinder affected by doubling the radius? How do you account for this result?

10. A silo 42 ft. in height has an inside diameter of 15 ft. What is its capacity in cubic feet?

11. How many cubic inches of tomatoes are there in a can whose diameter is 3 in. and depth $4\frac{1}{2}$ in.?

12. What would it cost to dig a cistern $3\frac{1}{2}$ ft. in diameter and 16 ft. deep at 65 cents per cubic yard?

13. What fractional part of a cubic foot is there in an iron pipe 14 ft. long and 2 in. across?

14. How many gallons are there in a barrel $3\frac{1}{2}$ ft. deep and 2 ft. across?

15. How many gallons of water will a street sprinkler hold if it is 15 ft. long and 4 ft. across?

16. An automobile gasoline tank that is cylindrical has a diameter of 12 in. and is 35 in. long. How many gallons will it hold when filled? How many are in it when the instrument on the dashboard shows the tank to be three-fourths full?

17. A lawn sprinkler has a $\frac{1}{8}$-in. stream and spreads about 100 gallons of water per hour. What will it cost to operate the sprinkler for three hours at the rate of \$1.50 per thousand cubic feet of water used?

18. What is the weight of a concrete roller $8\frac{1}{2}$ ft. long and 4 ft. in diameter if one cubic foot of concrete weighs 147 lb.?

19. Bring an empty cylindrical can from your home. Measure its diameter and height. How many square inches are in the bottom of the can? How many cubic inches of vegetables would be required to fill it? How many if the can were twice as tall? How many if the diameter of the can were doubled? How many if the height of the can and the diameter were both doubled?

20. How many cubic inches does a #1 can of corn contain, if it has a diameter of 3 in. and a height of $4\frac{5}{8}$ in.? How many cubic inches are there in a #$2\frac{1}{2}$ can of corn if the diameter is 4 in. and the height is $4\frac{11}{16}$ in.?

21. A #1 can contains 2 cups and a #$2\frac{1}{2}$ can contains $3\frac{1}{2}$ cups. If a #1 can of corn sells for 15 cents and a #$2\frac{1}{2}$ can of corn sells for 24 cents, which is more economical to purchase? What is the saving per cup?

22. (OPTIONAL) Make the measurements necessary to find the volume of a pipe. Find its volume. Do the same for a water tank, oil tank, or any other available cylinders.

163. The volume of a cone and pyramid. Figure 34 shows a cone and a cylinder of the same height, with bases of the same size. Estimate how many times as much will be contained in the cylinder as in the cone.

It is interesting to check your estimate by the following experiment. Obtain a cylinder of convenient size. Make a cone of stiff paper the same height and with its base the same size as that of the cylinder.

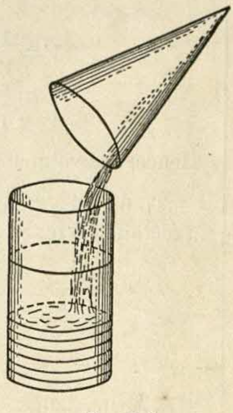


Fig. 34

Fill the cone with sand, sawdust, or any other suitable material and pour it into the cylinder. Continue filling the cone and pouring into the cylinder until the cylinder is filled. If the work is accurately done, the capacity of the cylinder will be found to be three times the capacity of the cone.

Since the volume of a cylinder is equal to the product of the area of the base and the altitude, we arrive at the following rule:

The volume of a cone is equal to one-third the product of the area of the base and the altitude.

The formula for this rule is:

$$V = \tfrac{1}{3}\,Bh$$

or,

$$V = \tfrac{1}{3}\,\pi r^2 h$$

A similar experiment may be tried with a prism and a pyramid that have the same bases and equal altitudes. It will be found that the capacity of the prism is three times that of the pyramid. Hence:

The volume of a pyramid is equal to one-third the product of the area of the base and the altitude.

That is:

$$V = \tfrac{1}{3}\,Bh$$

Examples

To find the volume of a cone whose height is 8 in. and radius of base $3\frac{1}{2}$ in., solve as follows:

$$V = \tfrac{1}{3}\pi r^2 h \qquad r = 3\tfrac{1}{2}\text{ in.}$$

$$V = \frac{1}{3} \times \frac{\overset{11}{\cancel{22}}}{\cancel{7}} \times \frac{\cancel{7}}{2} \times \frac{7}{\cancel{2}} \times \frac{\overset{4}{\cancel{8}}}{1} \qquad h = 8\text{ in.}$$

$$V = \tfrac{308}{3}$$

$$V = 102\tfrac{2}{3}.$$

Hence, the volume of the cone is $102\frac{2}{3}$ cu. in.

To find the volume of a pyramid whose altitude is 9 in. and whose base is a rectangle 4 in. by 3 in., solve as follows:

$$V = \tfrac{1}{3}\,Bh \qquad l = 4\text{ in.}$$

$$V = \frac{1}{3} \times 4 \times 3 \times 9 \qquad w = 3\text{ in.} \quad h = 9\text{ in.}$$

$$V = 36.$$

Hence, the volume of the pyramid is 36 cu. in.

EXERCISES

1. Find the volume of a cone the radius of whose base is 3 in. and whose altitude is 7 in.

2. How many cubic feet are there in a cone whose altitude is 15 ft. and whose diameter at the base is 7 ft.?

3. What is the volume of a pyramid whose altitude is 18 in. and whose base is a square 4 in. on each side?

4. How many cubic feet are there in a pyramid whose height is 5 ft. and whose base is a triangle with an area of 42 sq. ft.?

5. How many cubic feet of space are there in a conical tent whose height is 12 ft. and whose diameter on the ground is 14 ft.?

6. A stone pyramid of the dimensions in figure 35 contains how many cubic feet?

7. If a certain tepee has a diameter on the ground of 9 ft. and is 11 ft. in height, what is its volume?

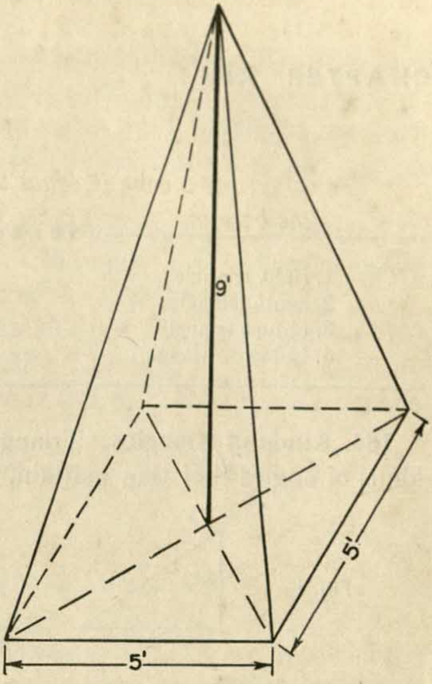


Fig. 35

8. Some putty is molded in the shape of a rectangular prism with a base 3 in. x 3 in. and an altitude of 5 in. How many pyramids with the same base and altitude can be molded from the same amount of putty? Why?

9. A cylindrical tank is topped with a cone as shown in figure 36. Find the capacity of the tank if the inside diameter is 15 ft. and the height of the cylinder and of the cone are 12 ft. and 2 ft., respectively.

10. The Great Pyramid of Egypt (see page 82) was originally 481 ft. high and its base was 756 ft. square. What was its volume?

Fig. 36

CHAPTER XIV

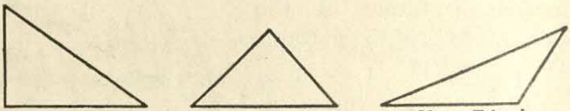
CONSTRUCTIONS

VOCABULARY

1. right triangle
2. acute triangle
3. obtuse triangle
4. isosceles triangle
5. equilateral triangle
6. compasses
7. bisect
8. △ symbol for triangle

164. Kinds of triangles. Triangles are classified according to the kinds of angles that they contain.

Fig. 1



Right Triangle   Acute Triangle   Obtuse Triangle

A *right triangle* is a triangle that contains one right angle.

An *acute triangle* is a triangle all the angles of which are acute.

An *obtuse triangle* is a triangle that contains one obtuse angle.

Figures 2 and 3 are special kinds of acute triangles. In figure 2, measure AC and CB. How do they compare in length? A triangle having two equal sides is called an *isosceles* triangle. Measure $\angle A$ and $\angle B$, the *base angles*. How do they compare in size?

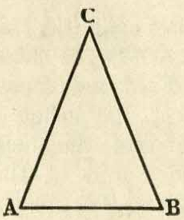


Fig. 2

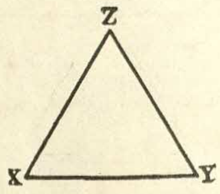


Fig. 3

In figure 3, measure and compare the lengths of XY, YZ, and XZ. A triangle having three equal sides is called an *equilateral*

triangle. What does the word "equilateral" mean? Compare the number of degrees in each angle. What is the size of each angle of an equilateral triangle?

165. The sum of the angles of a triangle. Draw a 2-in. line AB. At A draw a 50° angle and at B draw a 40° angle. Extend the sides of these angles until they meet, forming $\triangle ABC$. Measure the angle at C. What kind of triangle is ABC? Find the sum of the degrees in $\measuredangle A$, B, and C.

On a $2\frac{1}{2}$-in. line XY, draw $\measuredangle X$ and Y equal to 30° and 70°, respectively. Complete $\triangle XYZ$. Measure $\angle Z$. What kind of triangle is $\triangle XYZ$? Find the sum of the three angles.

Draw a line MN 3 in. long. At M draw an angle equal to 110° and at N draw an angle equal to 20°. Complete $\triangle MNO$. Measure $\angle O$. What kind of triangle has been formed? Find the sum of the angles in $\triangle MNO$.

What do you observe about the sum of the three angles of the above triangles?

An important geometric truth:

The sum of the angles of any triangle equals 180°.

EXERCISES

1. Cut from paper a triangle of any convenient size. Tear off the three angles and put them together, as in figure 4. How does this prove that the sum of the angles of a triangle equals 180°?

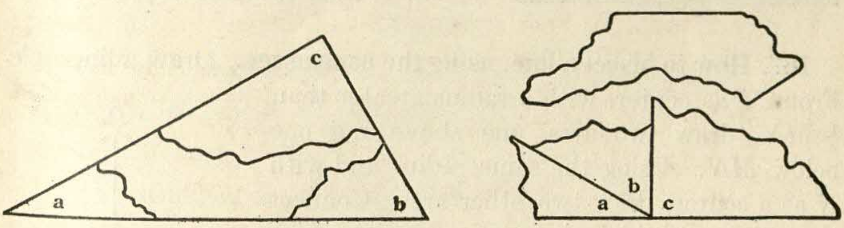


Fig. 4

2. If the sum of two angles of a triangle equals 70°, what is the size of the third angle?

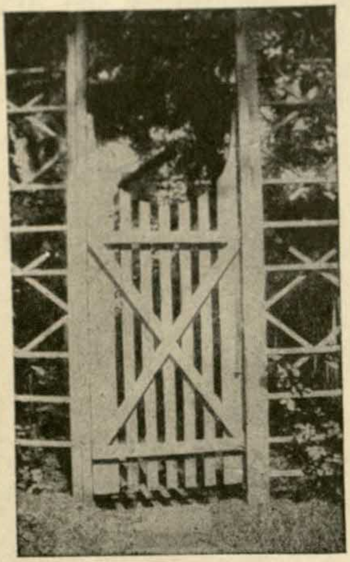
3. The sum of two angles in a triangle equals 148°. Find the size of the third angle.

4. In $\triangle XYZ$, $\angle X$ equals 60° and $\angle Y$ equals 35°; $\angle Z$ equals how many degrees?

5. In $\triangle MNO$, $\angle M = 48°$ and $\angle N = 75°$; $\angle O = ?$

6. Why can you not draw a triangle containing more than one right angle? More than one obtuse angle?

166. A practical use of triangles. You have no doubt noticed the many uses of triangles in construction work of carpenters and engineers. The trusses on bridges, brackets for shelves, diagonal pieces across box ends and gates, and the supports of a roof are all uses made of one or more triangles. The triangle is used in these instances because of its rigid form. Once a triangle is made, its shape or size cannot be changed by pressure applied to any of its sides or angles. This is not true of quadrilaterals. Fasten four sticks together with one nail at each corner and note how the size and shape of the quadrilateral thus formed can be changed by a little pressure on its sides or angles. Then fasten three sticks together with one nail at each corner and note how the triangle thus formed cannot be changed in size or shape by any amount of pressure. A triangle once formed retains its shape and hence is rigid. What instances have you noticed in which the triangle has been used to make objects rigid?



167. How to bisect a line, using the compasses. Draw a line MN. From M as center, with a radius greater than $\frac{1}{2}$ MN, draw two arcs, one above and one below MN. Using the same radius and with N as a center, draw two other arcs. Connect the points at which the arcs intersect. Line OP is the bisector of line MN. Prove by measuring. Why must the radius of the arcs be greater than $\frac{1}{2}$ MN?

The resulting line OP not only bisects line MN but is also perpendicular to it. You can prove this by measuring the angles at the intersection of the two lines.

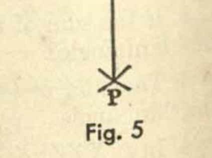


Fig. 5

Can you suggest practical uses for this construction?

EXERCISES

1. Draw a line 3 in. long. Using compasses, bisect the line. Measure the halves.

2. Bisect a 4-in. line, a 5-in. line, and a $4\frac{1}{2}$-in. line. Measure the parts in each case.

3. Divide a 6-in. line into 4 equal parts, using compasses.

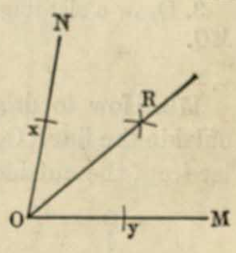
4. Draw an acute triangle. Erect a perpendicular bisector to each side of the triangle.

An important rule:

The perpendicular bisectors of the sides of any triangle intersect at the same point.

5. Draw an obtuse triangle. Erect a perpendicular bisector to each side of the triangle. What is true of the bisectors?

168. How to bisect an angle, using the compasses. To bisect an angle, as $\angle MON$, place the steel point of the compasses on the vertex O and draw an arc cutting the sides of the angle, as at x and y. From x and y as centers, draw two arcs intersecting, as at R. Join R and O with a straight line. The line OR bisects $\angle MON$. Prove by measuring $\angle$s MOR and RON.



EXERCISES

1. Draw a 90° angle. Bisect it. Test with a protractor.
2. Draw a 150° angle. Bisect and test.
3. Bisect an angle of 70°. Test.
4. How would you divide an angle into 4 equal parts?
5. Divide an angle of 160° into 4 equal parts, using compasses.
6. Draw an acute triangle. Bisect each angle.

An important rule:

The bisectors of the angles of any triangle intersect at the same point.

7. Draw an obtuse triangle. Bisect the angles. What is true of the bisectors?

169. How to erect a perpendicular to a line from a given point in the line. On the line AB it is required to erect a perpendicular at the point O. From O as a center draw arcs cutting AB in points X and Y, making $OX = OY$. From X and Y as centers draw arcs of equal radii intersecting at point Z. Draw ZO. ZO is perpendicular to the line AB at the point O. What kind of angles are ∡ YOZ and ZOX? Prove by measuring.

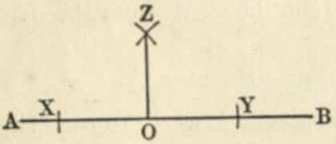


Fig. 6

EXERCISES

1. Draw a line MN. At a point O in MN erect a perpendicular line KO.

Fig. 7

2. Copy figure 7. At M erect a perpendicular line RM above the line AB. At N erect a perpendicular line SN below the line AB.

3. Draw a slanting line AB. At a point O on AB erect a perpendicular MO.

170. How to drop a perpendicular to a line from a given point outside the line. On the line AB it is required to drop a perpendicular from the outside point O. Using O as center, draw an arc intersecting the line AB in two points, x and y. With x and y as centers, draw two arcs of equal radii below the line AB, intersecting at point Z. Draw OZ. OZ is perpendicular to the line AB. Prove by measuring the angles thus formed.

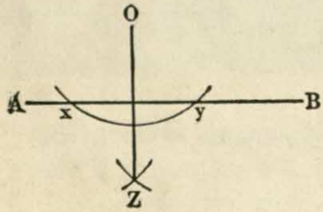


Fig. 8

EXERCISES

1. Draw a line MN. From a point O outside the line drop a perpendicular to MN.

2. Copy figure 9. Drop perpendiculars to the line AB from points M and N.

3. Copy figure 10. Drop perpendiculars to the line AB from points M and N.

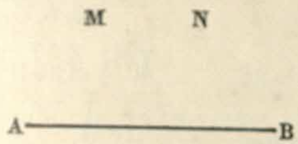


Fig. 9

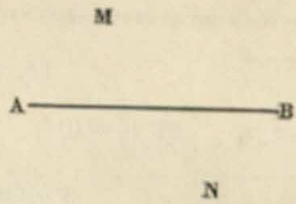


Fig. 10

4. Draw a slanting line and drop a perpendicular to it from any point outside the line.

ACHIEVEMENT TEST IV

Addition—4 minutes

1. 9	2. 87	3. 286	4. 42.66	5. $\frac{3}{4}$
7	46	375	.96	$\underline{\frac{7}{8}}$
5	32	421	3.77	
4	$\underline{59}$	189	$\underline{58.96}$	
8		674		
$\underline{6}$		$\underline{822}$		

6. $8.92 + .7 + 9$

7. $\frac{5}{9} + \frac{2}{3} + \frac{1}{6}$

8. $38\frac{1}{8}$	9. $37\frac{4}{5}$	10. 7 gal. 3 qt.
$\underline{62\frac{1}{6}}$	$\underline{68\frac{2}{3}}$	$\underline{\text{9 gal. 1 qt.}}$

Subtraction—4 minutes

11. 508	12. 7005	13. 815.9
$\underline{59}$	$\underline{786}$	$\underline{698.8}$

14. $3.17 − $.59

15. $2 − 35¢

16. $32\frac{1}{8} - 15.7$

17. $\frac{3}{8} - \frac{3}{16}$

18. $42\frac{2}{3}$	19. $81\frac{1}{4}$	20. 40° 30 min.
$\underline{18\frac{1}{6}}$	$\underline{38\frac{3}{5}}$	$\underline{\text{10° 40 min.}}$

Multiplication—4 minutes

21. 29	22. 204	23. 290	24. 7.9
$\underline{76}$	$\underline{708}$	$\underline{80}$	$\underline{.09}$

25. $\frac{1}{3} \times \frac{1}{5}$

26. $5\frac{1}{3} \times 21$

27. $\frac{1}{7} \times 4\frac{2}{3}$

28. $\frac{5}{6} \times 180$

29. $72\frac{3}{5}$	30. 2 hr. 10 min.
$\underline{35\frac{1}{3}}$	$\underline{8}$

Division—4 minutes

31. $4\overline{)387}$

32. $.19\overline{)1.216}$

33. $6.25 \div 100$

(Achievement Test IV—Continued)

34. $2.4\overline{)4.32}$

35. $\begin{array}{r}137\\ .057\overline{)7.809}\end{array}$

36. $\begin{array}{r}137\\ 5.7\overline{)78.09}\end{array}$

37. $\frac{3}{4} \div \frac{5}{8}$

38. $1\frac{2}{3} \div 10$

39. $14 \div 3\frac{1}{2}$

40. $\frac{1}{3}$ of 16 lb. 4 oz.

Per Cent—4 minutes

Write as fractions:

41. $66\frac{2}{3}\%$ **42.** 2%

Write as decimals:

43. 7% **44.** $1\frac{1}{4}\%$

Write as per cents:

45. $\frac{9}{25}$ **46.** .1

47. 40% of $125 = ?$

48. 14% of $? = 70$

49. $?\%$ of $45 = 15$

Arrange in order of size—largest first:

50. $.033\frac{1}{3}$, $3.33\frac{1}{3}$, $\frac{34}{100}$, $33\frac{1}{3}\%$

Record your scores on the Achievement Chart.

This is your last test. Calculate your percentage of improvement over the scores you made on Achievement Test I.

CHAPTER XV

INDIRECT MEASUREMENT OF LINES

VOCABULARY

1. indirect measurement
2. opposite side
3. square
4. square root
5. radical sign
6. hypotenuse

171. Distinguishing between direct and indirect measurement. Direct measurement is used whenever a ruler, tapeline, protractor, or any other measuring instrument is applied directly upon whatever is to be measured. Often it is inconvenient, and sometimes impossible, to measure a line or an angle by direct measurement. At such times some indirect method must be used. The width of a river, the breadth of a hill, and the height of trees, buildings, chimneys, flagpoles, and mountains are usually found by indirect measurement.

If one is familiar with certain truths, indirect measurement can easily be done by any one of several methods. In this chapter you will learn some of these truths and apply them in finding distances by indirect measurement.

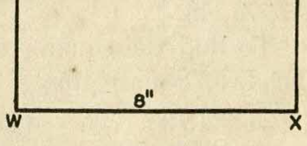


Fig. 1

172. Squares and square roots. Suppose that the square *WXYZ* measures 8 in. on a side. The area, then, is $(8 \text{ in.})^2$, or 64 sq. in.

Now, suppose you know the area to be 64 sq. in. and wish to know the length of one side. Since the area, 64 sq. in., is obtained by multiplying the length by the length, one side of the square can be found if we know what number multiplied by itself equals 64. Since $8 \times 8 = 64$, 8 in. is the length of one side of the square.

64 is the *square* of 8.
8 is the *square root* of 64.

The square root of a number is one of its two equal factors.

The sign of square root is $\sqrt{\ }$. It is called the *radical* sign.

Thus, $\sqrt{64} = 8$ and is read, "The square root of 64 equals 8."

EXERCISES

1. Write the squares of all the numbers from 0 through 15.

2. Find the square of 25, of 16, and of 18.

3. Find the value of 21^2, 17^2, 50^2, 24^2, 70^2.

4. (a) $(\frac{1}{3})^2 = \frac{1}{3} \times \frac{1}{3} = ?$ (b) $(\frac{2}{5})^2 = ?$
(c) $(\frac{3}{4})^2 = ?$ (d) $(\frac{7}{8})^2 = ?$ (e) $(\frac{4}{7})^2 = ?$

5. What is the square root of 16? Of 36? Of 81?

6. Find:

(a) $\sqrt{100}$ (e) $\sqrt{625}$ (i) $\sqrt{3600}$ (m) $\sqrt{\frac{1}{9}}$
(b) $\sqrt{225}$ (f) $\sqrt{121}$ (j) $\sqrt{2500}$ (n) $\sqrt{6\frac{1}{4}}$
(c) $\sqrt{144}$ (g) $\sqrt{196}$ (k) $\sqrt{\frac{4}{9}}$ (o) $\sqrt{11\frac{1}{9}}$
(d) $\sqrt{4900}$ (h) $\sqrt{6400}$ (l) $\sqrt{\frac{16}{25}}$ (p) $\sqrt{2\frac{1}{4}}$

7. What is the length of one side of a square if its area is 25 sq. in.?

8. Find the length of the sides of the squares whose areas are:

(a) 196 sq. in. (c) 1600 sq. rd. (e) 400 sq. in. (g) 1 sq. ft.
(b) 900 sq. ft. (d) 169 sq. yd. (f) .25 sq. ft. (h) 81 sq. yd.

To save time scientists, mechanics, and others often use a square and square root table, as on the next page. This table is shortened, giving the squares and square roots through 75 only.

To find the square of the number 23 in the table, locate the 23 in the first column headed "No." Then, in the column headed "Square," just to the right of the 23 is the number 529. $23^2 = 529$.

Using the table, find the squares of: 18, 24, 28, 33, 42, 48, 69, 57, 75, 46.

To find the square root of the number 23, look to the column headed "Square Root," the second column to the right of the 23. $\sqrt{23} = 4.795$. The square roots in this table are to thousandths but not necessarily to the nearest thousandths.

Find: $\sqrt{75}$, $\sqrt{43}$, $\sqrt{28}$, $\sqrt{51}$.

You can find the square root of the figures in the "Square" column by finding the number in the column headed "No." just to the left. For instance, the square root of 2116 is 46.

Find: $\sqrt{1849}$, $\sqrt{1156}$, $\sqrt{3969}$, $\sqrt{5329}$.

TABLE OF SQUARES AND SQUARE ROOTS

No.	*Square*	*Square Root*	*No.*	*Square*	*Square Root*	*No.*	*Square*	*Square Root*
1	1	1.000	26	676	5.099	51	2601	7.141
2	4	1.414	27	729	5.196	52	2704	7.211
3	9	1.732	28	784	5.291	53	2809	7.280
4	16	2.000	29	841	5.385	54	2916	7.348
5	25	2.236	30	900	5.477	55	3025	7.416
6	36	2.449	31	961	5.567	56	3136	7.483
7	49	2.645	32	1024	5.656	57	3249	7.549
8	64	2.828	33	1089	5.744	58	3364	7.615
9	81	3.000	34	1156	5.831	59	3481	7.681
10	100	3.162	35	1225	5.916	60	3600	7.746
11	121	3.316	36	1296	6.000	61	3721	7.810
12	144	3.464	37	1369	6.082	62	3844	7.874
13	169	3.605	38	1444	6.164	63	3969	7.937
14	196	3.741	39	1521	6.245	64	4096	8.000
15	225	3.873	40	1600	6.324	65	4225	8.062
16	256	4.000	41	1681	6.403	66	4356	8.124
17	289	4.123	42	1764	6.480	67	4489	8.185
18	324	4.242	43	1849	6.557	68	4624	8.246
19	361	4.358	44	1936	6.633	69	4761	8.306
20	400	4.472	45	2025	6.708	70	4900	8.366
21	441	4.582	46	2116	6.782	71	5041	8.426
22	484	4.690	47	2209	6.855	72	5184	8.485
23	529	4.795	48	2304	6.928	73	5329	8.544
24	576	4.899	49	2401	7.000	74	5476	8.602
25	625	5.000	50	2500	7.071	75	5625	8.660

A number whose square root is an integer is called a *perfect square.*

If you wish to find the square root of a number that is not in the table, you can compute the square root for yourself.

Example

Find $\sqrt{1049.76}$

	3 2 . 4
	10 49 . 76
3^2	9
60	1 49
62	1 24
640	25 76
644	25 76

1. Starting at the decimal point, and working to the left and to the right, set down the digits in pairs.

2. Place the decimal point in the quotient directly above the decimal point in the dividend. There will be as many digits in the quotient as there are pairs in the dividend. In this quotient there will be two digits to the left and one digit to the right of the decimal point.

3. Find the largest integer that, when squared, will be contained in 10. It is 3. $3^2 = 9$. ($4^2 = 16$ is too large.) Place the 3 in the quotient just above the zero in the first pair.

4. Put the 9 under the 10 and subtract, $10 - 9 = 1$. Bring down the next pair, 49. This makes the new dividend 149.

5. Trial divisor: To find the trial divisor, the part of the square root already found is always doubled and then multiplied by 10. This is the same as multiplying by 20. Hence, the part of the square root already found $(3) \times 20 = 60$, the trial divisor. Put the 60 on a line with and to the left of the 149. $149 \div 60 = 2+$. Place the 2 in the quotient above the 9 in the second pair. Then add 2 to the 60.

6. Real divisor: $60 + 2 = 62$, the real divisor. $62 \times 2 = 124$. Subtract and bring down the next pair. 2576 is the new dividend.

7. Trial divisor: $32 \times 20 = 640$. 2576 divided by $640 = 4+$. Put the 4 in the quotient above the 6 in the third pair. Add 4 to 640.

8. Real divisor: $640 + 4 = 644$, the real divisor. $644 \times 4 = 2576$. There is no remainder.

9. The square root of $1049.76 = 32.4$.

10. Check: $32.4 \times 32.4 = 1049.76$, the original square.

Find the square root of:

1. 14.44	6. 16.81	11. 1267.36	16. 10.7584
2. 26.01	7. 43.56	12. 17.2225	17. 9216
3. 12.96	8. 10.24	13. 46.24	18. 1049.76
4. 21.16	9. 50.41	14. 18.0625	19. 37.9456
5. 31.36	10. 72.25	15. 739.84	20. 219.04

Many numbers have no exact square roots, yet an approximate square root can be found for any number.

Example

Find $\sqrt{326.149}$

	1 8. 0 5
	3 26.14 90
1^2	1
20	2 26
28	2 24
360	2 14 90
3600	
3605	1 80 25
	34 65

1. In setting off 326.149 into pairs, there is only one digit (3) for the first pair to the left. This is all right. There is also just one digit (9) for the last pair to the right of the decimal point. In this case annex a zero, and the pair becomes 90.

2. Proceed as usual until the second trial divisor (360) will not be contained in the dividend (214). Put a zero in the quotient and bring down the next pair. The new trial divisor is 3600.

3. The square root to hundredths is 18.05. To the nearest tenth it is 18.1. The nearest hundredth can be found by carrying the quotient to thousandths. Annex a pair of zeros for each additional digit desired in the quotient.

4. Check: $18.05^2 = 325.8025$
Add remainder $= .3465$
Original square $= 326.1490$

EXERCISES

1. Find the square root to tenths:

(a) 147.06 (b) 238.84 (c) 7245.8
(d) 6925.6 (e) 573.1 (f) 6.6

2. Find to the nearest tenth:

(a) $\sqrt{64.8274}$ (b) $\sqrt{936.745}$ (c) $\sqrt{1512.94}$

3. Find to the nearest hundredth:

(a) $\sqrt{86.4528}$ (b) $\sqrt{3.7}$ (c) $\sqrt{25.623}$

4. Find the value of:

(a) $\sqrt{\frac{25}{64}} = \frac{5}{8}$. Why? (b) $\sqrt{\frac{3-\frac{3}{4}}{9}} = \sqrt{\frac{2.25}{9}} = \frac{1.5}{3} = .5$

(c) $\sqrt{\frac{49}{225}}$, (d) $\sqrt{\frac{324}{625}}$, (e) $\sqrt{\frac{81}{2500}}$, (f) $\sqrt{\frac{289}{576}}$, (g) $\sqrt{\frac{7-\frac{3}{4}}{16}}$.

5. Find to the nearest tenth after expressing as decimals:

(a) $\sqrt{\frac{4}{5}}$, (b) $\sqrt{\frac{1}{3}}$, (c) $\sqrt{\frac{1}{8}}$, (d) $\sqrt{1\frac{1}{2}}$, (e) $\sqrt{\frac{12+\frac{1}{4}}{6}}$.

6. If the area of a square is 5,184 sq. ft., what is the length of one side of the square?

7. A square garden contains 7,744 sq. ft. Find the length of one side of the garden.

8. A mechanic needs a square board whose cross-section area is 60.84 sq. in. What must be the length of the cross-section?

9. A square kitchen contains $90\frac{1}{4}$ sq. ft. How long is the kitchen?

10. A regulation baseball diamond is a square containing 8100 sq. ft. How far is it between consecutive bases?

11. (OPTIONAL) A square table cover contains 1,089 sq. in. How many inches of fringe will be needed around the edge? Allow $1\frac{1}{2}$ extra inches for each corner.

12. (OPTIONAL) The area of a square garden is 6,724 sq. ft. How much will fencing cost to inclose the garden at 39¢ per yard?

13. (OPTIONAL) A square field contains 40 acres. How many rods of fencing will be needed to inclose it?

173. Uses of square root. We have seen how square roots are used to find the length of one side of a square when the area is known. Square roots may be used also to find the length of one side of a right triangle when the lengths of the other two sides are known.

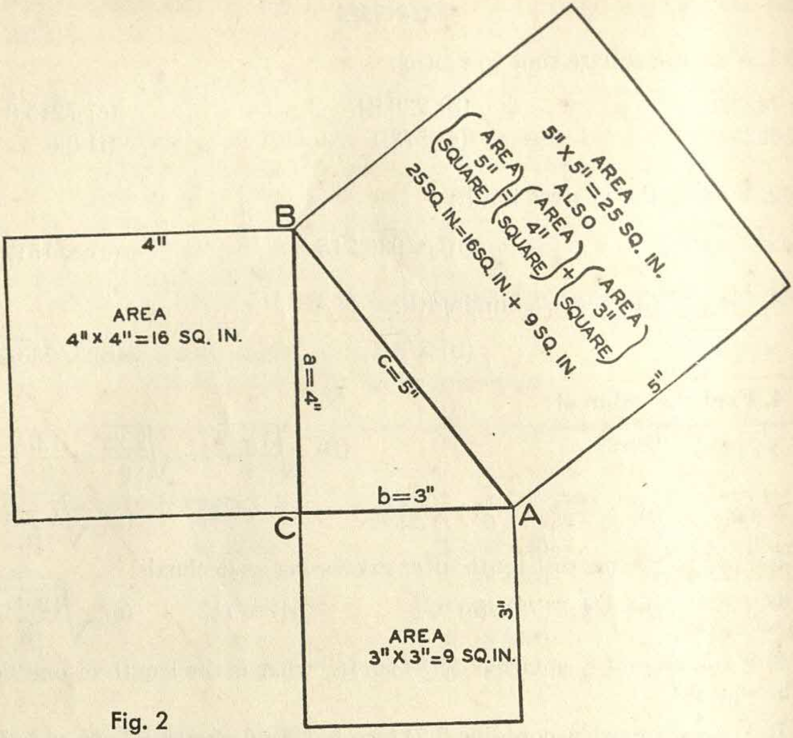


Fig. 2

More than two thousand years ago a Greek philosopher, Pythagoras, made the discovery that is illustrated in figure 2. Triangle ABC is a right triangle with a right angle at C. a represents the altitude, b the base, and c the side opposite the right angle, called the *hypotenuse.* What is the area of the square on a, the square on b, and the square on c? Add the areas of the squares on a and b. How does this sum compare with the area of the square on c?

In any right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Put into a formula, this truth becomes:

$$c^2 = a^2 + b^2$$

From this formula we get two other formulas:

$$a^2 = c^2 - b^2 \quad \text{and} \quad b^2 = c^2 - a^2$$

From these three formulas we get three other formulas, which we use in finding the hypotenuse, the base, and the altitude of a right triangle:

$$c = \sqrt{a^2 + b^2} \qquad a = \sqrt{c^2 - b^2} \qquad b = \sqrt{c^2 - a^2}$$

Example A

If your classroom is 36 ft. long and 27 ft. wide, what is the diagonal distance from one corner to the opposite corner?

$$c = \sqrt{a^2 + b^2} \qquad a = 36 \text{ ft.}$$
$$c = \sqrt{36^2 + 27^2} \qquad b = 27 \text{ ft.}$$
$$c = \sqrt{1296 + 729}$$
$$c = \sqrt{2025}$$
$$c = 45$$

Hence, the diagonal is 45 ft.

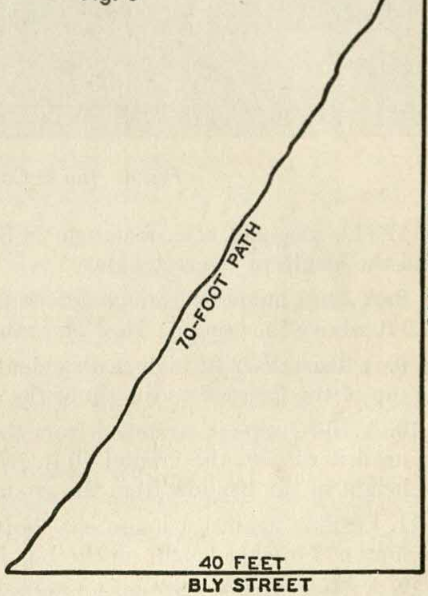
Example B

Bly and Oak streets are perpendicular to each other. See Fig. 3. There is a 70-ft. path cutting Bly Street 40 ft. from the corner. How far from the corner does the path cut Oak Street?

$$a = \sqrt{c^2 - b^2} \qquad c = 70 \text{ ft.}$$
$$a = \sqrt{70^2 - 40^2} \qquad b = 40 \text{ ft.}$$
$$a = \sqrt{4900 - 1600}$$
$$a = \sqrt{3300}$$
$$a = 57.4$$

Hence, Oak Street is cut approximately 57.4 ft. from the corner.

Fig. 3



EXERCISES

Sketch, label, and then solve:

1. The base of a right triangle is 21 ft. and its altitude is 28 ft. Solve for the hypotenuse.

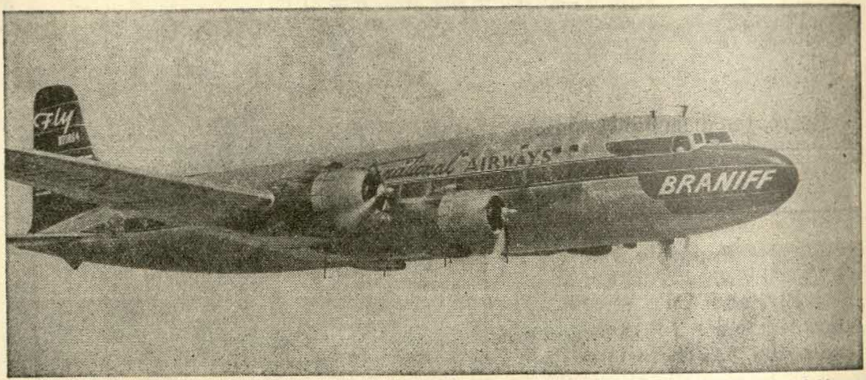
2. An iron gate which has a length of 10 ft. and a height of 4 ft. is to be braced diagonally from corner to corner. Find the length of the brace.

3. A lot is in the shape of a rectangle. Its length is 32 ft. and its diagonal distance from the northwest corner to the southeast corner is 40 ft. Find the width of the lot.

4. An automobile travels directly west 20 miles. If it then turns and travels directly north for 25 miles, how far is it from its starting point by a straight line?

5. On a regulation baseball diamond, it is 90 ft. from any base to the next. What is the distance from home plate to second base?

6. Plane A leaves an airport and flies directly south for 120 miles. Plane B leaves at the same time and flies directly east for 160 miles. How far apart are the planes by the shortest route?



Courtesy Braniff International Airways

Fig. 4. The El Conquistador.

7. The diagonal of a rectangle is 36 in. long and its width is 14 in. Find the length of the rectangle.

8. A 32-ft. ladder is leaning against the window of a barn. The window is 25 ft. above the ground. How far is the foot of the ladder from the barn?

9. A flagstaff 72 ft. high casts a shadow 30 ft. long. How far is it from the top of the flagstaff to the tip of the shadow?

10. A 75-ft. rope is stretched from the second-story window of a building until it reaches the ground 40 ft. from the base of the building. Find the height of the window from the ground.

11. Leaning against a house is a 24-ft. ladder whose base is 15 ft. from the house. To what height on the building does the ladder extend?

12. A 70-ft. brace wire on a telephone pole touches the ground 24 ft. from the foot of the pole. What is the approximate height of the pole?

13. A 180-ft. rope of a circus tent is stretched so that it is fastened to the ground at a point 100 ft. from the base of the center pole. Find the height of the pole.

14. (OPTIONAL) What is the diameter of the largest wheel that can be taken through a door that is $4\frac{1}{2}$ ft. wide and $8\frac{1}{2}$ ft. high?

15. (OPTIONAL) Measure the length and width of your classroom to the nearest foot. Find the length of the diagonal from one corner to the opposite corner of the floor. Check your result by measurement.

16. (OPTIONAL) Find the distance around a room, to the nearest tenth of a foot, if its length is 16 ft. and the distance between the opposite corners is 24 ft.

CHAPTER XVI

THE FORMULA

VOCABULARY

1. symbol	4. literal factor	7. similar terms
2. factor	5. coefficient	8. dissimilar terms
3. numerical factor	6. terms	9. evaluate

174. Writing formulas. In Chapter XIII you shortened geometric rules by the use of letters and other symbols. In the formula $A = lw$, the A, l, and w are called *symbols*. In this formula the w does not stand for the word "width"; but w does stand for the number of units in the width. The l stands for the number of units in the length. What does the A represent? The letters are used to represent numbers and not words.

The signs (symbols) for the fundamental operations in algebra are: $+$, $-$, $\times$, and $\div$, as used in arithmetic. However, the times sign, $\times$, is usually either replaced by a dot or omitted entirely, as in such expressions as $a \cdot x$, lw, $3b$, and $4xy$.

The division of numbers in algebra is more often expressed in the form of a fraction than by the use of the division sign $\div$. That is, $\frac{a}{b}$ is a more common form of division in algebra than $a \div b$.

Here, for example, are a few expressions stated in terms of symbols and numbers:

(a) The expression "2 more than a" may be written "$a + 2$" or "$2 + a$."
(b) The expression "5 times y" may be written "$5y$."
(c) The expression "the quotient of 2 and 3" may be written "$\frac{2}{3}$."
(d) The expression "the quotient of s and t" may be written "$\frac{s}{t}$."
(e) The expression "one-third x" may be written "$\frac{1}{3}x$" or "$\frac{x}{3}$."
(f) The expression "4 times a divided by 5" may be written "$\frac{4}{5}a$" or "$\frac{4a}{5}$."

EXERCISES

1. State each of the following expressions in terms of symbols and numbers:

(a) y increased by 4.
(b) The sum of m and n.
(c) 8 times r.
(d) The product of l and w.
(e) a less b.
(f) a divided by b.
(g) m divided by r.
(h) 4 y's added together.
(i) One-half g decreased by 3.
(j) t divided by 4.
(k) The sum of x, y, and z.
(l) The difference of c and d (c being the larger).
(m) The quotient of $3a$ and $4b$.
(n) 5 decreased by e.
(o) Three times the product of e and f.
(p) Twice x divided by 4.
(q) The square of x.
(r) Three times the square of a.
(s) The square of d minus the square of m.
(t) The product of the square of a and the square of b.
(u) The sum of the square of x and the square of y.
(v) The quotient of five times the square of w and 4 times the square of x.
(w) The product of b and c divided by the product of d and e.
(x) The square of r minus the product of s and t plus the square of x.
(y) Four times the product of w and x minus the sum of the square of y and the square of z.
(z) Twice the product of the square of x and the square of t divided by the sum of a and b.

2. Change the following formulas into sentences, orally:

$$A = bh \qquad V = lwh \qquad V = \pi r^2 h \qquad A = s^2$$
$$C = 2\pi r \qquad A = \pi r^2 \qquad P = 2b + 2h \qquad P = 4s$$

3. State the fundamental operations called for in the following expressions: bh, lwh, $2b$, $2h$, s^2, πr^2, $2\pi r$, $6xy$, $l + w$, $2l + 2w$, $a + b + c$, $\frac{bh}{2}$, $\frac{A}{w}$, $\frac{V}{lw}$, $c - e$, $\frac{n}{2}$, $\frac{9c}{2}$, $\frac{1}{2}y$, $2d - 3$.

4. State the following expressions in a shorter way: $l + l$, $s + s + s$, $2 \times a$, $3 \times b$, $a \times b$, $e \times e$, $\pi \times r \times r$, $e \times e \times e$, $2 \times w \times w$, $6 \times s \times s$, $A \div l$, $V \div wh$, $3c + 4c$, $\frac{1}{2}d$.

5. What is the difference in meaning between $l + w$ and lw? $a + b + c$ and abc? $2l$ and l^2? $2r$ and r^2? $3e$ and e^3? $3s$ and s^3?

6. If the length of a pencil is represented by l, what do the following expressions mean: $2l$, $3l$, $4l$, $5\frac{1}{2}l$, $\frac{1}{2}l$, $\frac{1}{4}l$, $2.5l$, $\frac{l}{2}$, $l+5$, $l+10$, $l-1$ in., $2l-3$ in., $l+\frac{l}{2}$, $l-\frac{1}{2}l$?

7. Let the width of the room be called w. What is the meaning of $8w$, $\frac{1}{2}w$, $3\frac{1}{3}w$, $1.5w$, $w+6$ ft., $w-3$ ft., $2w-1$ ft.?

8. Let h denote the height of a wall. What is the height of a wall 3 times as high? $\frac{1}{3}$ as high? $\frac{2}{3}$ as high? $\frac{1}{3}$ higher? $\frac{1}{3}$ lower? $\frac{2}{3}$ higher? $\frac{2}{3}$ lower? $\frac{1}{4}$ as high?

9. Draw a line the length of your thumb. Letter it l. What does $2l$ mean? Explain the meaning of $3l$. Of $2\frac{1}{2}l$. Draw each length.

10. Let d represent the diameter of a penny. Draw lines representing $2d$, $5d$, $1\frac{1}{2}d$, $3\frac{1}{2}d$, $\frac{d}{2}$, $d+1$ in., $2d-\frac{1}{2}$ in. $\frac{2d}{3}$.

175. Some common algebraic terms. In the simple exercise $3 \times 4 = 12$, what do you call the 12? The 3 and 4? As in arithmetic, you call the 12 the *product* and the 3 and the 4 the *factors*. In the formula $p = 4s$, p is called the product and 4 and s the factors; 4 is known as the *numerical factor* and s as the *literal factor*. Can you explain why? The numerical factor is usually called the *coefficient*. In such expressions as a, x, k, mn, yz, the coefficient or numerical factor is understood to be 1.

Algebraic expressions such as $4a$, $7b$, and $3x$ are called *terms*. Two or more terms having the same literal factor are said to be *similar*. For instance, $4m$, $5m$, and $2m$ are *similar terms* because each term contains the literal factor m.

Two or more terms having different literal factors are said to be *dissimilar*. $4x$, $5y$, $4x^2$, and $2z$ are *dissimilar terms*.

EXERCISES

1. State orally (1) the numerical factors, (2) the literal factors, of the following terms: $4a$, $7b$, $2k$, ax, $8r$, $\frac{1}{2}t$, $6s$, $3.5c$, $5.8b$, $3\frac{1}{4}m$, $6ab$, $2mn$, $\frac{1}{3}xy$, $4abc$, $8xyz$, mn, $.6c$, $1\frac{1}{2}az$, d^2, ef.

2. Name orally the coefficients in the following terms: $2l$, $5m$, $\frac{1}{10}n$, $7ab$, $5t$, y, $3.4r$, $.6f$, $.4e$, $1.2k$, $\frac{n}{6}$, $14axy$, $\frac{n}{6}$, $\frac{2n}{4}$, $2x^2$.

3. List the following terms in two groups, (1) those similar to $4a$, and (2) those dissimilar to $4a$: $7a$, $6x$, $5a$, $4a$, a, $3y$, $2n$, $6a^2$, $\frac{1}{2}a$, $2b$, $4a^3$, $3.2a$, $4ab$, $\frac{a}{2}$, $.4a$, $16a^2$, $8a$.

4. Is $6n$ similar or dissimilar to n? To n^2? To mn? To $5n$? To $6an$?

5. Make a list of five terms similar to $6x$ and five terms dissimilar to $6x$.

6. List five terms similar to $2mn$ and five terms dissimilar.

7. List five terms similar to $3l$ and five terms dissimilar.

8. List five terms similar to a^2 and five dissimilar to a^2.

9. List five terms similar to $25xyz$ and five dissimilar to $25xyz$.

176. Combining similar terms. You may add or subtract similar terms by finding the sum or difference of the coefficients of the given terms and multiplying this result by the common literal factor. Thus:

$4m + 5m + 2m = 11m$ (that is, $4 + 5 + 2 = 11$; $11 \times m = 11m$).
$8m - 2m = 6m$ (that is, $8 - 2 = 6$; $6 \times m = 6m$).

The addition and subtraction of terms that are not similar can only be indicated by the plus and minus signs. Thus, $4m$ added to $5n$ equals $4m + 5n$, and $3b$ subtracted from $2a$ equals $2a - 3b$.

If more than one set of similar terms appears in an expression, combine them as follows:

1. Group together the terms that are similar to each other.

2. Add or subtract separately each group of similar terms. For example:

$p = 4m + 2y + 3m + 5y + 2m$
$p = 4m + 3m + 2m + 2y + 5y$ (Grouping similar terms together)
$p = 9m + 7y$ (Adding similar terms)

EXERCISES

1. In the following exercises perform the necessary operations, letting r represent the result in each case:

(a) $3a + 5a + 7a$
Solution:
$r = 3a + 5a + 7a$
$r = 15a.$
(b) $4b + 8b + 6b + b$
(c) $80w + 70w + 40w - 20w$
(d) $100n - 16n + 26n - n$
(e) $60h + 24h - 30h$
(f) $\frac{1}{5}e + \frac{3}{10}e + \frac{1}{2}e$
(g) $\frac{1}{2}y + \frac{3}{8}y + \frac{5}{8}y - y$
(h) $3\frac{1}{4}x - 1\frac{1}{2}x + \frac{1}{4}x$
(i) $2\frac{1}{2}n + 8\frac{3}{4}n - 3\frac{1}{4}n - n$
(j) $8.4m - 2.3m + 4.1m$
(k) $700k - 150k - 275k - 25k$
(l) $.64y + .72y + .35y - .82y$
(m) $3.8z + .4z + 7z + z$
(n) $9.4a - 3.9a + 6.2a - .7a$
(o) $9x + 3\frac{1}{2}x + 7\frac{5}{6}x - 8\frac{1}{2}x$
(p) $42d + 78d + 65d + 37d$
(q) $5.6h + 7.8h - 3.5h + 1.4h$
(r) $7x + 2y + 3y + 8x - 9x - x$
(s) $6a + 3a + 2b + 5a - a - 6a$
(t) $6m + 4n - 2m - n + n$
(u) $7a + 5b - 3a + b + 4a$
(v) $4.6x + 3.2y + 5.2x - 1.4y$

(w) $\begin{array}{r}195c\\ \underline{+\,76c}\end{array}$	(x) $\begin{array}{r}7\frac{2}{3}w\\ \underline{+\,9\frac{1}{6}w}\end{array}$	(y) $\begin{array}{r}15.4l\\ \underline{-\,8.8l}\end{array}$	(z) $\begin{array}{r}24.3m\\ \underline{-\,16.7m}\end{array}$

2. The costs of four books are $3c$, $5c$, $8c$, and $2c$, respectively. Find the sum (s) of the costs.

3. The attendance during the first 5 months of a school year was $408a$, $412a$, $460a$, $454a$, and $472a$. Find the total (t) attendance for these months.

4. The savings of Lila, Jean, and Rose are $14s$, $28s$, and $45s$, respectively. Find their total savings (t). How much more are the savings of Rose than those of Lila? How many times the savings of Jean are those of Lila?

5. Find the total length (t) of the sides of a regular pentagon and a regular hexagon, if their perimeters are of length $8e$ and $10e$, respectively.

6. James makes 6 models of cubes from sticks. The sum of their edges is $2e$, $3e$, $4e$, $6e$, $7e$, and $8e$, respectively. What is the total length of sticks used for the 6 models?

7. Two boards the same length (l) are placed together, as in figure 1. What is the total length of the larger rectangle thus formed?

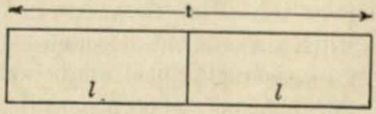


Fig. 1

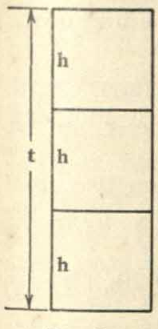


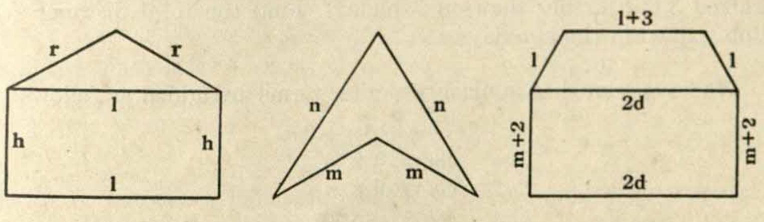
Fig. 2

8. John piled three blocks, all of which were the same height (h), one on top of the other, as in figure 2. What is the total height (t) of the three blocks?

9. Draw a regular hexagon. If the length of each side is l, write the formula for the perimeter of the hexagon.

10. Write the formula for the perimeter of a regular octagon. Of a regular pentagon. (Let l represent the length of one side in each case.)

11. Write the formula for the sum of the lines of each of the shapes in figure 3.

Fig. 3


12. (OPTIONAL) Draw, cut, or otherwise make models of various forms; letter the edges; write the formula for the length of the total number of edges in each case.

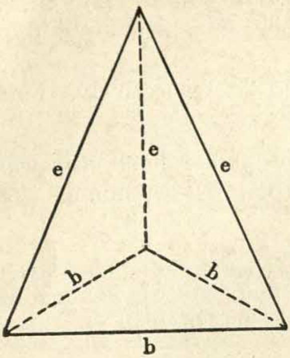


Fig. 4

13. In figure 4, the edges are represented by e and b, as pictured. What is the formula for the length of the total number of edges (t)?

14. The distance (d) equals the rate (r) multiplied by the time (t). Express this relationship in a formula. $d = ?$

15. The selling price (s) equals cost price (c) plus gain (g). $s = ?$

16. The loss (l) equals cost price (c) minus selling price (s). $l = ?$

17. The gain (g) equals selling price (s) minus cost (c). $g = ?$

18. Interest (i) equals principal (p) times rate (r) times time in years (t). $i = ?$

19. Amount (A) equals principal (p) plus interest (i). $A = ?$

20. The average (a) of a series of numbers is the sum (s) divided by the number (n) in the series. $a = ?$

21. If the cost of a telegram between two cities is 60¢ for 15 words plus $2\frac{1}{2}$¢ for each additional word, explain the formula $C = 60 + 2\frac{1}{2}(n - 15)$.

22. If the cost of a 15-word telegram between two stations is 75¢ plus $4\frac{1}{2}$¢ for each additional word, write the formula representing the cost.

23. If the parcel-post rate to the third zone is 13¢ for the first pound plus 3¢ for each additional pound, explain $C = 13 + 3(n - 1)$.

24. Write a formula representing the parcel-post charge for the sixth zone, if the rate is 16¢ for the first pound plus 8¢ for each additional pound.

25. (OPTIONAL) The cost of a 50-word night letter between two cities is 85¢. Five cents is charged for each 5 additional words. Write the formula.

26. (OPTIONAL) A bus fare (f) is 25¢ for the first mile and 10¢ for each mile (m) thereafter. $f = ?$

27. (OPTIONAL) On Monday, Bob walked m miles. On Tuesday, he walked 3 mi. farther than on Monday. Find the total distance (t) that Bob walked in the two days.

We have seen that in algebra similar terms are added as follows:

$$4m + 6m + 3m$$

$$\begin{aligned} 4m &= 4 \times m \\ 6m &= 6 \times m \\ 3m &= 3 \times m \\ \hline \text{Sum} &= 13 \times m = 13m \end{aligned}$$

In arithmetic, sums may be found in a like manner. Thus, if a salesman sells 142 books to one firm at 68¢ each, 134 books to another firm at

68¢ each, and 175 books to a third firm at 68¢ each, he may total his three sales as follows:

$$\begin{array}{l} 142 \times 68¢ \\ 134 \times 68¢ \\ \underline{175 \times 68¢} \\ 451 \times 68¢ = \$306.68 \end{array}$$

Solve the following exercises by the preceding method:

28. James sold from his garden 8 qt. of beans to Mr. A @ 15¢; 6 qt. to Mr. B @ 15¢; and 10 qt. to Mr. C @ 15¢. Find the total amount received from his sales.

29. Florence made doorstops, which she sold at 75¢ apiece. During September, October, November, and December she sold 12, 18, 20, and 32, respectively. Find the amount of her total sales for the four months.

30. Mr. Kelley borrowed \$1,280 from Mr. Morgan and \$1,500 from Mr. Witte. He paid 8% interest in each case. How much interest does he pay each year?

31. Mr. Ward bought his coal from three different firms, paying each \$12.50 per ton. Find the total cost of his coal if from one firm he purchased 4 T., from another 5 T., and from the third $3\frac{1}{2}$ T.

32. For the three weeks preceding Christmas, Jack sold toys that he had made in his workshop. The first week he sold 34, the second week 48, and the third week 65. If his profit on each was 25¢, how much was his total profit?

177. Making algebraic formulas for perimeters and areas. The perimeter of a figure is the distance around its outer boundary. Thus, in figure 5 the perimeter is $a + a + b + a + b + a$, or $4a + 2b$.

To find the entire amount of space inclosed within the lines (the area), you must add together the two areas inclosed. The area of the square in figure 5 is $a \times a = a^2$. The area of the rectangle in figure 5 is $a \times b = ab$. The area of the entire figure then is $a^2 + ab$.

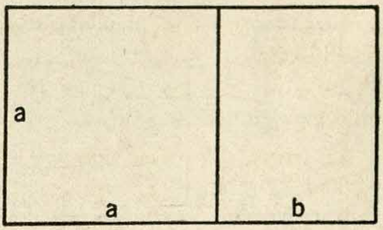


Fig. 5

EXERCISES

1. Draw or cut out a rectangle. Label its sides l and w to represent length and width, respectively. What formula will represent its perimeter; that is, $p = ?$ $A = ?$

2. Draw or cut out a triangle. Label its three sides x, y, and z. What formula will represent its perimeter?

3. In an equilateral triangle, let the length of each side be represented by s. Then $p = 3s$. Explain.

4. Draw an isosceles triangle. Represent each of the equal sides by a and the base by b. Write in two different ways the formula for the perimeter of the triangle.

5. Let e represent the length of each edge of a cube. Write the formula for the length of the total number of edges (t). Area of one face = ? Area of all faces = ?

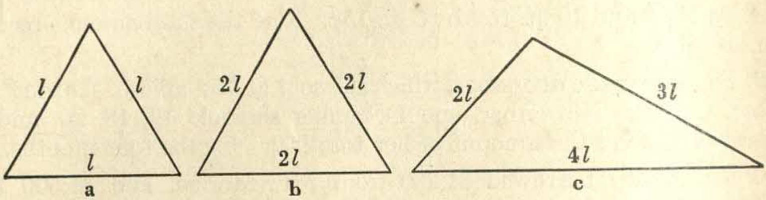


Fig. 6

6. Find the perimeter of each of the triangles in figure 6.

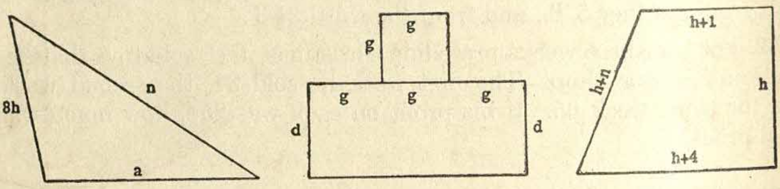


Fig. 7

7. Write the formula for the perimeter of each figure in figure 7.

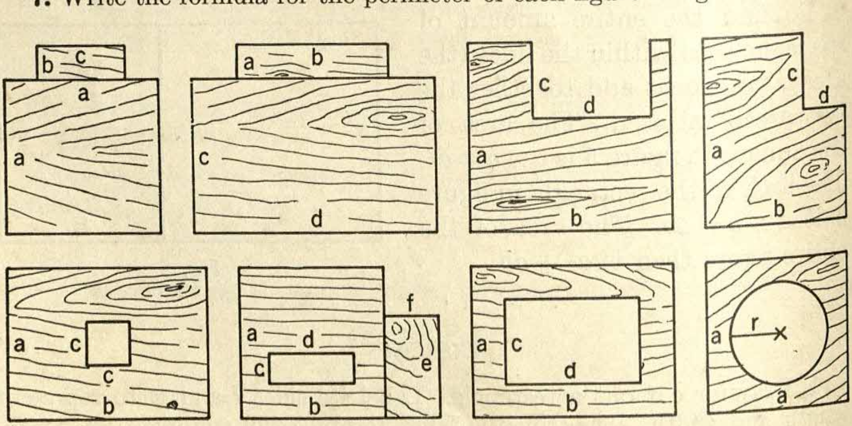


Fig. 8

8. Write the formula for the area of each board in figure 8.

178. Evaluating algebraic expressions. To *evaluate* is merely to put a number in place of a letter in an algebraic expression in order to find the numerical value of the whole expression.

Examples

A. If:
$$a = 4$$
$$b = 7$$
$$c = 5$$
Then:
$$a + b - c = 4 + 7 - 5$$
$$a + b - c = 6.$$

B. If:
$$x = 8$$
$$y = 3$$
Then:
$$x^2 + y^2 = 64 + 9$$
$$x^2 + y^2 = 73.$$

If an expression involves multiplication and division as well as addition and subtraction, *perform the operations of multiplication and division first, and then perform the operations of addition and subtraction.*

Examples

A. If:
$$x = 5$$
$$y = 3$$
Then: $4x + 2y = 4 \times 5 + 2 \times 3$
Multiplying: $4x + 2y = 20 + 6$
Adding: $4x + 2y = 26.$

B. If:
$$a = 2$$
$$b = 3$$
$$c = 5$$
Then: $6ab - 2bc = 6 \times 2 \times 3 - 2 \times 3 \times 5$
Multiplying: $6ab - 2bc = 36 - 30$
Subtracting: $6ab - 2bc = 6.$

EXERCISES

A. Oral

Find the numerical value of each of the following expressions; let $a = 10$, $b = 6$, $c = 5$, $x = 8$, $y = 2$:

1. $a + b$

2. $a + b - c$

3. $2a + b$

4. $x - y$

5. $3x - 2y$

6. $x^2 + y^2$

7. $a^2 - b^2$

8. $ab + c$

9. $abc + xy$

10. $bc - a$

11. $5x + xy$

12. $3ab - bc$

13. $b^2 + c^2$

14. $ax - by$

15. $2ay + bc - cy$

16. $\frac{a}{c} + \frac{x}{y}$

17. $\frac{c}{a} + \frac{y}{x}$

18. $\frac{y}{a} + \frac{y}{c}$

19. $\frac{2a}{c} + x$

20. $ay + bx + cy$

21. $\frac{ab}{c} - \frac{x}{y}$

22. $aby - bcy$

23. $\frac{y}{a} + \frac{x}{c} - \frac{c}{b}$

24. $\frac{12c}{a} - \frac{y}{b}$ 25. $\frac{y}{x} + \frac{c}{b}$ 26. $\frac{cxy}{2a} - \frac{3c}{b}$ 27. $\frac{2c}{2y} - \frac{2c}{2b} - \frac{y}{a}$

B. Written

Let $a = 3$, $b = 2$, $m = 5$, and $n = 4$. Find the value of each of the following expressions:

1. $4a - 2b + ab$
2. $3m + 8n - m$
3. $a^2 + 2n^2$
4. $3ab + 2mn$
5. $4a^2 - 3b^2$
6. $\frac{a}{b} + \frac{m}{n}$
7. $\frac{a}{m} + \frac{b}{n}$
8. $\frac{a^2b^2}{n} - \frac{bm}{a}$
9. $an + ab + 2mn$
10. $6mn - 4ab$
11. $m^2 - n^2 + mn$
12. $2amn - b^2m$
13. $a^2m^2 - 2b^2m^2$
14. $4am^2 - 5b^2n$
15. $\frac{a^2n}{2b} + \frac{bmn}{8}$
16. $5am - \frac{4ab}{an}$
17. $3b^2n + 2a^2m$
18. $10a^2 + 5b^2$
19. $m^2n - a^2b$
20. $10am^2 + 4bn^2$
21. $4abm + 5bmn$
22. $2a^2b^2m^2n^2$
23. $\frac{m^2n}{2b^2}$
24. $\frac{3a^2bn^2}{a^2b^2}$
25. $\frac{5mn^2}{b^2m^2}$
26. $10a^2b - 10b^2n$
27. $\frac{n^2m}{b^2} \div mn$
28. $\frac{20}{a} + \frac{20}{bn}$
29. $\frac{3a^2m}{m} + \frac{n^2}{m}$

Let $a = 3$, $b = 5$, $c = 1$, $n = 10$, $t = 4$, $x = 2$, $y = \frac{1}{2}$. Find the value of each of the following expressions:

1. $2a^2b$
2. $c^2 + cy$
3. y^2
4. $2x + x^2$
5. $6ty + \frac{n}{b}$
6. $\frac{n^2}{n} + \frac{a^2}{a} + \frac{b^2}{b}$
7. $\frac{1}{2}a + \frac{1}{2}c + \frac{1}{2}y$
8. $\frac{2ab}{n} + \frac{x^2}{t}$
9. $\frac{n^2}{b^2} - \frac{t^2}{x^2}$
10. $\frac{xy}{c}$
11. $\frac{a}{c} - \frac{n}{b}$
12. $\frac{xy + t}{b}$
13. $2a^2 + 2b^2 + 2c^2$
14. $\frac{ntx}{b} - 4x^2$
15. $2t^2 - 2t$
16. $3nx^2 - 2ny^2$
17. $abc + 5c$
18. $cy + ac - cx$
19. $\frac{2b^2x^2}{n^2}$
20. $\frac{a^2b^2c^2}{n} - xy$

179. Making formulas. The relationship between two numbers may be studied and then expressed in a formula. For instance, May deposits money in the school bank every Tuesday. Her total savings at the close of every Tuesday are shown in the following table:

Number of weeks (w)	1	2	3	4	5	6	10	15	20	52
Total savings in cents (s)	15	30	45	60	75	—	—	—	—	—

Each week the number representing the total savings is how many times the number representing the number of weeks? At this rate how much will May's total savings be for the remaining weeks indicated in the table? If s represents the total savings and w the number of weeks, $s = ?w$.

What is the relation of b to a in the following table? b is how much more than a? Then $b = a + ?$ Find the missing numbers.

$a =$	1	2	3	4	5	6	8	10	12	18
$b =$	4	5	6	7	—	—	—	—	—	—

EXERCISES

Write the formula expressing the relationship shown in each table from 1 to 8, and find the missing numbers:

1.

$x =$	1	2	3	4	5	6	7	8
$y =$	3	6	9	12	—	—	21	—

$y = ?x$ $\quad x = ?y$

2.

$a =$	4	8	12	16	20	—	—	—
$b =$	1	2	3	4	5	6	7	10

$a = ?b$ $\quad b = ?a$

3.

$s =$	5	6	7	10	15	20	25	28
$t =$	10	11	12	15	20	—	—	—

$t = s + ?$ $\quad s = ?$

4.

$m =$	8	10	12	15	18	20	22	30
$n =$	6	8	10	13	16	—	—	—

$n = ?$ $\quad m = ?$

5.

$l =$	1	2	3	4	5	6	8	10
$w =$	1	4	9	16	—	—	—	—

$w = ?$

6.

$c =$	1	2	3	4	5	6	7	8
$d =$	3	5	7	9	—	—	15	—

$d = 2c + ?$

7.

$w =$	2	4	5	6	8	10	12	15
$x =$	3	7	9	11	—	19	—	—

$x = ?$

8.

$r =$	3	4	6	8	10	11	12	20
$s =$	11	14	20	26	—	—	—	—

$s = ?$

9. Fred rides straight from his home to a camp, a distance of 50 mi., at the rate of 10 mi. per hour.

Hours traveled $(h) =$	1	2	—	—	—
Distance traveled $(d) =$	—	—	—	—	—

Fill in the preceding table. How long will it take Fred to reach the camp? Write the formula showing the relation between the distance traveled and the number of hours traveled ($d = ?h$; $h = ?d$).

10. George earned $5 a week. Make a table showing his earnings at the close of the first week, second week, fifth week, eighth week, tenth week, and fifty-second week. Write the formula showing the relation between earnings (e) and weeks worked (w).

11. Sketch plane figures of the following numbers of sides: 4, 5, 6, 7, and 8. Draw as many diagonals as possible from one vertex. How many diagonals are drawn in each figure? Make the table showing the relation between the number of sides (s) and number of diagonals (d). Write the formula.

12. How many diagonals can you draw from one vertex in a decagon? A 12-sided figure? A 20-sided figure?

180. Finding the value of a formula. Every symbol in a formula has a definite numerical value when applied to a particular problem. The values of the symbols vary with different problems. For instance, in the formula for the area of the rectangle, $A = lw$, when the rectangle has a length of 10 ft. and a width of 8 ft., $l = 10$ ft., $w = 8$ ft., and $A = 80$ sq. ft. If the rectangle is 7 in. long and 3 in. wide, $l = 7$ in., $w = 3$ in., and $A = 21$ sq. in.

When the numerical value of some one symbol in a formula is to be found, the numerical values of all the other symbols are usually known. To find the value of this one symbol, substitute the numerical values in place of the other symbols and perform the necessary operations of arithmetic.

Example 1

In a triangle the length of side $a = 2$ in., of $b = 4$ in., and of $c = 5$ in. Find the perimeter.

Formula for perimeter: $p = a + b + c$
Substituting: $p = 2 + 4 + 5$
Adding: $p = 11.$

Hence, the perimeter of the triangle is 11 in.

Example 2

In a parallelogram whose base equals 8 in. and height equals 3 in., find the area.

The formula for area of parallelogram is:

$$A = bh$$

Substituting: $A = 8 \times 3$

Multiplying: $A = 24.$

Hence, the area of the parallelogram is 24 sq. in.

EXERCISES

Write the formula for each of the following and solve as in the preceding examples.

1. Some Boy Scouts built a surfboard to be used on the lake at their camp. The board was 60 in. long and 30 in. wide. How many square inches did the board contain?

2. A garage roof is 18 ft. long and 12 ft. wide. How many square feet of roofing are needed to cover it?

3. A building contractor may estimate the number of bricks (n) needed for a construction by the formula $n = 22\,lwh$, if l, w, and h represent the length, width, and height of the building. How many bricks are needed to construct a wall 80 ft. long, $1\frac{1}{2}$ ft. wide, and 6 ft. high?

4. Some Girl Scouts bought a triangular piece of land for a camp. It had a base of 640 ft. and an altitude of 400 ft. How many square feet of land did the girls buy?

5. Clara made for her club a triangular pennant. It had a base of 18 in. and an altitude of 42 in. How many square inches were there in the pennant?

6. A rectangular swimming pool is 38 ft. wide and 54 ft. long. How many cubic feet of water are there in the pool when it is filled to a depth of 5 ft.?

7. A tin cup 4 in. across and 3 in. deep will hold how much water?

8. How many cubic feet are there in a conical pile of grain that is $3\frac{1}{2}$ ft. high and $4\frac{1}{2}$ ft. in diameter on the ground?

9. The base of a pyramid contains 90 sq. ft., and its height is 8 ft. How many cubic feet are there in the pyramid?

10. Frank's school attendance for five successive years was as follows: 180 days, 175 days, 200 days, 188 days, 198 days. Find his average annual attendance for the five years. $\left(a = \frac{s}{n}\right)$

11. For four consecutive weeks Paul's school lunch cost him as follows: \$1.00, \$.80, \$1.25, \$.75. Find his average weekly lunch expenditure.

12. The average score in a test taken by a class of 36 was 78. What was the sum of all the scores? $(s = a \times n)$

13. The average Red Cross contribution of the 38 pupils in a certain room was 27¢. What was the total Red Cross contribution?

The formula for determining the average mileage of an automobile tire is $m = 6545d$, m being the distance in miles the tire will probably travel and d the diameter of the tire in feet.

14. A tire whose diameter is 18 in. should withstand how many miles of travel? A tire of 24 in. should last how many miles of travel?

If $p =$ the number of pounds of pressure the air exerts upon the wings of a moving airplane and $V =$ the speed of the plane in miles per hour, then the formula is $p = .0005\,V^3$.

15. A plane flying 100 m. p. h. has how many pounds of pressure on its wings? What is the pressure at 150 m. p. h.?

16. How many square miles are there on the surface of the earth, if the earth is considered to have a diameter of 8,000 mi.? ($S = \pi d^2$)

17. How many cubic miles are there in the earth? ($V = \frac{4}{3}\pi r^3$)

18. To find the distance in feet (d) that an object will fall from a height in (t) seconds, use the formula $d = 16t^2$.

A brick falling from the top of a high building will in 5 sec. have fallen a distance of how many feet?

$$d = 16t^2$$
$$d = 16 \times 25$$
$$d = ?$$

19. Find the distance that a stone will fall from a tower in 3 seconds.

20. The following formulas are used in the business and scientific world:

a. $d = \frac{v^2}{2r}$. (a) Solve for d when $v = 30$ and $r = 10$.

(b) Solve for d when $v = 8$ and $r = 3$.

b. $p = \frac{2wh}{S+1}$. (a) Solve for p when $w = 1500$, $h = 28$, and $S = 3$.

(b) Solve for p when $w = 600$, $h = 15$, and $S = 2$.

c. $l = d + \frac{2r^2}{5d}$. (a) Find l if $d = 40$ and $r = 1.5$.

(b) Find l if $d = 6$ and $r = .3$.

d. $h = \frac{V}{b^2}$. (a) Find h when $V = 160$ and $b = 4$.

(b) Find h when $V = 200$ and $b = 5$.

e. $r = \frac{s^2 + h^2}{2h}$. (a) What is the value of r if $s = 4$ and $h = 3$?

(b) What is the value of r if $s = 15$ and $h = 10$?

f. $C = \frac{n(n-1)}{2}$. (a) Solve for C when $n = 20$.
(b) Solve for C when $n = 12$.

g. $R = \frac{l^2}{6d} + \frac{d}{2}$. (a) Find R if $l = 3$ and $d = 4$.
(b) Find R if $l = 10$ and $d = 8$.

The speed of an automobile determines the distance in which the automobile can be stopped in case of an emergency. The following formula is approximately true for good brakes and a dry concrete road: $d = .045\, r^2 + 1.1\, r$, where d = distance in feet required to stop and r = rate of speed (miles per hour).

21. (OPTIONAL) Charles is driving at a speed of 30 m. p. h. on a dry concrete road. About how many feet of road will he need to stop the car? When he is driving at 60 m. p. h., in how many feet can he stop?

22. (OPTIONAL) James notices that the Fahrenheit (F) thermometer records the temperature to be 86°. He wishes to know just what the Centigrade (C) thermometer records at the same time. In his science class he has learned that the relation between the two scales may be expressed by these two formulas:

$$\mathrm{C} = \frac{\mathrm{F} - 32^\circ}{9} \times 5, \qquad \text{and} \qquad \mathrm{F} = \frac{\mathrm{C}}{5} \times 9 + 32^\circ.$$

Determine the Centigrade temperature for him.

23. (OPTIONAL) On a hot summer day the Centigrade thermometer read 40°. What was the reading on the Fahrenheit thermometer?

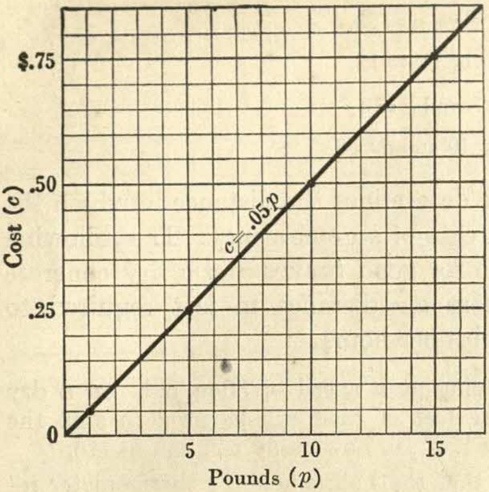
24. (OPTIONAL) Mabel opened a bank account with a \$5 birthday present. Every month thereafter she was able to save \$3 from her earnings. Write the formula representing her savings (s) for any number (n) of months, disregarding interest. Find her total savings 10 months later. One year later.

181. Graphing formulas. While a formula shows in symbols the relationship between numbers, the graph of a formula pictures this relationship and thereby gives us the answers to some questions without numerical computation.

Suppose that salt is selling at 5¢ a pound. The relationship between the cost (c) of the salt and the number of pounds (p) may be stated in the formula $c = .05p$. That is, the cost of any amount of salt is .05 times the number of pounds purchased.

To graph the formula $c = .05p$, prepare a table similar to the one at the right, showing the cost of the salt for various numbers of pounds. Then draw the axes for the graph. Mark the vertical guide line for the cost axis, letting one square represent \$.05.

Pounds	*Cost*
1	\$.05
5	.25
10	.50
15	.75



Mark the horizontal guide line for the amount axis, using one square to each pound. Next plot the pairs of corresponding numbers found in the table.

Since 1 lb. costs \$.05, place the first dot on the intersection of the 1-lb. line and the \$.05 line. Place the next dot on the intersection of the 5-lb. line and the \$.25 line. Where will the third dot be placed? The fourth dot? It is easily seen that the dots lie on a straight line. Let us now draw the straight line through these dots. This line is called "the graph of the formula $c = .05p$."

By graphing this formula showing the relationship between the cost of the salt and the number of pounds purchased, a picture is formed whereby the cost of any number of pounds of salt may be found by merely reading the cost from the graph. It is also possible to read from the graph the number of pounds of salt that can be bought for a given amount of money.

By reading the graph, find the cost of 4 lb. of salt, of 8 lb., and of 11 lb.

From the graph tell how many pounds of salt can be bought for \$.15, for \$.35, for \$.65.

Upon what does the cost (c) of the salt depend?

If the straight line is extended indefinitely, the cost of any number of pounds can be read directly from the graph.

Tom and Dick are traveling over the same highway. Tom is driving at the uniform rate of 30 mi. per hour; Dick, 40 mi. per hour. Tom starts at 6 A. M. and Dick at 7 A. M. from the same place. At what time will Dick overtake Tom, and how far will they have traveled?

Tom		Dick	
Hr.	*Mi.*	*Hr.*	*Mi.*
6 *A. M.*	0	7 *A. M.*	0
7	30	8	40
8	60	9	80
9	90	10	120
10	120	11	160
11	150	12 *Noon*	200
12 *Noon*	180	1 *P. M.*	240
1 *P. M.*	210	2	280
2	240		

A graph showing the relation between distance traveled and hours traveled by each boy makes this problem clear. (See figure 9.) The distance (d) that Tom has traveled by any certain hour (h) is obtained by the formula $d = 30h$. Why? Dick's distance from the starting point is $d = 40h$. Why?

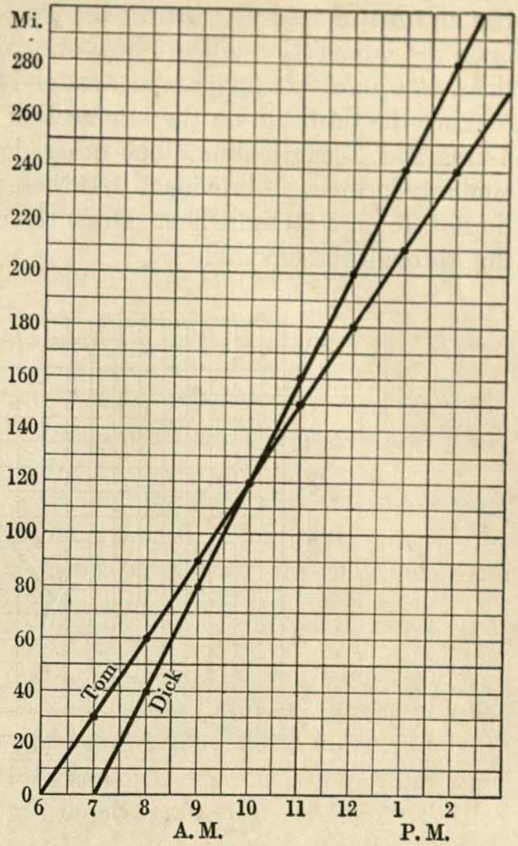


Fig. 9

In making the tables, consider that at 6 A.M. Tom is just starting; he is 0 mi. away. At 7 A. M. he is 30 mi. away. At 8 A. M., 60 mi. How far away is he at 9 A.M.? At 7 A. M. Dick is starting and is 0 mi. away, but by 8 A.M. he is 40 mi. on the road. How far away is he at 9 A. M.? At what hour do the lines meet? Hence, at what time do the boys meet? For how many hours has each been traveling? How many miles is this from the starting point? Consider the rates traveled and prove that each boy will actually be 120 mi. away at 10 A. M. How far apart are they at noon? At 1 P. M.? At 2 P. M.? Which boy is ahead before 10 A.M.? After 10 A. M.? At what time is each boy 240 mi. away? 180 mi. away?

Not all formula graphs form straight lines. Figure 10 illustrates a curved-line graph. It is the graph of the formula for the area of a square, $A = s^2$.

To graph this formula, use the table on page 300. How are the values of A found? The areas, A, are represented on the vertical axis and the lengths, s, on the horizontal axis. A square 0 ft. long evidently contains 0 sq. ft.; hence, the first dot is on the intersection of the 0 area and 0 length lines. A square 1 ft. long

has an area of 1 sq. ft.; hence, the second dot is on the intersection of the 1-area and 1-length lines. A square 2 ft. long has an area of 4 sq. ft., making the third dot on the intersection of the 4-area and 2-length lines. The other dots are similarly located. It is evident that these points do not lie on a straight line. Draw the curved line through these points.

Length (s) in ft.	*Area (A) in sq. ft.*
0	0
1	1
2	4
3	9
4	16

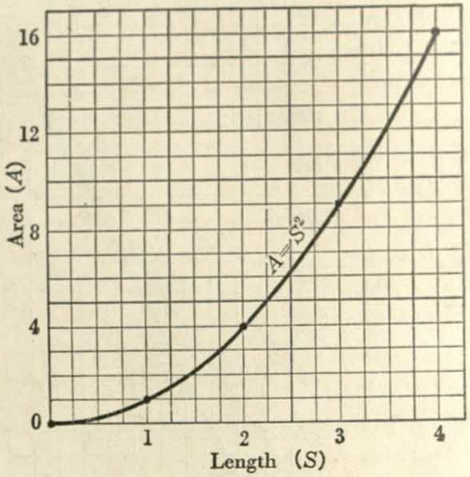


Fig. 10

Estimate from the graph the area of a square 1.5 ft. on an edge, a square $2\frac{1}{2}$ ft. on an edge, a square $3\frac{1}{4}$ ft. long, and a square $\frac{1}{2}$ ft. long.

By squaring the given lengths, determine how close you came to the correct answer for each of the above areas.

Find from the graph the approximate lengths of squares with the following areas: 6 sq. ft., 8 sq. ft., and 14 sq. ft.

Square each of these approximate lengths and compare the results with the given areas to determine how close you came to the correct answer in each case.

EXERCISES

1. Write the formula, make the table, and draw the graph to represent the cost (c) of apples at 12¢ per pound (p).

2. From the above graph find the cost of 2 lb. of apples. Of $1\frac{1}{2}$ lb. Of 2.5 lb.

3. From the above graph determine how many pounds of apples can be bought for 36¢. For 6¢. For 15¢. For 30¢.

4. Let the formula $p = 5l$ represent the perimeter (p) of a regular pentagon each side of which is of length l. Prepare the table and make the graph of this formula.

5. From the above graph determine the perimeter of a regular pentagon each side of which is of length 3 ft. Of length 6 ft. Of length $4\frac{1}{2}$ ft.

6. According to the above graph, if the perimeter is 20 ft., what is the length of one side of the pentagon? If the perimeter is 15 ft.? $12\frac{1}{2}$ ft.? $22\frac{1}{2}$ ft.?

7. Cecil earns $20 a month working after school. Let the formula $w = 20m$ represent his wages (w) after any number of months (m). Draw the graph showing his wages after 1 mo., 2 mo., 3 mo., 4 mo., and so on.

8. From the above graph find the amount of Cecil's wages after 6 mo. After 8 mo.; $3\frac{1}{2}$ mo.; $4\frac{1}{2}$ mo.; $7\frac{1}{2}$ mo.

9. Determine from the above graph how long Cecil will have to work to earn $80; $50; $110; $90; $175.

10. Frank and Harold are both starting savings accounts. Frank saves $4 a month and Harold $8 a month. Frank starts in January and Harold in March. Draw the graph showing when Harold will have as much saved as Frank.

11. (OPTIONAL) Write the formula and draw a graph of the following table:

$t=$	0	2	3	4	6	9
$d=$	0	40	60	80	120	180

CHAPTER XVII

THE EQUATION

VOCABULARY

1. equation
2. known number
3. unknown number
4. left member (equation)
5. right member (equation)
6. balanced (equation)

182. The meaning and purpose of equations. In algebra, as in arithmetic, we are mostly concerned with finding the value of some *unknown number*. This is done by the use of certain numbers whose values are *known*. For instance, in the problem of finding the miles traveled in 4 hr. at 30 mi. per hour, the miles traveled is the unknown number; to find it we multiply 30 mi. by 4. (The 30 mi. and the 4 hr. are the known numbers.) The unknown distance or number is found to be 120 mi.

In the formula $P = a + b + c$, P is the unknown number and a, b, and c are the known numbers. The value of P is found if the known values of a, b, and c are added.

The study of *equations* in algebra is concerned with finding the value of the unknown number. By means of equations, the most difficult problems that confront the businessman, the scientist, and the engineer are often greatly simplified.

An equation is a statement in numbers or symbols that two expressions are equal. You have long been familiar with such simple equations as the following: $? + 4 = 6$, $3 \times ? = 12$, $7 - ? = 5$, and $? \div 2 = 5$. Formulas are illustrations of equations. The expressions $3x = 21$, $\frac{1}{2}y = 4$, $m + 2 = 16$, and $n - 1 = 12$ are four simple forms of equations used in the solution of everyday problems.

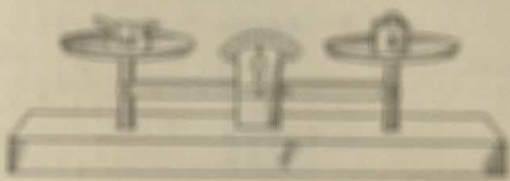
183. Terms used in equations. In an equation the expression on the left-hand side of the equals sign is called the *left member* and that on the right-hand side is called the *right member*.

The symbol whose value is to be found is called the *unknown number*, or merely the *unknown*. When equations are written, the

letter a, n, y, or any other symbol may be used to represent the unknown number.

In the equation $x - 3 = 6$, $x - 3$ is the left member and 6 is the right member. The symbol x is the unknown number.

184. Writing and solving simple equations. Since many equations cannot be solved orally, it is necessary to learn a few truths that will aid in the solution of the more complex equations.



An equation is like balanced scales. If the scales are to remain perfectly balanced, a change made on one arm must be made also on the other arm. It is the same with equations; a change made in one member of an equation must be made also in the other member.

1. In the figure, **if the salt is divided so that only one-half of it remains on the left arm, in order that the balance may be maintained the weight must also be divided so that only one-half of the weight remains on the right arm.** One and one-half pounds of salt will balance a $1\frac{1}{2}$-lb. weight.

2. **If 2 lb. of salt are taken from the left arm of the scales, 2 lb. in weight must be taken from the right arm or the balance will not be maintained.** One pound of salt will balance a 1-lb. weight.

3. **If 2 lb. of salt are added to the left arm of the scales, a 2-lb. weight must be added to the right arm in order that the balance may be maintained.** That is, the 5 lb. of salt and the 5-lb. weight will still balance the scales.

4. **If the amount of salt on the left arm is doubled, the weight on the right arm must also be doubled to keep the balance.** Six pounds of salt will balance a 6-lb. weight.

The truths involved in the preceding illustrations hold good for the solving of equations; that is, these truths are used in finding the value of the unknown.

The equation $4x = 20$ means that some unknown number times 4 equals 20. It is evident that there is 4 times x (the unknown) in the left member of the equation. It is important in solving an equation to have only the unknown number in the left member. In order to have x (no more and no less) in the left member, it is necessary to divide the left member by 4. To keep the left and right members of the equation still equal—*balanced*—it is also necessary to divide the right member by 4.

Solution

The equation is:	$4x = 20$
Dividing by 4:	$4x \div 4 = 20 \div 4$
Hence:	$x = 5.$
Check:	$4 \times 5 = 20$
	$20 = 20.$

This illustration leads us to the first truth that we shall need in solving equations:

Truth I: *If both members of an equation are divided by the same amount, the members remain equal.*

EXERCISES

Solve and check the following equations, using Truth I:

1. $4a = 20$

2. $6w = 18$

3. $7x = 28$

4. $5b = 100$

5. $8r = 8$

6. $15c = 900$

7. $10k = 35$

8. $12m = 72$

9. $20d = 800$

10. $6s = 1800$

11. $4y = 25$

12. $25z = 650$

13. $6t = 3$

14. $7e = 32$

15. $30l = 10$

16. $3n = 1$

17. $5a = 18$

18. $4c = 6$

19. $9c = 35$

20. $.6m = 12$

(*Hint:* Multiply the equation by 10 to clear of decimals.)

21. $.12a = 36$

22. A certain number multiplied by 4 equals 84. What is the number?

Solution

Let:	$n =$ the number
The equation is:	$4n = 84$
Dividing by 4:	$n = 21.$
Hence, the number is 21.	
Check:	$4 \times 21 = 84$
	$84 = 84.$

23. Forty-eight is 3 times a certain number. Find the number.

24. The perimeter of a square is 64 ft. Find the length of one side.

25. Charles said, "My number multiplied by 3 equals 27." Find Charles's number.

26. Jack's home room was selling tickets for a school entertainment. Said Jack, "Four times my sales amounts to 108 tickets." How many tickets has Jack sold?

27. Jean has been earning money for the Junior Red Cross. On returning to school one morning she said, "If I multiply my earnings by 8, I shall have $1.28." How much money has Jean earned?

28. In 6 hours an airplane flew a distance of 840 miles. How many miles did it average per hour?

The equation $x + 4 = 16$ means that some unknown number plus 4 equals 16. It is evident that there is 4 more than x (the unknown) in the left member of the equation. In order to have only x in the left member, it is necessary to subtract 4 from it. To keep the left and right members of the equation still equal, 4 must also be subtracted from the right member, that is, subtract 4 from each member of the equation.

Solution

The equation is:	$x + 4 = 16$
Subtracting 4:	$x + 4 - 4 = 16 - 4$
Hence:	$x = 12.$
Check:	$12 + 4 = 16$
	$16 = 16.$

This illustration leads us to the second truth that we shall need in solving equations:

> ***Truth II:*** *If the same amount is subtracted from both members of an equation, the members remain equal.*

EXERCISES

In each of the following exercises, find the value of x, using Truth II. Check results.

1. $x + 10 = 18$	**8.** $5 + x = 15$	**15.** $121 + x = 612$
2. $x + 32 = 67$	**9.** $8 + x = 28$	**16.** $a + 14 = 15$
3. $x + 12 = 45$	**10.** $16 + x = 22$	**17.** $3 + a = 3$
4. $x + 9 = 26$	**11.** $x + 218 = 476$	**18.** $12 + a = 13$
5. $x + 14 = 32$	**12.** $x + 142 = 385$	**19.** $a + 2 = 9$
6. $x + 46 = 78$	**13.** $x + 74 = 191$	**20.** $9 + a = 48$
7. $x + 38 = 43$	**14.** $43 + x = 164$	**21.** $37 + a = 41$

22. A certain number plus 20 equals 35. What is the number?

Solution

Let:	$x =$ the number
The equation is:	$x + 20 = 35$
Subtracting 20:	$x = 15.$
Hence, the number is 15.	
Check:	$15 + 20 = 35$
	$35 = 35.$

23. Sixteen cents added to Mary's money gives 38¢. Let x stand for Mary's money. Form an equation and solve.

24. If John had 12 more rabbits, he would have 116 rabbits. Form an equation and solve.

25. Joe said, "If I can win 75 points this game, I will have 500 points." How many points has Joe?

26. What number added to 188 equals 391?

27. Said Carol, "If you add 35¢ to my earnings, I will have 62¢." How much money has Carol?

28. Jane said, "I am thinking of a number; if you add 256 to it, the sum is 428. What is the number?"

29. Mr. Brown received an increase in salary of \$96, making his salary \$1500. How much was his salary before the increase?

30. A house sold for \$8,500. If the contractor made \$1,750 on it, what did it cost him?

31. During the last year Robert gained $3\frac{1}{2}$ inches in height. He is now 62 inches tall. What was his height a year ago?

In the equation $x - 4 = 16$, it is evident that there is 4 less than x in the left member. In order to have only x in the left member, we must add 4 to it. We must also add 4 to the right member in order to keep the balance. That is, add 4 to each member of the equation.

Solution

The equation is:	$x - 4 = 16$
Adding 4:	$x - 4 + 4 = 16 + 4$
Hence:	$x = 20.$
Check:	$20 - 4 = 16$
	$16 = 16.$

This illustration leads us to the third truth needed in solving equations:

***Truth III:** If the same amount is added to both members of an equation, the members remain equal.*

EXERCISES

In each of the following exercises, find the value of n. Check results.

1. $n - 2 = 12$
2. $n - 8 = 25$
3. $n - 7 = 15$
4. $n - 14 = 23$
5. $n - 10 = 37$
6. $n - 24 = 62$
7. $n - 38 = 72$
8. $n - 46 = 59$
9. $n - 80 = 112$
10. $n - 76 = 120$
11. $n - 72 = 171$
12. $n - 118 = 212$
13. $n - 400 = 609$
14. $n - 108 = 130$
15. $n - 107 = 211$
16. $n - 1 = 1$
17. $n - 12 = 0$
18. $n - 1 = 9$
19. $n - 17 = 18$
20. $n - 4 = 2$
21. $n - 18 = 3$

22. If you subtract 14 from Ruth's grade, she will have 72. What is Ruth's grade?

Solution

Let:	$g =$ Ruth's grade
The equation is:	$g - 14 = 72$
Adding 14:	$g = 86.$
Hence, Ruth's grade is 86.	
Check:	$86 - 14 = 72$
	$72 = 72.$

23. What number less 41 equals 96?

24. Jack said, "I am thinking of a number; if you subtract 161 from it, the result is 316." What is the number?

25. A group of boys climbed Pike's Peak. After climbing 10,190 ft. they had 4,218 ft. to go to reach the top. What is the height of the peak?

26. The net profit of the Black Coal Co. was \$2,458 and its expenses were \$3,824. Find its gross income.

27. Clyde's height is $2\frac{1}{2}$ in. less than Jim's height. Jim is 61 in. tall. What is Clyde's height?

28. A farmer after selling 482 bushels of wheat had 376 bushels unsold. How many bushels did he have at first?

In the equation $\frac{1}{3}x = 4$, there is only $\frac{1}{3}$ of x in the left member. In order to have x in the left member of the equation, multiply $\frac{1}{3}x$ by 3. To keep the left and right members equal, also multiply the right member by 3.

Solution

The equation is:	$\frac{1}{3}x = 4$
Multiplying by 3:	$3 \cdot \frac{1}{3}x = 3 \cdot 4$
Hence:	$x = 12.$
Check:	$\frac{1}{3} \times 12 = 4$
	$4 = 4.$

The fourth truth used in solving equations follows:

> ***Truth IV:*** *If both members of an equation are multiplied by the same amount, the members remain equal.*

EXERCISES

Using Truth IV, solve the following equations; check results:

1. $\frac{1}{3}x = 2$
2. $\frac{1}{2}d = 5$
3. $\frac{1}{6}c = 4$
4. $\frac{1}{8}b = 7$
5. $\frac{1}{5}k = 12$
6. $\frac{1}{10}y = 8$
7. $\frac{n}{4} = 6$
8. $\frac{a}{5} = 10$
9. $\frac{1}{12}r = 6$
10. $\frac{m}{3} = 17$
11. $\frac{n}{8} = 14$
12. $\frac{1}{15}m = 8$
13. $\frac{1}{2}a = 1$
14. $\frac{a}{8} = 8$
15. $\frac{1}{3}a = 3$
16. $\frac{n}{12} = 0$
17. $\frac{b}{6} = 9$
18. $\frac{s}{2} = 5$

19. One-fourth of a certain number equals 18. Find the number

Solution

Let:	$n =$ the number
The equation is:	$\frac{n}{4} = 18$
Multiplying by 4:	$n = 72.$
Hence, the number is 72.	
Check:	$\frac{72}{4} = 18$
	$18 = 18.$

20. One-eighth of what number equals 16?

21. Gerald spent one-third of his money. He spent \$15. How much money had he?

22. A school sold in one day 520 tickets to an entertainment. Those tickets represented one-third of the seating capacity of the auditorium. How many people would the auditorium seat?

23. A motorist had car trouble 18 miles from his starting point, which was one-sixth of the distance to his destination. How far was his starting point from his destination?

24. I saved one-twelfth of my income during the year. During this time my savings amounted to \$180. How much was my income?

25. A crew was laying a pipeline between two cities. The engineer's report showed that during the first month they had laid the pipe for a distance of 128 miles or one-third of the distance between the cities. How far apart are the cities?

26. A firm's report showed a sales increase of one-eighth over the preceding year. This was an increase of \$28,000. What was the amount of the sales during the preceding year?

ALL PROCESSES

Solve each of the following, using the correct truth:

1. $n-80=1$
2. $w+60=82$
3. $\frac{a}{3}=14$
4. $5k=13$
5. $s+2=18$
6. $m-200=150$
7. $33y=165$
8. $4+b=18$
9. $\frac{1}{2}k=7\frac{1}{2}$
10. $w-2\frac{1}{2}=7\frac{1}{2}$
11. $m+1\frac{1}{4}=3\frac{3}{4}$
12. $n+2.4=6.5$
13. $2\frac{1}{2}a=10$
14. $\frac{1}{3}d=2$
15. $h-4.2=8.7$
16. $m-.4=7.2$
17. $92+n=183$
18. $3.1n=9.3$
19. $s+6=7$
20. $s-7=6$
21. $9a=3$
22. $16+b=35$
23. $\frac{n}{9}=9$
24. $1\frac{1}{2}n=3$
25. $12+w=21$
26. $m-8=0$
27. $\frac{n}{17}=2$
28. $\frac{1}{3}c=3$
29. $18s=6$
30. $3n=4$
31. $s-9=2$
32. $24+a=30$
33. $.06a=6$
34. $.7x=21$
35. $.001c=1$
36. $2.8m=5.6$

37. A square has a perimeter of 72 in. How long is one side?

38. The perimeter of a regular hexagon is 96 in. Find the length of one side.

39. Mr. Clark bought a used car. After repairing the car he sold it for \$125 more than he paid for it. The selling price was \$680. How much did he pay for the car?

40. Cecil made doorstops to sell in order to earn money for Christmas. After selling 18, he had 24 remaining unsold. How many had he made altogether?

41. A home room has collected one-fourth of its Red Cross quota. It has collected \$2.25. What is its quota?

42. Paul had 72 in. of wire with which to make a wire model of an equilateral triangle. How long can he make each side?

43. Sam said, "If I earn 46 honor points this semester, I shall have 120 points." How many honor points has Sam?

44. (OPTIONAL) The sun is about 93,000,000 miles from the earth. Light travels about 186,000 miles per second. How many seconds does it take for light to reach the earth from the sun?

45. (OPTIONAL) After paying one-sixth of the cost of his bicycle, James had paid \$5.75. What was the cost of his bicycle?

185. Solving equations by using more than one of the preceding truths. The four truths may be combined into the following single truth: *If both members of an equation are increased, decreased, multiplied, or divided by the same amount, the members remain equal.*

When the use of more than one truth is necessary, follow the illustrated order of procedure.

Examples

A. The equation is: $3n - 2 = 10$
Adding 2: $3n = 12$
Dividing by 3: $n = 4.$
Hence, the number is 4.
Check: $3 \cdot 4 - 2 = 10$
$12 - 2 = 10$
$10 = 10.$

B. The equation is: $\frac{a}{4} + 6 = 9$
Subtracting 6: $\frac{a}{4} = 3$
Multiplying by 4: $a = 12.$
Hence, the number is 12.
Check: $\frac{12}{4} + 6 = 9$
$3 + 6 = 9$
$9 = 9.$

C. The equation is: $\frac{2}{5}x = 6$
Multiplying by 5: $5 \cdot \frac{2}{5}x = 5 \cdot 6$
$2x = 30$
Dividing by 2: $x = 15.$
Hence, the number is 15.
Check: $\frac{2}{5} \cdot 15 = 6$
$6 = 6.$

EXERCISES

Solve and check each of the following equations:

1. $3n + 4 = 19$
2. $4n + 2 = 18$
3. $8n + 6 = 30$
4. $5x + 3 = 23$
5. $7x + 1 = 36$
6. $6x - 3 = 15$
7. $2a - 5 = 19$
8. $3a - 12 = 9$
9. $4a - 8 = 20$
10. $5a - 7 = 13$
11. $\frac{n}{2} + 4 = 5$
12. $\frac{n}{4} + 7 = 10$
13. $\frac{n}{3} - 5 = 9$
14. $\frac{a}{5} - 6 = 15$

15. $\frac{a}{6} + 2 = 11$

16. $4a + 2 = 10$

17. $5 + 3a = 14$

18. $7n - 1 = 13$

19. $\frac{k}{3} - 4 = 16$

20. $\frac{m}{10} + 8 = 14$

21. $3y + 7 = 22$

22. $\frac{2}{3}n = 12$

23. $\frac{3}{4}n = 15$

24. $\frac{2}{5}k = 8$

25. $3n + 2n = 15$
Hint: $5n = 15$

26. $4a + 2a = 24$

27. $8x - 3x = 20$

28. $7n - n = 18$

29. $\frac{1}{2}n + \frac{1}{4}n = 6$

30. $2.5n + 3.5n = 12$

31. $1.4n + 1.1n = 10$

32. $.1y + .5y = 7.2$

33. $.3a + .4a = 70$

34. $.02b + .03b = 2$

35. $2.3n + 1.3n = 72$

36. $4.2x - 1.2x = 9$

37. One-third of Sam's age plus 8 equals 14 years. How old is Sam? $\left(\textit{Hint:}\text{ Let } a = \text{Sam's age. Then, } \frac{a}{3} + 8 = 14.\right)$

38. Three times Albert's money plus 12¢ is 60¢. How much money has Albert?

39. Edna said, "I am thinking of a number. Four times this number plus 8 is 20." What is the number?

40. Helen said, "Subtract 6 from two times a certain number and the result is 22." What is the number?

41. Marjorie said, "One-half of my money increased by \$75 is \$120." How much money had she?

42. Four times my age increased by 3 times my age is 84 years. How old am I?

43. One-half of a number less one-third of the number equals 20. Find the number.

44. Mr. Karr bought a piece of land. After improving it, he sold it for \$80 more than twice what he paid for it. He sold it for \$2,600. What did Mr. Karr pay for the land?

45. A graduating class of 165 pupils decided to make a class gift costing \$44. The class had \$11 in the treasury. How much should each member contribute in order to pay for the gift?

46. After the last cutting of a field of alfalfa, a farmer had 90 tons of alfalfa. This was 12 tons more than one-half of last year's yield. How many tons of alfalfa did he have last year?

47. Fred sold 76 papers today, which was 9 less than one-third of yesterday's sales. How many papers did he sell yesterday?

48. Clark's arithmetic grade is 84 today. This is seven-eighths of yesterday's grade. What was Clark's grade yesterday?

49. Clara solved $\frac{4}{5}$ as many problems as Jane. Clara solved 12 problems. How many problems were solved by Jane?

50. The length of Larry's step is $\frac{4}{7}$ as long as his father's step. Larry's step is 24 inches long. What is the length of his father's step?

186. Difficult arithmetic problems made easy by the use of equations. Most of the preceding problems could have been as easily solved by arithmetic as by equations. However, many problems are very difficult to solve by arithmetic but easy to solve by equations. Let us consider some of them.

Examples

1. A class of 36 pupils has twice as many girls as boys. How many of each are there in the class? (If more than one unknown number is to be found, it is usually better to let n, or some other symbol, represent the smallest number and express the others in terms of n, or whatever symbol is used.)

Solution

Let:	$n =$ number of boys in class
Then:	$2n =$ number of girls in class
Total in class is:	$n + 2n = 36$
Adding the n's:	$3n = 36$
Dividing by 3:	$n = 12$
	$2n = 24.$

Hence, there are 12 boys and 24 girls in the class.

Check: (a) $12 + 24 = 36$ (number of pupils in the class)
(b) $12 \times 2 = 24$ (There are twice as many girls as boys.)

2. In a school election, 45 votes were cast. Of these Don received 5 more than Richard. How many votes did each receive?

Solution

Let:	$n =$ number of Richard's votes
Then:	$n + 5 =$ number of Don's votes
Total votes are:	$n + n + 5 = 45$
Adding the n's:	$2n + 5 = 45$
Subtracting 5:	$2n = 40$
Dividing by 2:	$n = 20$
	$n + 5 = 25.$

Hence, Richard received 20 votes and Don 25 votes.

Check: (a) $20 + 25 = 45$ (number of votes cast)
(b) $25 - 20 = 5$ (Don received 5 more votes than Richard.)

3. Emily, Norma, and Lorene's ages are one year apart. Emily is the youngest, Norma is a year older, and Lorene is a year older than Norma. How old is each girl if their ages total 33 years?

Solution

Let:	$n =$ Emily's age
Then:	$n + 1 =$ Norma's age
And:	$n + 1 + 1 =$ Lorene's age
Total ages:	$n + n + 1 + n + 1 + 1 = 33$
Adding the n's and the 1's:	$3n + 3 = 33$
Subtracting 3:	$3n = 30$
Dividing by 3:	$n = 10$
	$n + 1 = 11$
	$n + 1 + 1 = 12.$

Hence, Emily is 10 years old, Norma is 11, and Lorene is 12.

Check: (a) $10 + 11 + 12 = 33$ (total ages of the girls)
(b) $10 + 1 = 11$ (Norma is one year older than Emily.)
(c) $11 + 1 = 12$ (Lorene is one year older than Norma.)

EXERCISES

1. Kenneth gave twice as much money to the Red Cross as did Clarence. The sum of their gifts was 63¢. How much did each give?

2. Charles worked three times as many days during the summer as Paul. If they worked a total of 92 days, how many days did each work?

3. Louise picked 8 more boxes of strawberries than Norene. They picked a total of 52 boxes. How many boxes did each pick?

4. May's age is three times Jane's age. The sum of their ages is 24 years. Find the age of each girl.

5. A certain recipe calls for 4 times as much flour as sugar. If 10 cups of the mixture are to be made, how many cups of each should be used?

6. A mixture calls for crushed stone, sand, and cement in the ratio of 4, 3, and 1, respectively. If a total of 24 barrels of this mixture is to be used, how many barrels of each are needed?

7. Nell sold two books at a total profit of 60¢. The profit on one book was 30¢ more than that on the other. What was the profit on each?

8. One-third of a number equals 3. What is the number?

9. In one month Fred earned three times as much as Bill. Together they earned $2.80. How much did each earn?

10. Mary bought a bathing suit and a bathing cap for $3.84. The suit cost fifteen times as much as the cap. What did each cost?

11. A pan and a lid cost $1.10. The pan cost $1 more than the lid. What did each cost?

12. Two castings weigh 46 lb. One casting weighs 1 lb. more than twice the other casting. What does each weigh?

13. A large airplane weighs 17,600 lb. If the plane weighs four times the weight of the engine, what does each weigh?

14. An arithmetic book and a history book together cost \$3.50. The history costs \$1 more than the arithmetic. Find the cost of each.

15. The sum of the ages of Nancy, Kathleen, and Natalie is 27 years. Nancy is the youngest, Kathleen is one year older than Nancy, and Natalie is one year older than Kathleen. How old is each girl?

16. George had 75¢ to spend. He spent four times as much of it for a game as for candy. How much did he spend for each?

17. Bertha has 39 in. of lace. She wishes to cut it so that one piece will be 5 in. longer than the other piece. How long will she cut each piece?

18. Mr. Adams wishes to divide a 300-ft. piece of land so that one piece will be 20 ft. shorter than the other. How shall he divide it?

19. Allen wishes to divide a 17-in. line into three parts so that the second part will be 3 in. longer than the first part and the third part will be 5 in. longer than the first part. How long will he make each part?

20. Grace drew a triangle so that $\angle B$ was 10° more than $\angle A$, and $\angle C$ was 20° more than $\angle A$. How many degrees did she make each angle of the triangle?

21. Bob drew a triangle so that $\angle B$ was three times as large as $\angle A$, and $\angle C$ was twice as large as $\angle B$. How many degrees were there in each angle?

22. Mr. Carlson sold his farm for \$4,500. He gained one-fifth of its cost. Find the cost.

23. Ben sold his bicycle for \$18. He lost one-third of its cost. What was the cost?

24. There are 2 more than twice as many girls as boys in a class of 41 pupils. How many girls and how many boys are in the class?

25. The length of a rectangular field is three times its width. Its perimeter is 480 rd. What are the dimensions of the field?

26. Mr. Coe told his church friends that for every dollar they contributed toward church repairs he would give two dollars. The total fund amounted to \$2,100. How much of this amount was contributed by Mr. Coe?

187. Finding the value of a symbol in the right member of an equation. If the symbol of unknown value is on the right-hand side of the equality sign, solve as follows:

(a) Substitute for the known values as usual.

(b) Exchange the left member of the equation for the right member. (This exchange can be made, for, if $3 + 5 = 6 + 2$, it is also true that $6 + 2 = 3 + 5$. And if $A = lw$, it is just as correct to write $lw = A$.)

Examples

1. The area of a rectangular lot is 120 sq. rd. Its length is 24 rd. Find its width.

Solution

Formula:	$A = lw$
Substituting:	$120 = 24w$
Exchanging sides:	$24w = 120$
Dividing by 24:	$w = 5$

Hence, the width of the rectangular lot is 5 rd.

2. A circular lot contains 15,400 sq. ft. Find its radius.

Solution

Formula:	$A = \pi r^2$
Substituting:	$15{,}400 = \frac{22}{7}r^2$
Exchanging sides:	$\frac{22}{7}r^2 = 15{,}400$
Dividing by $\frac{22}{7}$:	$r^2 = 4{,}900$
Finding square root:	$r = 70$

Hence, the radius of the circular lot is 70 ft.

3. The perimeter of a triangle is 16 in. Sides a and b are 4 and 5 in., respectively. What is the length of the third side?

Solution

Formula:	$P = a + b + c$
Substituting:	$16 = 4 + 5 + c$
Adding:	$16 = 9 + c$
Exchanging sides:	$9 + c = 16$
Subtracting 9:	$c = 7$

Hence, the length of the third side of the triangle is 7 in.

EXERCISES

1. A rectangular lot contains 4,800 sq. ft. It has a length of 60 ft. What is its width?

2. The area of a square playground is 6,400 sq. ft. Find the length of one side.

3. The perimeter of a rectangle is 56 in. The length is 20 in. Find the width.

4. What principal at 6% will yield $180 interest in 2 yr.? $(i = prt)$

5. In what time will $500 yield $40 interest at 4%?

6. At what rate will $1,200 yield $90 interest in $1\frac{1}{2}$ yr.?

7. The class average in a test is 82. If 35 pupils took the test, find the sum of their grades. $\left(a = \frac{s}{n}\right)$

8. Find the value of the symbol in each of the following:

(a) $27 = 18k$

(b) $168 = 4x + 32$

(c) $45 = 9 + 2a + 17$

(d) $48 = \frac{w}{9}$

(e) $175 = 25 + 10b$

(f) $9 + 4 = \frac{1}{3}x - 3$

(g) $1 = 6a$

(h) $16 = 3s - 2$

(i) $6 = 9h - 39$

(j) $0 = 3w - 39$

(k) $32 = 18p + 14$

(l) $3 = \frac{n}{9}$

(m) $27 + 19 = 12 + 2c$

(n) $\frac{1}{4}a + 1 = 5$

CHAPTER XVIII

SIGNED NUMBERS

VOCABULARY

1. signed numbers	3. negative number
2. positive number	4. absolute value

188. The meaning of signed numbers. Alaska has a wide range of temperature. The excessive cold in winter has made highway construction extremely difficult. Also, the difficulty of preventing automobile engines from freezing at 60° below zero has made the dog sled and airplane popular. The highest summer temperature is 90° *above zero*. From 90° above zero to 60° below zero is a difference of 150° in temperature.

The statement of such a fact as this is frequently made more brief by the use of what are known as *signed numbers*. It has been agreed to indicate all thermometer readings above zero as *plus numbers* (numbers with a plus sign prefixed); for instance, $+6°$ (read "plus 6°") means 6° above zero. And it has been agreed to denote all temperatures below zero as *minus numbers*; thus, $-6°$ (read "minus 6°") means 6° below zero. The plus and minus signs as used in denoting temperatures show a difference in direction, the plus numbers extending above zero degrees and the minus numbers below zero degrees.

Numbers that are preceded by the plus or minus sign are called *signed numbers*. Those that are preceded by the plus sign, like $+8$, are called *positive numbers*; those preceded by the minus sign, like -8, are called *negative numbers*.

The plus sign is usually omitted before a positive number, but the minus sign is never omitted before a negative number. That is, "positive 5" may be written "5" or "+5"; but "negative 5" is always written "− 5." Arithmetic deals only with positive numbers and does not prefix the plus sign. Algebra deals with both positive and negative numbers.

The plus (+) and minus (−) signs placed in front of numbers as in the preceding illustrations must not be confused with the + and − signs of addition and subtraction. Their use is entirely different.

The 0 meridian (the prime meridian at Greenwich, near London, England) is the guide line from which longitude is measured. The location of a place 40° east of the 0 meridian might be designated as being in + 40° longitude; and a place west of the zero meridian as being in − 40° longitude. These locations are said to be opposite in direction. One is east and the other west from the zero line until the 180th meridian is reached on the opposite side of the globe.

While the above locations are in opposite directions from the zero line, they are equally distant from this line in that each is 40° from it. These distances of + 40° and − 40° measure the same; that is, the *absolute values* of the plus 40° and the minus 40° are the same; namely, 40°. The *absolute value* of a signed number is the value of the number without regard to its sign.

189. Using signed numbers to indicate opposites. Bob has saved \$5. Gene not only has saved no money, but has gone in debt \$5. Bob's money may be represented by + \$5, as he has \$5 more than zero (no) dollars. Gene's money may be represented by − \$5, as he has \$5 less than zero (no) dollars. The plus and minus signs indicate that the numbers are *opposite* in meaning. In each case the absolute value of the numbers is \$5.

EXERCISES

1. Represent the following numbers as positive or negative and give the absolute value of each:

(a) Example: A \$3 gain and a \$3 loss.

Positive	*Negative*	*Absolute Value*
+ \$3 (gain)	− \$3 (loss)	\$3

(b) Locations of 20° N. latitude and 20° S. latitude.
(c) Temperatures of 15° above zero and 15° below zero.
(d) \$25 deposited in a bank and \$25 withdrawn.
(e) 6 games won and 6 games lost.
(f) 14 mi. east and 14 mi. west.
(g) 500 ft. above sea level and 500 ft. below sea level.
(h) \$100 spent and \$100 earned.

2. State the opposite of each of the following numbers and write each one as a signed number:

(a) Example: $10 profit.

Positive	*Negative*
+ $10 (profit)	− $10 (loss)

(b) 4 ft. east.
(c) $12 spent.
(d) 3 ft. up.
(e) 1000 A. D.
(f) 30° below zero.
(g) 40 points won.

3. Represent the following as signed numbers:

(a) 300 ft. above sea level and 50 ft. below sea level.
(b) 5 steps above a landing and 3 steps below a landing.
(c) 3 floors above the main floor and 1 floor below the main floor.
(d) 10 steps forward and 7 steps backward.
(e) 1492 A. D. and 30 B. C.

4. What do these signed numbers mean?

(a) A longitude of + 90°.
(b) A longitude of − 30°.
(c) The year − 76.
(d) Points made in a game are + 140.
(e) A temperature of − 8°.
(f) Result of a sale is − $25.
(g) Location of an elevator is + 8.

5. What does 0° mean in connection with longitude? Latitude? Temperature?

6. What does elevation 0 mean with reference to sea level?

7. What does the score 0 mean in a game?

8. (OPTIONAL) Give additional examples of signed numbers derived from familiar experiences.

190. Using signed numbers to indicate changes. Signed numbers are often used to indicate various types of changes. For instance, advances and declines in stock prices from day to day are frequently designated by positive and negative numbers. If the price of the stock of a certain company was quoted as closing at 82 yesterday and 84 today, the change in closing prices would be + 2, as the closing price today is $2 per share more than the closing price yesterday. On the other hand, if the price closed at 80 today, the change would be − 2, as the price would then have closed today at $2 per share less than yesterday.

The average daily attendance of a certain class is $34\frac{1}{2}$. On a day when the attendance is 35, the change is indicated by $+\frac{1}{2}$, as there is present $\frac{1}{2}$ pupil more than the average attendance. An attendance of 34 pupils shows a change of $-\frac{1}{2}$, as this is $\frac{1}{2}$ pupil less than the average attendance.

EXERCISES

1. In the following stock quotations the net change from the close of one day to the close of the next day is indicated by means of + and − signs:

Name of Stock	*Range of Stocks in New York*	
	Closing Price	*Net Change*
Otis Elev.	39	$+1\frac{3}{4}$
Am. Airlines.	$15\frac{5}{8}$	$-\frac{1}{4}$
Walgreen	$25\frac{5}{8}$	$-\frac{1}{8}$
Braniff Airw.	$13\frac{1}{2}$	$+\frac{1}{2}$
Allied Mills	$31\frac{1}{2}$	$-\frac{1}{4}$

How does the closing price of each of the above shares of stock compare with its closing price the previous day? Find the closing price on the previous day for each.

2. Copy the following quotations and fill in the missing numbers:

Name of Stock	*Closing Price*		*Net Change*
	Today	*Yesterday*	
Am. Exp.	17	$17\frac{1}{2}$	$-\frac{1}{2}$
Corn Prod.	80	$79\frac{3}{8}$	—
McCall	$20\frac{3}{4}$	$20\frac{1}{4}$	—
Am. Tel. & Tel.	156	$154\frac{5}{8}$	—
Sun Oil	—	$86\frac{3}{4}$	$+1\frac{1}{8}$
Goodyear	$98\frac{1}{4}$	—	$-\frac{1}{4}$
Pet Milk	$51\frac{1}{2}$	—	$+\frac{7}{8}$
Penney (J. C.).	—	$68\frac{1}{4}$	$+1\frac{3}{4}$
Swift & Co.	$40\frac{1}{2}$	$40\frac{1}{2}$	—
Motorola	48	$48\frac{1}{2}$	—

3. A 12-yr.-old girl was told that she should weigh 78 lb. What was her actual weight for each of five consecutive weeks if the changes in her weight with reference to normal weight were: $-1, +2, -\frac{1}{2}, +1\frac{1}{2}$?

4. The average temperatures for a certain city during the first six months of the year are 20°, 28°, 34°, 45°, 50°, and 62°, respectively. During a recent year the averages for these six months varied from the usual averages as follows: $+3°, -7°, -2°, +1°, -1\frac{1}{2}°, +2°$. Determine the average temperature for each of these months for this particular year.

5. Every home room of a certain school was asked to sell a definite number of tickets for a school benefit. Interpret the results obtained with reference to the quota in the home rooms whose final reports read as follows: + 50 tickets, − 80 tickets, − 62 tickets, and + 100 tickets.

6. The average test score made by a certain class was 82. Tom's score was 88, Clara's 80, and Roy's 72. Express the variation in these scores from the class average by means of signed numbers.

7. By means of signed numbers compare the following enrollment of a school from 1944 to 1950 with the enrollment for 1950:

Year	1944	1945	1946	1947	1948	1949	1950
Enrollment	795	789	804	796	747	692	791

8. (OPTIONAL) From any available source gather statistics on changes in prices, population, amount of sales, and any other suitable subjects; show changes by means of signed numbers as in the above exercises.

191. Adding positive and negative numbers with like signs. Ralph earned 20¢ and 45¢. How much did he earn altogether? If we add as in arithmetic, 65¢ is immediately seen to be the total amount earned. To add as signed numbers, think of the amount earned as $+20¢$ and $+45¢$. The sum is $+65¢$. The 65 represents the sum of the absolute values and the plus sign indicates that it is money earned. This sum may be written in two ways:

$$\text{(a) } (+20¢) + (+45¢) = +65¢ \quad \textit{or} \quad \text{(b) } \begin{array}{r} +20¢ \\ \underline{+45¢} \\ +65¢ \end{array}$$

To obtain this result, add the absolute values of the numbers and prefix the plus sign to the sum.

Kate spent 15¢ for a game and 20¢ for a book. How much did she spend altogether? By arithmetic, the sum spent is seen to be 35¢. To add as signed numbers, think of the amounts spent as $-15¢$ and $-20¢$. The sum or total amount spent is $-35¢$. The 35 represents the sum of the absolute values and the minus sign indicates that it is money spent, the opposite of money earned. This sum may be written in two ways:

$$\text{(a) } (-15¢) + (-20¢) = -35¢ \quad \textit{or} \quad \text{(b) } \begin{array}{r} -15¢ \\ \underline{-20¢} \\ -35¢ \end{array}$$

To obtain this sum, add the absolute values and prefix the minus sign to the sum.

From these illustrations we derive the following rule:

To find the sum of numbers with like signs, add their absolute values and to the sum prefix the common sign.

EXERCISES

A. Oral

State the sum of each of the following:

1. $\begin{array}{r} +3 \\ \underline{+7} \end{array}$

2. $\begin{array}{r} +4 \\ \underline{+5} \end{array}$

3. $\begin{array}{r} +8 \\ \underline{+8} \end{array}$

4. $\begin{array}{r} -7 \\ \underline{-2} \end{array}$

5. $\begin{array}{r} -6 \\ \underline{-7} \end{array}$

6. $\begin{array}{r} -10 \\ \underline{-\ 8} \end{array}$

7. $\begin{array}{r} +9 \\ \underline{+5} \end{array}$

8. $\begin{array}{r} -5 \\ \underline{-8} \end{array}$

9. $\begin{array}{r} -7 \\ \underline{-1} \end{array}$

10. $\begin{array}{r} +9 \\ \underline{+6} \end{array}$

11. $\begin{array}{r} -17 \\ \underline{-\ 4} \end{array}$

12. $\begin{array}{r} -40 \\ \underline{-40} \end{array}$

13. $\begin{array}{r} -18 \\ \underline{-10} \end{array}$

14. $\begin{array}{r} -8\frac{1}{2} \\ \underline{-7\frac{1}{2}} \end{array}$

15. $\begin{array}{r} +14\frac{1}{2} \\ \underline{+17\frac{1}{4}} \end{array}$

16. $\begin{array}{r} +8 \\ +7 \\ +6 \\ \underline{+2} \end{array}$

17. $\begin{array}{r} -4 \\ -6 \\ -8 \\ \underline{-3} \end{array}$

18. $\begin{array}{r} -4 \\ -5 \\ -8 \\ \underline{-3} \end{array}$

19. $(+6)+(+2)$

20. $(+5)+(+5)$

21. $(+7)+(+2)$

22. $(-3)+(-4)$

23. $(-8)+(-3)$

24. $(-6)+(-5)$

25. $(+7)+(+9)$

26. $(-4)+(-8)$

27. $(-6)+(-6)$

B. Written

28. Paul walked 12 blocks east, was overtaken by a friend in a car, and rode 16 blocks farther east with the friend. How many blocks east did Paul travel? Express the distance as the sum of two signed numbers.

29. Andy lost 25 points and 16 points in a game. What was his total loss? Express his loss in terms of signed numbers.

30. Mr. Andrews withdrew the following amounts from his bank account in one month: \$50, \$75, and \$60. Total his withdrawals in terms of signed numbers.

Copy each of the following and add:

31. $\begin{array}{r} +789 \\ \underline{+156} \end{array}$

32. $\begin{array}{r} -192 \\ \underline{-463} \end{array}$

33. $\begin{array}{r} -198 \\ \underline{-464} \end{array}$

34. $\begin{array}{r} +186 \\ +782 \\ \underline{+925} \end{array}$

35. $\begin{array}{r} -784 \\ -980 \\ \underline{-507} \end{array}$

36. $\begin{array}{r} -475\frac{3}{4} \\ \underline{-196\frac{1}{2}} \end{array}$

37. $\begin{array}{r} +672\frac{1}{3} \\ \underline{+599\frac{1}{2}} \end{array}$

38. $\begin{array}{r} +788\frac{3}{5} \\ \underline{+687\frac{3}{10}} \end{array}$

39. $\begin{array}{r} -148\frac{3}{8} \\ -106\frac{3}{4} \\ \underline{-\ \ 58\frac{1}{2}} \end{array}$

40. $\begin{array}{r} -700\frac{5}{6} \\ -148\frac{2}{3} \\ \underline{-905\frac{3}{4}} \end{array}$

41. $\begin{array}{r} +42.17 \\ \underline{+16.13} \end{array}$

42. $\begin{array}{r} -48.9 \\ \underline{-72.6} \end{array}$

43. $\begin{array}{r} -7.48 \\ \underline{-\ \ .39} \end{array}$

44. $\begin{array}{r} +\ 1.67 \\ +28.03 \\ \underline{+\ \ \ .58} \end{array}$

45. $\begin{array}{r} -33.4 \\ -72.45 \\ \underline{-\ 8.23} \end{array}$

46. $\begin{array}{r} -561\frac{2}{3} \\ \underline{-897\frac{3}{4}} \end{array}$

47. $\begin{array}{r} +93.77 \\ \underline{+49.75} \end{array}$	48. $\begin{array}{r} -4863\frac{1}{2} \\ \underline{-\ 879\frac{3}{8}} \end{array}$	49. $\begin{array}{r} +38.19 \\ +\ 9.73 \\ \underline{+26.45} \end{array}$	50. $\begin{array}{r} -4593 \\ -\ 609 \\ \underline{-2846} \end{array}$

Express every number in the following exercises as a signed number:

51. Fred's deposits for January in the school bank were \$2.10, \$1.85, \$.78, and \$.48. Find his total deposit for January.

52. Dorothy's lunch for four consecutive weeks cost her as follows: \$1.20, \$.90, \$1.28, and \$.97. What did Dorothy's lunches total for the four weeks?

53. Mr. Kirk bought two lots which he later sold, one at a loss of \$176 and the other at a loss of \$218. Find his total loss.

54. On a certain morning the thermometer registered 18° below zero. Later in the day the mercury fell 7° below this point. What did it then register?

55. Howard earned \$8.25, \$6.45, \$7.20, \$5.35, and \$4.60. What were his total earnings?

192. Adding positive and negative numbers with unlike signs. George made 50 points in a game and then lost 20 points. What was his final score? It is easily seen that George's final score was 30 points.

Writing the above facts using the plus and minus signs, we have $+50$ points made and -20 points lost. The final score is $+30$ points, since George made 30 points more than he lost. This sum may be written in two ways:

$$\text{(a) } (+50) + (-20) = +30 \quad \textit{or} \quad \text{(b) } \begin{array}{r} +50 \\ \underline{-20} \\ +30 \end{array}$$

We find this result by subtracting the absolute values of the numbers and prefixing the plus sign (the sign of the number having the greater absolute value).

Milton made 10 points in a game and then lost 25 points. Determine his final score. We readily see that his final score was -15 points, since he lost 15 points more than he made. This sum may be written in two ways:

$$\text{(a) } (+10) + (-25) = -15 \quad \textit{or} \quad \text{(b) } \begin{array}{r} +10 \\ \underline{-25} \\ -15 \end{array}$$

We obtain this result by subtracting the absolute values of the

numbers and prefixing to the difference the minus sign (the sign of the number having the greater absolute value).

From these illustrations we derive the following rule:

To find the sum of numbers with unlike signs, subtract their absolute values and to the difference prefix the sign of the number having the greater absolute value.

EXERCISES

A. Oral

Using the preceding rule, state the sum of each of the following:

1. $\begin{array}{r} +8 \\ \underline{-3} \end{array}$	**6.** $\begin{array}{r} +5 \\ \underline{-8} \end{array}$	**11.** $\begin{array}{r} +3 \\ \underline{-3} \end{array}$	**16.** $\begin{array}{r} -3 \\ \underline{+9} \end{array}$	**21.** $\begin{array}{r} +56 \\ \underline{-32} \end{array}$
2. $\begin{array}{r} +9 \\ \underline{-2} \end{array}$	**7.** $\begin{array}{r} +2 \\ \underline{-6} \end{array}$	**12.** $\begin{array}{r} +6 \\ \underline{-9} \end{array}$	**17.** $\begin{array}{r} +1 \\ \underline{-8} \end{array}$	**22.** $\begin{array}{r} -120 \\ \underline{120} \end{array}$
3. $\begin{array}{r} +16 \\ \underline{-\ 7} \end{array}$	**8.** $\begin{array}{r} +4 \\ \underline{-4} \end{array}$	**13.** $\begin{array}{r} +8 \\ \underline{-7} \end{array}$	**18.** $\begin{array}{r} +10 \\ \underline{-\ 8} \end{array}$	**23.** $\begin{array}{r} -68 \\ \underline{-\ 6} \end{array}$
4. $\begin{array}{r} -8 \\ \underline{+2} \end{array}$	**9.** $\begin{array}{r} -2 \\ \underline{+8} \end{array}$	**14.** $\begin{array}{r} +2 \\ \underline{-2} \end{array}$	**19.** $\begin{array}{r} -12 \\ \underline{+\ 7} \end{array}$	**24.** $\begin{array}{r} 36 \\ \underline{-26} \end{array}$
5. $\begin{array}{r} -7 \\ \underline{+1} \end{array}$	**10.** $\begin{array}{r} -9 \\ \underline{+8} \end{array}$	**15.** $\begin{array}{r} -3 \\ \underline{+1} \end{array}$	**20.** $\begin{array}{r} -7 \\ \underline{+2} \end{array}$	**25.** $\begin{array}{r} 83 \\ \underline{79} \end{array}$

26. $(+7) + (-5)$ **28.** $(-6) + (+5)$ **30.** $(-1) + (+1)$

27. $(+3) + (-8)$ **29.** $(-4) + (+8)$ **31.** $(+2) + (-9)$

32. Mr. Winn owed \$5,000 and had \$2,600. State his financial standing in terms of signed numbers.

33. Herbert made 18 tie racks and sold 15 of them. How many does he still own?

34. If at 6 A. M. the temperature reads 18° above zero and by 9 A. M. it has fallen 12°, what does it then register?

35. May had 170 points in a game and then lost 200 points. What was her final score?

B. Written

Copy each of the following and add:

36. $\begin{array}{r} +316 \\ \underline{-477} \end{array}$	**37.** $\begin{array}{r} -941 \\ \underline{+182} \end{array}$	**38.** $\begin{array}{r} -158 \\ \underline{+279} \end{array}$	**39.** $\begin{array}{r} 89 \\ \underline{-166} \end{array}$

40. $\begin{array}{r} +781 \\ \underline{-106} \end{array}$	43. $\begin{array}{r} +268 \\ \underline{-942} \end{array}$	46. $\begin{array}{r} -868 \\ \underline{+868} \end{array}$	49. $\begin{array}{r} 1581 \\ \underline{-2967} \end{array}$
41. $\begin{array}{r} -401 \\ \underline{589} \end{array}$	44. $\begin{array}{r} -767 \\ \underline{+592} \end{array}$	47. $\begin{array}{r} +587 \\ \underline{-499} \end{array}$	50. $\begin{array}{r} -67.81 \\ \underline{49.33} \end{array}$
42. $\begin{array}{r} +108 \\ \underline{-670} \end{array}$	45. $\begin{array}{r} -380 \\ \underline{+964} \end{array}$	48. $\begin{array}{r} -7810 \\ \underline{+3927} \end{array}$	51. $\begin{array}{r} 170.8 \\ \underline{-263.5} \end{array}$

Express each number in the following exercises as positive or negative and solve:

52. A stock was quoted as worth \$124 during a certain morning. In the afternoon it fell \$2½. What was it then worth?

53. Mr. Black bought two lots. He sold one at a profit of \$150 and the other at a loss of \$320. What was the net result of the sales?

54. A man had \$138 in his checking account. He wrote a check for \$150. Because the man was a good customer, the bank cashed the check. Express his balance in terms of signed numbers.

55. An airplane reached an elevation of 2,480 ft. and then descended 860 ft. What was its new elevation?

193. Adding several positive and negative numbers. In a series of games Clara scored as follows: won 30 points, won 25 points, lost 20 points, won 15 points, and lost 18 points. Determine Clara's final score. First add the points won (the positive numbers); next add the points lost (the negative numbers); then add the two sums.

$$\begin{array}{r} +30 \\ +25 \\ \underline{+15} \\ +70 \end{array} \qquad \begin{array}{r} -20 \\ \underline{-18} \\ -38 \end{array} \qquad \begin{array}{r} +70 \\ \underline{-38} \\ +32 \end{array}$$

Hence, Clara's final score is $+32$.

From the above illustration we derive the following rule:

To find the sum of several numbers with unlike signs, add the positive and negative numbers separately and then add the two sums.

EXERCISES

Collect the signed numbers in each of the following exercises:

$$\begin{array}{rrrrr}
1.\ +8 & 2.\ -7 & 3.\ -4 & 4.\ +6 & 5.\ -18 \\
-10 & -6 & -5 & -8 & +27 \\
\underline{+7} & +2 & \underline{+3} & +4 & -34 \\
 & \underline{+5} & & \underline{-3} & \underline{+36}
\end{array}$$

$$\begin{array}{rrrrr}
6.\ -18 & 7.\ -74 & 8.\ +148 & 9.\ +312 & 10.\ -873 \\
+12 & +26 & +263 & -748 & +625 \\
\underline{-23} & -13 & \underline{-152} & -69 & -466 \\
 & \underline{+35} & & \underline{+457} & \underline{+585}
\end{array}$$

Use signed numbers in solving the following exercises:

11. An airplane reached an altitude of 4,892 ft., descended 1,280 ft., ascended 460 ft., and then descended 812 ft. What was the final elevation?

12. Two partners in a game won 150, then won 120, then lost 100, then lost 80. What was their total score?

13. Mr. White's cash record showed the following balances for six consecutive months: − \$150, − \$60, + \$75, − \$80, − \$100, and + \$50. Find his gain or loss for the 6 mo.

14. Marion rented three lots for garden space. From the first lot he sold enough vegetables to gain \$18.20, on the second lot he lost \$6.50, and on the other lot he gained \$10.80. Find his net gain or loss.

194. Subtraction of signed numbers. By making use of the rules of addition, subtraction of signed numbers can be easily learned.

To subtract signed numbers, change the sign of the subtrahend (mentally), and then add.

The following examples illustrate subtraction of signed numbers:

$$\begin{array}{rrrr}
+13 & +13 & -13 & -13 \\
\underline{-10} & \underline{+10} & \underline{-10} & \underline{+10} \\
+23 & +3 & -3 & -23
\end{array}$$

EXERCISES

A. Oral

Subtract:

$$\begin{array}{rrrr}
1.\ +16 & 3.\ +43 & 5.\ -47 & 7.\ -81 \\
\underline{-32} & \underline{+31} & \underline{-34} & \underline{-27}
\end{array}$$

$$\begin{array}{rrrr}
2.\ +37 & 4.\ 65 & 6.\ -36 & 8.\ +72 \\
\underline{-18} & \underline{36} & \underline{45} & \underline{-93}
\end{array}$$

9. -154 $\underline{-\ 42}$	11. -77 $\underline{-48}$	13. -353 $\underline{+174}$	15. -658 $\underline{-107}$
10. -98 $\underline{72}$	12. $+68$ $\underline{-91}$	14. 427 $\underline{132}$	16. 362 $\underline{-170}$

B. Written

Solve the following:

17. -869 $\underline{-368}$	20. $+6.86$ $\underline{-4.07}$	23. -4.64 $\underline{-2.9}$	26. -5386 $\underline{-6497}$
18. 957 $\underline{659}$	21. -4037 $\underline{-7835}$	24. 75.8 $\underline{-68.3}$	27. -9053 $\underline{6403}$
19. -749 $\underline{+694}$	22. 67.4 $\underline{16.5}$	25. -5495 $\underline{+4709}$	28. 5219 $\underline{4737}$

29. James had a score of -240 in a certain game at the end of the first half of the game. At the close of the game his score was $+380$. How much better off was he at the close of the game than at the end of the first half?

30. A certain stock on the market opened at $+87$ and closed at $+78$. How much was the change?

31. On a certain day when the temperature of a northern city was $-25°$, the temperature in a southern city was $+76°$. What was the difference in temperature of the two cities?

32. On a certain morning the thermometer registered $-17°$. An hour later it registered 23°. Find the difference in the two readings.

195. Graphing positive and negative numbers. The graph on page 328 shows Eloise's financial record at the close of each day for one week. Points on the zero line represent no money on hand and no debts to be paid. Points above zero are positive and represent the amount of money on hand; points below zero are negative and denote the amount of money owed to others.

At the close of which day did Eloise have most money and how much did she have? When was she most in debt, and how much? On which two days did she have the same amount of money? How much? What was her financial condition Thursday? Saturday? Sunday? Which day shows the greatest decrease from the preceding day?

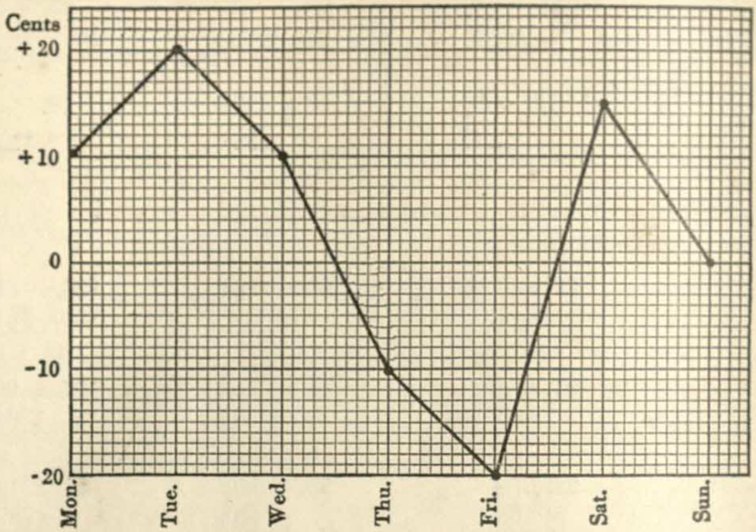


Fig. 1

EXERCISES

1. Using squared paper, make a graph from the following facts representing Harold's financial standing:

Jan.	$10 on hand	May	$5 in debt	Sept.	$5 on hand
Feb.	$15 on hand	June	$15 on hand	Oct.	$5 in debt
Mar.	$5 on hand	July	$20 on hand	Nov.	$15 in debt
April	$10 in debt	Aug.	$25 on hand	Dec.	$5 on hand

2. Make a graph of the following temperatures for one winter day:

6 A.M.	$-10°$	9 A.M.	$-6°$	Noon	$+2°$	3 P.M.	$+1°$
7 A.M.	$-9°$	10 A.M.	$-2°$	1 P.M.	$+3°$	4 P.M.	$-2°$
8 A.M.	$-8°$	11 A.M.	$0°$	2 P.M.	$+5°$	5 P.M.	$-4°$

3. 80 is the normal score for a certain kind of test. Graph Frank's monthly scores as positive and negative.

September	70	December	80	March	84
October	72	January	82	April	85
November	78	February	79	May	85

4. Using sea level as zero, make a graph showing the outline of the land near the shore at points A, B, C, and so on, which are equidistant and have elevations as follows:

Point	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Elevation	100′	80′	20′	Sea Level	− 20′	− 10′	− 20′	− 60′

5. (OPTIONAL) Bring to class and explain any graphs found in books or magazines that illustrate a practical use of signed numbers.

APPENDIX A

REMEDIAL PROGRAM—PRACTICE EXERCISES

PRACTICE EXERCISE 1a

Important Addition Combinations

Study these combinations until you *know* them so well that you can give the correct sums quickly. Add:

7	5	6	2	8	4	3	8	7	9	
5	4	7	5	4	0	7	2	2	8	(10)
6	7	3	9	7	5	7	6	6	7	
5	0	5	6	7	5	9	0	3	8	
5	3	8	3	7	9	7	2	6	5	
9	4	9	6	6	0	4	7	4	3	(30)
3	4	5	7	2	7	9	8	4	5	
3	2	7	1	6	3	2	6	4	0	
3	5	9	8	4	0	5	8	6	4	
9	8	3	5	7	8	6	3	9	3	(50)
6	4	9	6	5	3	4	1	2	0	
2	9	1	8	2	8	5	6	9	0	
2	8	9	6	9	4	9	4	8	9	
8	7	7	6	5	6	9	8	8	4	(70)

PRACTICE EXERCISE 1b

Horizontal Addition by Endings

The exercise 24 + 8 should cause you to see "32" without saying "8 + 4 = 12, put down the two and carry the one," and so forth. Study these exercises till you can give the sums quickly. Add:

16 + 8 = 15	17 + 6 = 14	19 + 8 = 18	16 + 9 =	12 + 3 =	
25 + 6 = 13	29 + 6 = 17	24 + 7 = 14	22 + 9 = 13	23 + 8 = 13	(10)

39 + 9 =	39 + 3 =	38 + 8 =	39 + 4 =	37 + 4 =	
45 + 9 =	44 + 8 =	43 + 7 =	47 + 6 =	42 + 9 =	(20)
56 + 8 =	59 + 4 =	58 + 5 =	59 + 7 =	57 + 8 =	
69 + 8 =	63 + 6 =	66 + 9 =	69 + 0 =	66 + 7 =	(30)
74 + 8 =	77 + 6 =	72 + 9 =	71 + 7 =	74 + 5 =	
89 + 4 =	86 + 5 =	82 + 6 =	87 + 7 =	88 + 3 =	(40)

PRACTICE EXERCISE 1c

Single Columns

Add and prove:

7	6	1	7	9	8	1	7	
3	5	7	6	8	9	9	8	
8	9	6	9	7	2	7	5	
4	3	4	4	2	6	2	3	
5	8	5	8	3	4	6	9	
8	7	9	3	5	5	8	7	(8)

5	7	1	7	5	8	2	4	
8	6	0	4	3	7	6	7	
2	3	9	3	2	4	6	6	
6	4	8	8	6	3	8	5	
4	8	6	6	9	2	8	7	
3	7	4	7	4	8	3	9	(16)

PRACTICE EXERCISE 1d

Carrying

Add and prove:

497	127	264	169	183	603	
672	548	357	139	209	268	
350	266	208	108	176	529	
728	546	527	219	728	177	
549	322	140	169	323	267	
288	728	226	545	157	486	(6)

230	121	326	246	179	700	
582	290	318	393	258	329	
727	313	507	179	549	436	
638	245	292	244	413	289	
459	766	387	307	537	176	
175	397	294	206	641	295	(12)

306	242	808	126	645	649	
459	267	790	462	728	382	
379	214	646	784	543	714	
428	432	729	396	826	622	
624	766	384	458	903	513	
283	822	297	727	728	828	
109	786	458	128	142	126	(18)

Try *Inventory Test 1* again. You should show much improvement.

PRACTICE EXERCISE 2a

Important Products

Study these multiplications till you know them so well that you can give the products quickly. Multiply:

7	5	6	2	9	7	8	3	
5	9	8	5	8	2	2	7	(8)
7	6	6	7	5	7	9	3	
8	3	0	9	5	7	6	5	
7	6	8	0	5	6	2	7	
0	5	4	4	3	4	7	4	(24)
9	7	3	8	3	5	5	4	
0	6	6	9	4	4	0	4	
8	9	7	2	7	5	4	3	
6	2	3	6	1	7	2	3	(40)
4	6	8	5	0	4	8	9	
3	9	3	6	8	7	5	3	
5	3	5	2	1	4	3	5	
8	9	0	9	6	5	8	2	(56)
6	9	4	6	2	8	9	6	
7	1	9	2	8	7	7	6	
9	8	4	9	4	9	2	0	
4	8	8	9	6	5	3	9	(72)

PRACTICE EXERCISE 2b

Zero Endings

Example A:

```
    728
   2300
 218400
1456
1674400
```

Any number times zero, or zero times any number, is zero. For example, $4 \times 0 = 0$ and $0 \times 4 = 0$ and $120 \times 20 = 2{,}400$.

Double zeros in the multiplier should be multiplied as illustrated in Examples A and B.

Example B:

```
    624
   2003
   1872
 124800
1249872
```

Multiply and then prove the following by dividing the product by the multiplier:

450	760	180	430	590
20	30	90	80	70

800	700	100	900	600	
60	50	10	40	90	(10)
106	308	705	509	608	
10	40	80	90	70	
870	407	560	980	706	
60	80	90	70	90	(20)

426	716	326	612	
1500	4600	1700	2400	(24)

PRACTICE EXERCISE 2c

Center Zero

Multiply and prove by interchanging the multiplier and the multiplicand:

102	206	309	105	704	
305	504	902	408	309	
708	308	608	209	605	
105	707	109	307	605	(10)
507	709	208	404	707	
208	606	802	808	908	
607	908	207	508	101	
706	908	704	708	101	(20)

PRACTICE EXERCISE 2d

Large Numbers

Multiply and prove:

328	428	547	719	
76	77	68	83	(4)
627	546	818	137	
37	59	61	80	(8)
726	293	188	356	
58	86	27	46	(12)
427	185	6257	3657	
1000	308	67	82	(16)
802	58	9134	7503	
27	706	64	24	(20)

Try *Inventory Test 2* again. You should show much improvement.

PRACTICE EXERCISE 3a –

Study these exercises until you can give the remainders quickly. Subtract:

7 5	5 4	8 4	4 0	8 2	7 2	9 8	6 5	7 0	9 6	(10)
7 7	5 5	6 0	6 3	7 6	9 0	7 4	6 4	5 3	3 3	(20)
4 2	7 1	7 3	9 2	8 6	4 4	5 0	9 3	8 5	8 3	(30)
4 3	6 2	9 4	5 2	3 0	8 7	9 7	6 6	9 5	9 9	(40)
9 1	8 0	8 8	0 0	7 6	6 1	1 0	5 1	4 1	8 1	(50)

PRACTICE EXERCISE 3b –

Subtract and prove:

27 7	37 5	44 2	54 3	19 1	15 4	
25 5	17 1	26 2	18 0	28 4	36 0	(12)
47 3	29 4	18 8	29 3	15 2	17 0	
19 6	29 5	37 6	18 8	49 0	32 2	(24)
11 7	10 6	12 5	14 6	15 7	11 4	
26 8	37 8	11 9	46 7	24 7	11 5	(36)
55 8	62 7	17 9	23 8	21 8	11 3	
10 3	10 1	20 8	12 7	22 6	14 8	(48)

PRACTICE EXERCISE 3c –

Zero in Minuend

Subtract and prove:

306 28	107 34	290 156	720 264	103 86	102 35	(6)

408 29	610 70	105 28	200 18	100 45	300 102	
404 107	202 27	101 88	400 72	304 108	500 101	(18)
620 188	310 47	204 68	510 34	706 309	104 83	
180 77	120 46	308 21	503 18	200 87	300 75	(30)

PRACTICE EXERCISE 3d

Large Numbers

Subtract and prove:

74026 15634	16924 8795	35496 18728	50628 13978	
24104 13726	51807 21426	33333 16666	72222 10948	(8)
15213 8764	26104 18702	31110 28794	17961 3948	
56728 17438	33101 28064	45202 17606	50925 18628	(16)
63956 51787	98646 19584	65232 47111	44200 26741	
18505 2487	23666 18999	47142 21847	57128 9072	(24)

If you scored well on this test, try *Inventory Test 3* again.

PRACTICE EXERCISE 4a

One-figure Divisors

Divide:

$\begin{array}{r}7\\9\overline{)63}\end{array}$	$\begin{array}{r}4\\8\overline{)32}\end{array}$	$\begin{array}{r}4\\9\overline{)36}\end{array}$	$\begin{array}{r}0\\8\overline{)0}\end{array}$	
$\begin{array}{r}2\\8\overline{)16}\end{array}$	$\begin{array}{r}9\\9\overline{)72}\end{array}$	$\begin{array}{r}8\\7\overline{)56}\end{array}$	$\begin{array}{r}3\\6\overline{)18}\end{array}$	(8)
$\begin{array}{r}4\\7\overline{)28}\end{array}$	$\begin{array}{r}4\\6\overline{)24}\end{array}$	$\begin{array}{r}2\\9\overline{)18}\end{array}$	$\begin{array}{r}5\\8\overline{)40}\end{array}$	

9)27	8)64	7)63	7)49	(16)
8)48	6)30	9)0	9)54	
9)45	7)14	8)56	6)0	(24)
8)72	6)12	7)42	7)35	
6)36	8)24	5)0	6)42	(32)
5)25	4)36	6)48	5)15	
4)24	3)27	4)28	3)18	(40)

PRACTICE EXERCISE 4b

Divide and then prove by multiplying the quotient and the divisor and adding the remainder to the product:

4)840	5)705	7)714	8)404	9)1000	
6)846	5)600	8)912	6)624	7)8248	(10)
4)728	6)635	9)684	5)206	8)8080	
8)645	6)304	7)645	8)724	4)4090	(20)
6)582	8)928	2)505	9)806	7)9177	
3)725	9)906	4)325	9)459	6)6274	(30)
4)726	8)909	7)626	6)542	9)5648	
2)101	4)301	8)604	9)248	7)5386	(40)

PRACTICE EXERCISE 4c

Long Division

Copy these exercises on paper. Divide and then prove by using the quotient as the divisor:

18)450	33)891	21)1596	38)3116	
16)384	32)896	52)2496	23)2162	(8)
19)418	17)306	45)3645	67)2278	
16)304	35)525	28)2016	34)1768	(16)
18)288	81)972	90)3240	25)1250	
69)966	17)714	37)2368	73)2482	(24)

PRACTICE EXERCISE 4d

Long Division

Copy exercises on paper. Divide and then prove:

$42\overline{)15036}$	$36\overline{)72036}$	$72\overline{)84509}$	
$63\overline{)45089}$	$24\overline{)37426}$	$18\overline{)17258}$	(6)
$51\overline{)64145}$	$35\overline{)28696}$	$90\overline{)80603}$	
$38\overline{)42122}$	$16\overline{)32048}$	$27\overline{)28972}$	(12)
$23\overline{)16560}$	$29\overline{)11484}$	$17\overline{)44710}$	
$132\overline{)16848}$	$182\overline{)72056}$	$124\overline{)36746}$	(18)

Try *Inventory Test 4* again. You should pass it now.

How do you score in the four processes with integers since solving the *Practice Exercises* and retaking the *Inventory Tests*?

You no doubt show a much improved skill. However, if your skill is not satisfactory, you should make it *your* job to continue your practice until you do master these processes.

ADDITION OF COMMON FRACTIONS

You can add common fractions if you first change them to a common denominator. The examples show the fractions changed to their *least common denominator*. The example at the left shows the addition of fractions arranged vertically. The example at the right shows the fractions arranged horizontally.

Example

$$\frac{3}{4}=\frac{9}{12}$$
$$\frac{5}{6}=\frac{10}{12}$$
$$\frac{19}{12}$$
$$\text{or } 1\frac{7}{12}$$

Example

$$\frac{3}{4}+\frac{5}{6}=$$
$$\frac{9}{12}+\frac{10}{12}=$$
$$\frac{19}{12} \text{ or } 1\frac{7}{12}$$

PRACTICE EXERCISE 5a

Add these fractions:

$\frac{4}{5}+\frac{3}{10}$	$\frac{7}{8}+\frac{3}{4}$	$\frac{1}{3}+\frac{1}{4}$	$\frac{1}{8}+\frac{1}{6}$	(4)
$\frac{1}{2}+\frac{1}{4}$	$\frac{5}{6}+\frac{2}{3}$	$\frac{2}{3}+\frac{1}{2}$	$\frac{1}{5}+\frac{1}{3}$	(8)

$\frac{1}{3}$	$\frac{3}{8}$	$\frac{1}{5}$	$\frac{2}{3}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{3}{4}$	
$\frac{3}{4}$	$\frac{1}{4}$	$\frac{3}{10}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{1}{2}$	(15)
$\frac{5}{6}$	$\frac{7}{16}$	$\frac{9}{10}$	$\frac{6}{7}$	$\frac{7}{12}$	$\frac{15}{16}$	$\frac{4}{9}$	
$\frac{7}{8}$	$\frac{3}{8}$	$\frac{4}{15}$	$\frac{5}{6}$	$\frac{5}{8}$	$\frac{7}{8}$	$\frac{7}{12}$	(22)

$\frac{3}{4}+\frac{7}{8}+\frac{1}{2}$	$\frac{7}{9}+\frac{2}{3}+\frac{5}{6}$	$\frac{3}{8}+\frac{4}{5}+\frac{7}{20}$	(25)

PRACTICE EXERCISE 5b

Vertical Addition

Add and express each sum in lowest terms:

$\frac{1}{4}$	$\frac{3}{8}$	$\frac{2}{5}$	$\frac{1}{3}$	$\frac{3}{8}$	$\frac{5}{6}$	$\frac{7}{10}$	
$\frac{1}{6}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{7}{8}$	$\frac{7}{12}$	$\frac{2}{3}$	$\frac{4}{5}$	(7)
$\frac{3}{16}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{5}{12}$	$\frac{3}{8}$	$\frac{1}{3}$	
$\frac{7}{8}$	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{1}{6}$	$\frac{11}{12}$	(14)
$\frac{2}{5}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{7}{10}$	$\frac{2}{3}$	$\frac{7}{12}$	
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{5}{6}$	$\frac{1}{2}$	
$\frac{7}{10}$	$\frac{1}{2}$	$\frac{5}{12}$	$\frac{5}{7}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{5}{6}$	(21)
$\frac{5}{8}$	$\frac{3}{5}$	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{9}$	
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{5}{7}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{2}{3}$	
$\frac{7}{8}$	$\frac{1}{5}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{5}{6}$	$\frac{5}{8}$	$\frac{5}{6}$	(28)

PRACTICE EXERCISE 5c

Horizontal Addition

Add and express all sums in lowest terms:

$\frac{2}{3}+\frac{5}{6}$	$\frac{3}{4}+\frac{1}{2}$	$\frac{7}{8}+\frac{2}{3}$	$\frac{4}{5}+\frac{3}{8}+\frac{3}{20}$	(4)
$\frac{6}{7}+\frac{1}{2}$	$\frac{4}{9}+\frac{5}{6}$	$\frac{1}{2}+\frac{1}{12}$	$\frac{1}{3}+\frac{3}{4}+\frac{1}{6}$	
$\frac{3}{14}+\frac{2}{7}$	$\frac{1}{12}+\frac{5}{8}$	$\frac{9}{10}+\frac{5}{6}$	$\frac{3}{4}+\frac{1}{2}+\frac{1}{4}$	(12)
$\frac{3}{5}+\frac{5}{6}$	$\frac{1}{9}+\frac{1}{3}$	$\frac{3}{4}+\frac{5}{8}$	$\frac{1}{5}+\frac{7}{10}+\frac{2}{15}$	
$\frac{7}{12}+\frac{5}{6}+\frac{1}{2}$		$\frac{9}{12}+\frac{3}{8}+\frac{4}{4}$	$\frac{5}{12}+\frac{3}{5}+\frac{2}{3}$	(19)
$\frac{3}{10}+\frac{3}{4}+\frac{2}{5}$		$\frac{5}{8}+\frac{2}{5}+\frac{1}{10}$	$\frac{15}{16}+\frac{5}{8}+\frac{3}{4}$	
$\frac{4}{9}+\frac{2}{3}+\frac{1}{9}$		$\frac{1}{2}+\frac{1}{8}+\frac{7}{16}$	$\frac{7}{12}+\frac{1}{6}+\frac{2}{3}$	(25)

PRACTICE EXERCISE 5d

Addition of Mixed Numbers

Add and write each sum in its lowest terms:

$3\frac{2}{3}$	$4\frac{3}{5}$	$7\frac{5}{9}$	$8\frac{4}{5}$	$7\frac{1}{8}$	$8\frac{3}{4}$	
$7\frac{1}{2}$	$8\frac{2}{3}$	$6\frac{2}{3}$	$9\frac{1}{2}$	$9\frac{2}{3}$	$7\frac{1}{3}$	(6)
$9\frac{1}{2}$	$6\frac{5}{8}$	$9\frac{3}{7}$	$8\frac{5}{12}$	$7\frac{1}{3}$	$8\frac{4}{5}$	
$8\frac{1}{3}$	$7\frac{3}{4}$	$9\frac{1}{2}$	$7\frac{3}{4}$	$7\frac{2}{9}$	3	(12)
$3\frac{1}{4}$	$6\frac{5}{6}$	$2\frac{3}{16}$	9	$5\frac{2}{5}$	$9\frac{7}{8}$	
$2\frac{1}{3}$	$6\frac{1}{12}$	$8\frac{3}{4}$	$8\frac{4}{9}$	$7\frac{3}{5}$	$5\frac{1}{16}$	(18)

$5\frac{1}{8}+2\frac{1}{3}$	$7\frac{1}{2}+6\frac{2}{3}$	$4\frac{1}{4}+6\frac{1}{6}$	$7\frac{1}{8}+6\frac{1}{4}$	(22)
$7\frac{1}{9}+6\frac{2}{9}$	$2\frac{1}{5}+6\frac{4}{5}$	$3\frac{1}{3}+5\frac{1}{5}$	$1\frac{1}{2}+1\frac{1}{2}$	(26)
$9\frac{1}{3}+7\frac{2}{5}$	$4\frac{11}{12}+5\frac{1}{2}$	$2\frac{3}{4}+7\frac{1}{8}$	$5\frac{1}{4}+5\frac{2}{3}$	(30)

PRACTICE EXERCISE 5e

Addition of Mixed Numbers

Add and reduce all fractions to lowest terms:

$\begin{array}{r} 17\frac{1}{4} \\ 24\frac{1}{12} \\ \hline \end{array}$	$\begin{array}{r} 43\frac{5}{6} \\ 28\frac{2}{3} \\ \hline \end{array}$	$\begin{array}{r} 74\frac{1}{2} \\ 82\frac{1}{3} \\ \hline \end{array}$	$\begin{array}{r} 69\frac{2}{5} \\ 73\frac{1}{4} \\ \hline \end{array}$	$\begin{array}{r} 86\frac{7}{9} \\ 23\frac{2}{3} \\ \hline \end{array}$	(5)
$\begin{array}{r} 87\frac{1}{3} \\ 33\frac{2}{3} \\ \hline \end{array}$	$\begin{array}{r} 47\frac{3}{8} \\ 26\frac{1}{8} \\ \hline \end{array}$	$\begin{array}{r} 66\frac{1}{2} \\ 38\frac{1}{4} \\ \hline \end{array}$	$\begin{array}{r} 94\frac{2}{5} \\ 76\frac{1}{3} \\ \hline \end{array}$	$\begin{array}{r} 29\frac{1}{6} \\ 46\frac{2}{3} \\ \hline \end{array}$	(10)
$\begin{array}{r} 28\frac{1}{16} \\ 5\frac{5}{8} \\ \hline \end{array}$	$\begin{array}{r} 13\frac{11}{12} \\ 96\frac{3}{4} \\ \hline \end{array}$	$\begin{array}{r} 28\frac{3}{5} \\ 77\frac{4}{5} \\ \hline \end{array}$	$\begin{array}{r} 55\frac{7}{8} \\ 66\frac{2}{3} \\ \hline \end{array}$	$\begin{array}{r} 29\frac{1}{8} \\ 45\frac{5}{24} \\ \hline \end{array}$	(15)

$18\frac{1}{4} + 16\frac{1}{3}$	$24\frac{1}{2} + 16\frac{1}{4}$	$38\frac{1}{5} + 17\frac{1}{8}$	
$26\frac{3}{7} + 18\frac{4}{7}$	$39\frac{2}{3} + 16\frac{1}{4}$	$47 + 18\frac{5}{6}$	(21)

$\begin{array}{r} 124\frac{3}{5} \\ 173\frac{1}{4} \\ \hline \end{array}$	$\begin{array}{r} 167\frac{1}{2} \\ 238\frac{1}{3} \\ \hline \end{array}$	$\begin{array}{r} 343\frac{3}{5} \\ 196\frac{1}{2} \\ \hline \end{array}$	$\begin{array}{r} 189\frac{4}{5} \\ 276\frac{2}{5} \\ \hline \end{array}$	$\begin{array}{r} 172\frac{2}{9} \\ 188\frac{2}{3} \\ \hline \end{array}$	(26)
$\begin{array}{r} 724\frac{1}{6} \\ 823\frac{2}{3} \\ \hline \end{array}$	$\begin{array}{r} 485\frac{1}{7} \\ 317\frac{1}{14} \\ \hline \end{array}$	$\begin{array}{r} 168\frac{1}{5} \\ 728\frac{1}{3} \\ \hline \end{array}$	$\begin{array}{r} 450\frac{2}{3} \\ 706\frac{1}{2} \\ \hline \end{array}$	$\begin{array}{r} 276\frac{5}{6} \\ 488\frac{3}{4} \\ \hline \end{array}$	(31)

If you have scored well on this exercise, you should have no trouble passing *Inventory Test 5*. Try it again.

SUBTRACTION OF COMMON FRACTIONS

Example A

$\frac{7}{8} - \frac{3}{4}$

$\frac{7}{8} - \frac{6}{8} = \frac{1}{8}$

You can subtract common fractions if you first change them to a common denominator. As in the addition of fractions, the arrangement of the fractions may be horizontal or vertical.

Example B

$$\begin{array}{r} \frac{2}{3} = \frac{4}{6} \\ -\frac{1}{6} = \frac{1}{6} \\ \hline \frac{3}{6} = \frac{1}{2} \end{array}$$

PRACTICE EXERCISE 6a

Vertical Subtraction of Fractions

Subtract:

$\begin{array}{r} \frac{1}{2} \\ \frac{1}{4} \\ \hline \end{array}$	$\begin{array}{r} \frac{1}{3} \\ \frac{1}{5} \\ \hline \end{array}$	$\begin{array}{r} \frac{3}{4} \\ \frac{1}{2} \\ \hline \end{array}$	$\begin{array}{r} \frac{3}{4} \\ \frac{3}{5} \\ \hline \end{array}$	$\begin{array}{r} \frac{2}{3} \\ \frac{1}{2} \\ \hline \end{array}$	$\begin{array}{r} \frac{3}{4} \\ \frac{3}{8} \\ \hline \end{array}$	
$\begin{array}{r} \frac{7}{8} \\ \frac{1}{4} \\ \hline \end{array}$	$\begin{array}{r} \frac{5}{6} \\ \frac{1}{3} \\ \hline \end{array}$	$\begin{array}{r} \frac{4}{5} \\ \frac{1}{3} \\ \hline \end{array}$	$\begin{array}{r} \frac{7}{9} \\ \frac{2}{3} \\ \hline \end{array}$	$\begin{array}{r} \frac{6}{7} \\ \frac{1}{2} \\ \hline \end{array}$	$\begin{array}{r} \frac{7}{12} \\ \frac{3}{16} \\ \hline \end{array}$	(12)
$\begin{array}{r} \frac{11}{12} \\ \frac{1}{6} \\ \hline \end{array}$	$\begin{array}{r} \frac{3}{4} \\ \frac{1}{8} \\ \hline \end{array}$	$\begin{array}{r} \frac{2}{3} \\ \frac{1}{2} \\ \hline \end{array}$	$\begin{array}{r} \frac{3}{4} \\ \frac{1}{3} \\ \hline \end{array}$	$\begin{array}{r} \frac{1}{2} \\ \frac{1}{6} \\ \hline \end{array}$	$\begin{array}{r} \frac{5}{8} \\ \frac{1}{3} \\ \hline \end{array}$	
$\begin{array}{r} \frac{3}{5} \\ \frac{1}{6} \\ \hline \end{array}$	$\begin{array}{r} \frac{11}{16} \\ \frac{3}{8} \\ \hline \end{array}$	$\begin{array}{r} \frac{7}{10} \\ \frac{2}{5} \\ \hline \end{array}$	$\begin{array}{r} \frac{5}{12} \\ \frac{3}{8} \\ \hline \end{array}$	$\begin{array}{r} \frac{5}{6} \\ \frac{7}{12} \\ \hline \end{array}$	$\begin{array}{r} \frac{7}{8} \\ \frac{3}{16} \\ \hline \end{array}$	(24)

PRACTICE EXERCISE 6b

Horizontal Subtraction of Fractions

Subtract:

$\frac{3}{4}-\frac{1}{4}$	$\frac{7}{8}-\frac{1}{2}$	$\frac{4}{5}-\frac{1}{5}$	$\frac{5}{6}-\frac{1}{3}$	
$\frac{3}{8}-\frac{1}{6}$	$\frac{3}{4}-\frac{1}{2}$	$\frac{9}{16}-\frac{1}{4}$	$\frac{7}{8}-\frac{5}{16}$	(8)
$\frac{7}{12}-\frac{5}{12}$	$\frac{1}{2}-\frac{1}{10}$	$\frac{2}{3}-\frac{1}{4}$	$\frac{3}{4}-\frac{1}{3}$	
$\frac{7}{9}-\frac{2}{3}$	$\frac{3}{5}-\frac{1}{4}$	$\frac{2}{3}-\frac{3}{8}$	$\frac{9}{10}-\frac{1}{2}$	(16)
$\frac{7}{8}-\frac{2}{3}$	$\frac{8}{9}-\frac{1}{6}$	$\frac{6}{8}-\frac{1}{3}$	$\frac{4}{5}-\frac{1}{4}$	
$\frac{1}{2}-\frac{1}{3}$	$\frac{1}{8}-\frac{1}{16}$	$\frac{7}{8}-\frac{3}{4}$	$\frac{11}{12}-\frac{3}{8}$	(24)

SUBTRACTION OF MIXED NUMBERS

If the fraction in the minuend is less than the fraction in the subtrahend, add 1 to the fraction in the minuend; then subtract as usual. In Example A the 1 that is to be added to the fraction $\frac{3}{9}$ is expressed as $\frac{9}{9}$. The result is $\frac{12}{9}$. The $\frac{5}{9}$ subtracted from $\frac{12}{9}$ is $\frac{7}{9}$. The remainder is then $1\frac{7}{9}$.

Example A

$$\begin{array}{rrr} 9\frac{1}{3} & \frac{3}{9} & \frac{12}{9} \\ -7\frac{5}{9} & \frac{5}{9} & \frac{5}{9} \\ \hline 1\frac{7}{9} & & \frac{7}{9} \end{array}$$

Example B

$$\begin{array}{rr} 8 & \frac{7}{7} \\ -3\frac{5}{7} & \frac{5}{7} \\ \hline 4\frac{2}{7} & \frac{2}{7} \end{array}$$

In Example B there is no fraction in the minuend. Express the 1 that is to be added as $\frac{7}{7}$ and proceed as usual.

PRACTICE EXERCISE 6c

Subtract:

$8\frac{2}{3}$	$9\frac{1}{6}$	$7\frac{3}{5}$	$5\frac{2}{3}$	6	$7\frac{1}{3}$	
$2\frac{1}{3}$	$3\frac{1}{8}$	$1\frac{9}{10}$	$2\frac{3}{4}$	$5\frac{1}{5}$	$2\frac{4}{5}$	
$8\frac{7}{8}$	$9\frac{1}{2}$	$7\frac{3}{8}$	$5\frac{3}{16}$	8	$7\frac{1}{4}$	
2	$4\frac{7}{8}$	$1\frac{7}{8}$	$\frac{3}{8}$	$2\frac{5}{7}$	$3\frac{3}{4}$	(12)
5	$7\frac{1}{8}$	$9\frac{2}{5}$	$9\frac{4}{7}$	$7\frac{5}{12}$	$4\frac{2}{3}$	
$1\frac{3}{5}$	$2\frac{1}{6}$	$1\frac{1}{2}$	5	$3\frac{3}{4}$	$1\frac{2}{3}$	
$6\frac{11}{12}$	7	$1\frac{1}{4}$	$6\frac{1}{5}$	8	$7\frac{3}{5}$	
$1\frac{1}{5}$	$2\frac{1}{12}$	$\frac{7}{8}$	$2\frac{1}{10}$	$2\frac{3}{16}$	$1\frac{3}{4}$	(24)

PRACTICE EXERCISE 6d

Subtract:

$18\frac{2}{5}$	$26\frac{1}{3}$	$37\frac{1}{2}$	$45\frac{1}{8}$	$48\frac{1}{4}$	76	
$7\frac{1}{5}$	$9\frac{1}{3}$	$16\frac{7}{8}$	$28\frac{1}{4}$	$19\frac{1}{3}$	$18\frac{4}{5}$	
$38\frac{1}{16}$	$94\frac{5}{6}$	$76\frac{8}{9}$	$46\frac{2}{3}$	$11\frac{3}{4}$	$20\frac{1}{4}$	
$10\frac{1}{2}$	$\frac{3}{8}$	19	$8\frac{5}{6}$	$8\frac{7}{10}$	$7\frac{3}{8}$	(12)

$47\frac{1}{6}$	$29\frac{1}{5}$	$40\frac{1}{2}$	127	$208\frac{1}{3}$	$480\frac{1}{6}$	
$19\frac{1}{2}$	$18\frac{1}{2}$	$16\frac{5}{8}$	$88\frac{4}{7}$	$194\frac{2}{3}$	$89\frac{3}{4}$	
$185\frac{7}{9}$	$181\frac{2}{3}$	$707\frac{1}{3}$	$398\frac{4}{5}$	$161\frac{1}{8}$	101	
110	$79\frac{1}{8}$	$109\frac{1}{5}$	$109\frac{1}{2}$	$97\frac{3}{4}$	$89\frac{3}{8}$	(24)

If you scored well on these exercises, you should be able to pass *Inventory Test 6* now. Try it.

MULTIPLICATION OF FRACTIONS

In general, we find the product of two or more fractions by multiplying their numerators together for the numerator of the product and the denominators together for the denominator of the product.

Example: $\frac{4}{5}\times\frac{3}{7}=\frac{12}{35}$.

If there is a common factor in the numerator and the denominator, cancel before multiplying.

Example: $\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}}\times\frac{\overset{\overset{1}{\cancel{2}}}{\cancel{6}}}{5}\times\frac{7}{\underset{\underset{2}{\cancel{4}}}{\cancel{8}}}=\frac{7}{10}$

If there are one or more mixed numbers in the multiplication, they are usually changed to improper fractions.

Example: $\frac{4}{5}\times 3\frac{1}{3}=\frac{4}{\underset{1}{\cancel{5}}}\times\frac{\overset{2}{\cancel{10}}}{3}=\frac{8}{3}=2\frac{2}{3}$

PRACTICE EXERCISE 7a

Multiplication of Two or More Fractions

Multiply:

$\frac{2}{3}\times\frac{2}{3}$	$\frac{4}{5}\times\frac{3}{5}$	$\frac{7}{8}\times\frac{4}{7}$	$\frac{6}{7}\times\frac{7}{9}$	
$\frac{2}{5}\times\frac{15}{22}$	$\frac{1}{2}\times\frac{1}{2}$	$\frac{4}{9}\times\frac{6}{11}$	$\frac{1}{4}\times\frac{1}{4}$	(8)
$\frac{5}{9}\times\frac{6}{7}$	$\frac{1}{2}\times\frac{2}{3}$	$\frac{2}{3}\times\frac{1}{4}$	$\frac{15}{16}\times\frac{28}{25}$	
$\frac{21}{25}\times\frac{10}{21}$	$\frac{2}{3}\times\frac{7}{8}$	$\frac{3}{8}\times\frac{3}{8}$	$\frac{4}{9}\times\frac{27}{28}$	(16)
$\frac{1}{3}\times\frac{2}{5}\times\frac{9}{10}$	$\frac{2}{3}\times\frac{2}{5}\times\frac{3}{4}$		$\frac{5}{6}\times\frac{1}{10}\times\frac{8}{21}$	
$\frac{17}{18}\times\frac{4}{5}\times\frac{20}{51}$	$\frac{7}{8}\times\frac{1}{2}\times 4$		$6\times\frac{2}{3}\times\frac{1}{5}$	(22)

PRACTICE EXERCISE 7b

Multiplication of Integer and Fraction

Multiply:

$6\times\frac{3}{4}$	$7\times\frac{3}{14}$	$8\times\frac{5}{6}$	$9\times\frac{2}{3}$	(4)
$\frac{7}{8}\times 4$	$\frac{5}{6}\times 9$	$2\frac{2}{3}\times 6$	$\frac{5}{9}\times 12$	
$\frac{7}{9}\times 15$	$\frac{4}{5}\times 4$	$\frac{3}{8}\times 6$	$2\frac{4}{9}\times 15$	(12)
$24\times 1\frac{3}{8}$	$72\times\frac{5}{9}$	$78\times\frac{1}{2}$	$45\times 3\frac{5}{6}$	
$\frac{3}{4}$ of 50	$\frac{7}{8}$ of 60	$\frac{3}{7}$ of 42	$\frac{5}{12}$ of 84	(20)

PRACTICE EXERCISE 7c

Multiplication of Mixed Numbers

Multiply:

$2\frac{1}{3} \times 3\frac{1}{7}$	$4\frac{1}{2} \times 3\frac{1}{3}$	$1\frac{1}{2} \times 1\frac{1}{4}$	$1\frac{1}{2} \times 1\frac{1}{2}$	
$7\frac{1}{3} \times 2\frac{1}{11}$	$3\frac{3}{4} \times 3\frac{1}{5}$	$7\frac{1}{2} \times 3\frac{1}{3}$	$2\frac{1}{4} \times 2\frac{1}{4}$	(8)
$4\frac{1}{5} \times 2\frac{1}{2}$	$8\frac{1}{3} \times 1\frac{7}{8}$	$8\frac{3}{4} \times 2\frac{1}{7}$	$3\frac{1}{3} \times 3\frac{1}{3}$	
$4\frac{1}{6} \times 1\frac{1}{5}$	$3\frac{2}{3} \times 4\frac{1}{2}$	$1\frac{1}{8} \times 1\frac{1}{3}$	$1\frac{1}{2} \times 3\frac{1}{3}$	(16)
$1\frac{4}{5} \times 2\frac{1}{2}$	$1\frac{5}{6} \times 1\frac{1}{5}$	$12\frac{1}{2} \times 12\frac{1}{2}$	$7\frac{1}{2} \times 7\frac{1}{2}$	
$1\frac{2}{3} \times 4\frac{3}{8}$	$16\frac{2}{3} \times 7\frac{3}{5}$	$18\frac{3}{4} \times 2\frac{2}{5}$	$4\frac{3}{8} \times 2\frac{4}{7}$	(24)

If your products are correct, try *Inventory Test 7* again.

DIVISION OF FRACTIONS

To divide with fractions, invert the divisor and multiply.

Example A

$\frac{4}{9} \div \frac{5}{12}$

$$\frac{4}{\underset{3}{\cancel{9}}} \times \frac{\overset{4}{\cancel{12}}}{5} = \frac{16}{15} = 1\,\frac{1}{15}$$

Example B

$\frac{6}{7} \div 4$

$$\frac{\overset{3}{\cancel{6}}}{7} \times \frac{1}{\underset{2}{\cancel{4}}} = \frac{3}{14}$$

Example C

$12 \div \frac{9}{7}$

$$\frac{\overset{4}{\cancel{12}}}{1} \times \frac{7}{\underset{3}{\cancel{9}}} = \frac{28}{3} = 9\,\frac{1}{3}$$

Example D

$4\frac{2}{3} \div 1\frac{3}{4}$

$$\frac{\overset{2}{\cancel{14}}}{3} \times \frac{4}{\underset{1}{\cancel{7}}} = \frac{8}{3} = 2\,\frac{2}{3}$$

PRACTICE EXERCISE 8a

Division of a Fraction by a Fraction

Divide; then reduce all fractions in quotients to lowest terms:

$\frac{7}{9} \div \frac{2}{3}$	$\frac{8}{15} \div \frac{1}{5}$	$\frac{5}{6} \div \frac{2}{3}$	$\frac{8}{9} \div \frac{1}{6}$	
$\frac{7}{12} \div \frac{3}{4}$	$\frac{8}{9} \div \frac{1}{2}$	$\frac{5}{9} \div \frac{5}{9}$	$\frac{3}{4} \div \frac{1}{8}$	(8)
$\frac{1}{2} \div \frac{2}{3}$	$\frac{7}{8} \div \frac{3}{4}$	$\frac{1}{3} \div \frac{1}{6}$	$\frac{2}{3} \div \frac{3}{4}$	
$\frac{2}{3} \div \frac{4}{9}$	$\frac{5}{8} \div \frac{1}{2}$	$\frac{1}{2} \div \frac{1}{2}$	$\frac{3}{5} \div \frac{3}{4}$	(16)
$\frac{1}{2} \div \frac{3}{8}$	$\frac{1}{4} \div \frac{1}{8}$	$\frac{7}{9} \div \frac{5}{6}$	$\frac{3}{16} \div \frac{7}{8}$	
$\frac{1}{9} \div \frac{1}{6}$	$\frac{1}{3} \div \frac{1}{4}$	$\frac{1}{2} \div \frac{5}{7}$	$\frac{2}{3} \div \frac{4}{7}$	(24)

PRACTICE EXERCISE 8b

Division of Fractions and Integers

Divide; then reduce all fractions in quotients to lowest terms:

$3 \div \frac{1}{12}$	$4 \div \frac{4}{7}$	$6 \div \frac{8}{9}$	$\frac{11}{12} \div 3$	(4)
$12 \div \frac{1}{6}$	$18 \div \frac{2}{3}$	$20 \div \frac{4}{5}$	$24 \div \frac{1}{2}$	
$\frac{2}{3} \div 2$	$\frac{5}{6} \div 8$	$\frac{7}{9} \div 14$	$\frac{6}{7} \div 12$	(12)
$\frac{7}{8} \div 21$	$\frac{4}{9} \div 6$	$\frac{5}{8} \div 15$	$\frac{5}{16} \div 12$	
$\frac{4}{5} \div 12$	$\frac{5}{8} \div 10$	$16 \div \frac{8}{9}$	$26 \div \frac{13}{15}$	(20)

PRACTICE EXERCISE 8c

Division of Mixed Numbers

Divide; then reduce all fractions in quotients to lowest terms:

$2\frac{1}{3} \div 4\frac{1}{2}$	$2\frac{1}{2} \div 3\frac{1}{3}$	$6\frac{1}{4} \div 1\frac{2}{3}$	$1\frac{1}{8} \div 1\frac{1}{4}$	(4)
$7\frac{1}{2} \div 2$	$6\frac{2}{3} \div 4$	$3\frac{3}{4} \div 3$	$4\frac{2}{5} \div 2\frac{1}{5}$	
$8 \div 1\frac{1}{4}$	$4\frac{2}{3} \div 1\frac{1}{3}$	$9 \div 1\frac{1}{8}$	$7\frac{2}{3} \div 1\frac{2}{3}$	(12)
$12 \div 3\frac{3}{4}$	$16 \div 2\frac{1}{4}$	$6\frac{1}{2} \div \frac{7}{8}$	$9\frac{1}{3} \div 4\frac{2}{3}$	
$5\frac{1}{2} \div 3\frac{2}{3}$	$21 \div 1\frac{7}{8}$	$10\frac{2}{3} \div 2\frac{2}{3}$	$12\frac{4}{5} \div 2\frac{1}{8}$	(20)

If you have a good score on this test, try *Inventory Test 8* again.

ADDITION AND SUBTRACTION OF DECIMAL FRACTIONS

Arrange the numbers to be added or subtracted in a vertical column with the decimal points directly below each other so that tenths will be under tenths, hundredths under hundredths, and so on. In any integer, the decimal point belongs at the right of the last digit. Zeros may be filled in after the decimals to prevent a ragged appearance.

Example A
Addition

3.17	3.17
2.8	2.80
12.	12.00
17.97	17.97

Example B
Subtraction

24.6	24.600
− 18.452	− 18.452
6.148	6.148

The decimal point in the sum (or in the remainder) is placed directly below the decimal points in the column.

If a common fraction and a decimal fraction are to be added or subtracted, either express the decimal as a common fraction or the common fraction as a decimal; then proceed as usual.

Example C

$3.2 + 1\frac{1}{4}$

$$\begin{array}{r} 3.20 \\ +\,1.25 \\ \hline 4.45 \end{array}$$

Example D

$5\frac{1}{2} - 2.37\frac{1}{2}$

$$\begin{array}{r} 5\frac{1}{2} \\ -\,2\frac{3}{8} \\ \hline 3\frac{1}{8} \end{array}$$

PRACTICE EXERCISE 9a

Addition of Decimals—Vertical

Add:

1.	2.	3.	4.	5.
.4	1.6	4.56	18.167	$.7\frac{1}{4}$
.6	2.3	9	2.95	2.6
.3	7.8	7.2	3.08	3.9
.5	9	.8	.9	.45

6.	7.	8.	9.	10.
48.67	29.6	.83	29.7	.85
4.38	7.4	2.9	$83.\frac{1}{2}$	2.6
7.9	.92	.7	7.64	.325
.36	3.1	4.857	.234	$8.1\frac{1}{2}$
$1.7\frac{1}{2}$	.675	$.3\frac{1}{4}$	$6.\frac{1}{8}$	7.45

PRACTICE EXERCISE 9b

Addition of Decimals—Horizontal

Add:

1. $.6 + 1.8 + 3.24$
2. $.02 + 3.7 + 7.8 + 12.78$
3. $3.86 + 4.027 + .9$
4. $.002 + 1.076 + .25 + 1.6$
5. $.9 + .8 + .7 + .6$
6. $.82 + .9 + .74 + 8$
7. $1.6 + 2.9 + 1\frac{1}{2}$
8. $32.8 + 67 + .92 + 2\frac{1}{4}$
9. $241.6 + 9.075$
10. $203.8 + 24.07 + 645$
11. $.767 + 14.9$
12. $6.7 + .406 + 30.9 + .8$
13. $10.25 + 16\frac{1}{5}$
14. $27.8 + 52\frac{1}{3} + 46.2\frac{1}{4}$
15. $7.5 + 8\frac{2}{3} + 6.4$
16. $1.9 + 7\frac{1}{8} + 2.93\frac{1}{2} + .7\frac{1}{4}$
17. $2.6 + 3.7 + 27 + .9$
18. $.97 + 1.09 + 3 + 97.948$

PRACTICE EXERCISE 9c

Subtraction of Decimals—Vertical

Subtract:

1.	2.	3.	4.	5.
.87	7.6	5.1	26.85	7.871
.45	3.8	2.8	18.79	1.792

6.	7.	8.	9.	10.
1.06	3.191	9.01	24.1	75.
.78	1.06	1.872	9.27	18.23

11.	12.	13.	14.	15.
33.	7.	18.	3.03	1.
9.176	.1	.018	.86	.39

16.	17.	18.	19.	20.
17.06	.213	761.8	.8	1.2
7.0848	.018	4.76	.148	.8745

PRACTICE EXERCISE 9d

Subtraction of Decimals—Horizontal

Subtract:

1. 9.6 − 1.4	**2.** .38 − .07	**3.** 4.8 − 3.27
4. 18.8 − 1.9	**5.** 1.76 − .39	**6.** 8 − 1.4
7. 8.7 − 3.9	**8.** 7.87 − 6.3$\frac{1}{4}$	**9.** 24 − .8
10. 62.7$\frac{1}{2}$ − 3.9	**11.** 35.18 − .719	**12.** 721.32 − 146.7
13. 82$\frac{1}{2}$ − 2.4	**14.** 12.6 − 8$\frac{1}{3}$	**15.** 100 − .37
16. 7.14 − 3.872	**17.** 200 − 4.5	**18.** 150 − .368

You should be able to pass *Inventory Test 9* now.

MULTIPLICATION OF DECIMALS

Multiplication of decimal fractions. Decimal fractions are multiplied exactly like integers. After multiplying, determine the location of the decimal point in the product as follows:

Beginning at the right-hand side of the product, point off as many decimal places as the sum of the number of decimal places in the multiplicand and the multiplier.

Example

$$\begin{array}{rl} 3.46 & \text{(2 decimal places)} \\ \underline{2.8} & \text{(1 decimal place)} \\ 2768 & \\ \underline{692} & \\ 9.688 & \text{(3 decimal places)} \end{array}$$

PRACTICE EXERCISE 10a

Multiply:

1. .067 × 8	**2.** 1.7 × .9	**3.** 2.68 × .01	**4.** 8.4 × .001	**5.** 7.8 × 100
6. 60.1 × .1	**7.** 39.2 × 1000	**8.** 1.865 × .08	**9.** 6.7 × .04	**10.** 7.57 × .09
11. 8.64 × .28	**12.** .723 × .46	**13.** 9.36 × .83	**14.** 45.72 × 7.2	**15.** 7.03 × 2.08
16. .175 × 400	**17.** .0675 × 8.2	**18.** .004 × .002	**19.** 1.725 × .28	**20.** 7.86 × 10
21. 7.48 × 100	**22.** .764 × 3.7	**23.** 1.04 × 20.7	**24.** 72.5 × 4.9	**25.** 6.78 × 3.7

PRACTICE EXERCISE 10b

Multiply:

1. 10 × .74	**2.** 42 × .001	**3.** 3.15 × 1000
4. 8 × .082	**5.** .125 × .01	**6.** 7.8 × .8
7. 3.9 × 10	**8.** 1.9 × .1	**9.** 74 × .001

10. $.65 \times 1000$	11. $100 \times .1$	12. 16.16×100
13. $30 \times .3$	14. $78 \times .44$	15. 2.5×2.5
16. $.6 \times 6$	17. $.139 \times 100$	18. $1.08 \times .2$
19. 1.02×10.2	20. $80 \times .80$	21. $.006 \times 10$
22. 1000×7.2	23. 18×1.8	24. 2.3456×100
25. $.009 \times .08$	26. $1.05 \times .105$	27. $.11 \times .11$
28. $.127 \times .06$	29. $10 \times .867$	30. $52.64 \times .08$

You should pass *Inventory Test 10* now.

DIVISION OF DECIMALS

The division of decimals may be performed in the same manner as the division of integers. Multiply the divisor by some multiple of 10 that will make the divisor an integer. The dividend must be multiplied by that same multiple of 10. Multiplying both the divisor and the dividend by the *same* number does not affect the value of the quotient.

Place the decimal point in the quotient directly above the decimal point in the dividend.

Example A

$450 \div .72$

$.72 \times 100 = 72$

$450 \times 100 = 45000$

$$\begin{array}{r} 625 \\ 72\overline{)45000} \\ \underline{432} \\ 180 \\ \underline{144} \\ 360 \\ \underline{360} \end{array}$$

Example B

$.876 \div 2.8$

$2.8 \times 10 = 28$

$.876 \times 10 = 8.76$

$$\begin{array}{r} .31\frac{2}{7} \\ 28\overline{)8.76} \\ \underline{8\,4} \\ 36 \\ \underline{28} \\ \frac{8}{28} = \frac{2}{7} \end{array}$$

The same result may be obtained as follows: Before beginning the process of division, (1) move the decimal point of the divisor to the right of the divisor; (2) move the decimal point of the dividend to the right the same number of places, adding any necessary zeros; (3) place the decimal point in the quotient directly above the decimal point in the dividend.

For each place the decimal point is moved to the *right*, the value of the decimal is multiplied by 10.

Example C

$450 \div .72$

$$\begin{array}{r} 625. \\ 72.\overline{)45000.} \\ \underline{432} \\ 180 \\ \underline{144} \\ 360 \\ \underline{360} \end{array}$$

Example D

$.876 \div 2.8$

$$\begin{array}{r} .31\frac{2}{7} \\ 2\,8.\overline{)8.76} \\ \underline{8\,4} \\ 36 \\ \underline{28} \\ \frac{8}{28} = \frac{2}{7} \end{array}$$

PRACTICE EXERCISE 11a

Short Division of Decimals

Estimate each quotient and then divide:

1. $4\overline{)7.2}$	2. $.3\overline{)8.4}$	3. $.04\overline{)6.4}$	4. $20\overline{)8}$
5. $.03\overline{).6}$	6. $.003\overline{).06}$	7. $.3\overline{).006}$	8. $300\overline{).6}$

9. $.04\overline{)8}$ 10. $.4\overline{).8}$ 11. $40\overline{).8}$ 12. $4\overline{).08}$

13. $400\overline{)8}$ 14. $400\overline{).8}$ 15. $4000\overline{).8}$ 16. $.004\overline{).8}$

17. $5\overline{).1}$ 18. $.5\overline{).01}$ 19. $.05\overline{).1}$ 20. $.05\overline{)1}$

PRACTICE EXERCISE 11b

Short Division of Decimals

Estimate the quotient and then divide:

1. $.6\overline{)2526}$ 2. $.02\overline{)846}$ 3. $.7\overline{)7.49}$ 4. $.1\overline{).09}$

5. $.200\overline{).0938}$ 6. $.9\overline{)24.016}$ 7. $.04\overline{)1.124}$ 8. $.2\overline{).8}$

9. $9\overline{)708.3}$ 10. $300\overline{).012}$ 11. $30\overline{)4.62}$ 12. $.3\overline{).009}$

13. $.6\overline{)12.12}$ 14. $20\overline{).08}$ 15. $.3\overline{)4.62}$ 16. $.06\overline{).2418}$

17. $8\overline{)88.8}$ 18. $.005\overline{).02}$ 19. $.09\overline{)27.27}$ 20. $.4\overline{)288}$

PRACTICE EXERCISE 11c

Long Division of Decimals

Point off the quotients. Prefix or annex zeros if needed. Divide:

1. $\begin{array}{r} 274 \\ .034\overline{)9.316} \end{array}$ 2. $\begin{array}{r} 127 \\ 2.12\overline{)2.6924} \end{array}$ 3. $\begin{array}{r} 343 \\ 41.2\overline{)141.316} \end{array}$

4. $\begin{array}{r} 25 \\ 7.17\overline{)1792.5} \end{array}$ 5. $\begin{array}{r} 34 \\ 162\overline{)5.508} \end{array}$ 6. $\begin{array}{r} 324 \\ 2.12\overline{)68.688} \end{array}$

7. $\begin{array}{r} 509 \\ 23.2\overline{)1180.88} \end{array}$ 8. $\begin{array}{r} 128 \\ 38.1\overline{)4876.8} \end{array}$ 9. $\begin{array}{r} 286 \\ .723\overline{)206.778} \end{array}$

10. $\begin{array}{r} 1283 \\ 8.4\overline{)1077.72} \end{array}$ 11. $\begin{array}{r} 3107 \\ .72\overline{)22.3704} \end{array}$ 12. $\begin{array}{r} 91 \\ .86\overline{).7826} \end{array}$

13. $\begin{array}{r} 78 \\ 8.8\overline{)68.64} \end{array}$ 14. $\begin{array}{r} 101 \\ 7.3\overline{)737.3} \end{array}$ 15. $\begin{array}{r} 674 \\ .023\overline{)1.5502} \end{array}$

16. $7.98 \div 10$ 17. $469 \div 100$ 18. $.06 \div 10$

19. $34.6 \div 1000$ 20. $.5 \div 100$ 21. $56.78 \div 100$

22. $.05 \div 100$ 23. $7.4 \div 100$ 24. $500 \div 1000$

You should be able to pass *Inventory Test 11* now.

PRACTICE EXERCISE 12a

Changing decimal fractions to per cents. *Per cent* is merely another word for *hundredths*. Hundredths include the first two places to the right of the decimal point; likewise, per cent includes the first two places to the right of the decimal point. Hence, to change a decimal fraction to a per

cent, express the decimal fraction as hundredths, omit the word *hundredths*, and write *per cent* to the right of the number. The following examples illustrate the process:

$$.35 = 35 \text{ hundredths} = 35 \text{ per cent} = 35\%$$
$$.09 = 9 \text{ hundredths} = 9 \text{ per cent} = 9\%$$
$$.1 = .10 = 10 \text{ hundredths} = 10 \text{ per cent} = 10\%$$
$$.375 = 37.5\ (37\tfrac{1}{2}) \text{ hundredths} = 37\tfrac{1}{2} \text{ per cent} = 37\tfrac{1}{2}\%$$
$$.1125 = 11\tfrac{1}{4} \text{ hundredths} = 11\tfrac{1}{4} \text{ per cent} = 11\tfrac{1}{4}\%$$
$$1.7 = 1.70 = 170 \text{ hundredths} = 170 \text{ per cent} = 170\%$$

Change these decimal fractions to per cents:

1. .18	**2.** .75	**3.** .42	**4.** .64	**5.** .03
6. .05	**7.** .01	**8.** .08	**9.** .175	**10.** .256
11. .625	**12.** .5625	**13.** .8	**14.** .1	**15.** .7275
16. 1.35	**17.** 1.18	**18.** 2.0625	**19.** 3.125	**20.** 4.0
21. 1.3	**22.** 2.5	**23.** $.9\frac{1}{2}$	**24.** $.6\frac{1}{4}$	**25.** $1.3\frac{1}{3}$
26. $.14\frac{1}{2}$	**27.** $.3\frac{1}{8}$	**28.** 2.2	**29.** .002	**30.** 1.005
31. .166	**32.** 1.08	**33.** 1.15	**34.** .2	**35.** $.12\frac{1}{2}$
36. .375	**37.** 1.625	**38.** 1.5	**39.** .025	**40.** 2.25

PRACTICE EXERCISE 12b

Changing common fractions to per cents. To change a common fraction to a per cent, first express the common fraction as hundredths in decimal form and then as per cent. For example:

$$\tfrac{1}{4} = .25 = 25\% \qquad \tfrac{4}{5} = .80 = 80\%$$
$$\tfrac{1}{16} = .06\tfrac{1}{4} = 6\tfrac{1}{4}\% \qquad \tfrac{5}{9} = .55\tfrac{5}{9} = 55\tfrac{5}{9}\%$$
$$1\tfrac{1}{2} = 1.50 = 150\%$$

Change each of these common fractions and mixed numbers to a per cent:

1. $\frac{1}{2}$	**2.** $\frac{3}{4}$	**3.** $\frac{1}{5}$	**4.** $\frac{3}{10}$	**5.** $\frac{7}{20}$
6. $\frac{1}{8}$	**7.** $\frac{6}{25}$	**8.** $\frac{1}{3}$	**9.** $\frac{4}{9}$	**10.** $\frac{1}{6}$
11. $\frac{3}{16}$	**12.** $\frac{7}{8}$	**13.** $\frac{4}{15}$	**14.** $\frac{1}{50}$	**15.** $\frac{1}{7}$
16. $\frac{2}{3}$	**17.** $\frac{4}{7}$	**18.** $\frac{5}{8}$	**19.** $\frac{5}{6}$	**20.** $\frac{7}{10}$
21. $\frac{3}{8}$	**22.** $\frac{3}{5}$	**23.** $\frac{3}{7}$	**24.** $\frac{1}{9}$	**25.** $1\frac{1}{2}$
26. $2\frac{2}{5}$	**27.** $1\frac{1}{4}$	**28.** $1\frac{5}{16}$	**29.** $2\frac{1}{3}$	**30.** $1\frac{2}{3}$
31. $1\frac{3}{4}$	**32.** $1\frac{2}{5}$	**33.** $1\frac{7}{9}$	**34.** $\frac{5}{12}$	**35.** $\frac{7}{16}$
36. $\frac{5}{7}$	**37.** $2\frac{2}{3}$	**38.** $\frac{1}{30}$	**39.** $\frac{4}{25}$	**40.** $1\frac{1}{8}$

PRACTICE EXERCISE 12c

Changing per cents to decimal fractions. To change a per cent to a decimal fraction, think of the per cent as hundredths and then write it, using the decimal point to indicate hundredths.

$65\% = 65$ hundredths $= .65$ $\quad$ $3\frac{1}{2}\% = 3\frac{1}{2}$ hundredths $= .03\frac{1}{2} = .035$
$5\% = 5$ hundredths $= .05$ $\quad$ $150\% = 150$ hundredths $= 1.50$
$.7\% = .7$ hundredth $= .007$ $\quad$ $\frac{1}{4}\% = \frac{1}{4}$ hundredth $= .00\frac{1}{4} = .0025$

A fraction smaller than 1 per cent is often thought of as that fractional part of 1%:

$\frac{1}{2}\%$ is really $\frac{1}{2}$ of 1% $\quad$ .8% is .8 of 1%
$1\% = .01$ $\quad$ $.8\% = .8$ of $.01 = .008$
$\frac{1}{2}\% = \frac{1}{2}$ of $.01 = .00\frac{1}{2} = .005$

Change these per cents to decimal fractions:

1. 50%	**2.** 75%	**3.** 18%	**4.** 20%
5. 10%	**6.** 2%	**7.** 9%	**8.** 7%
9. $2\frac{1}{2}\%$	**10.** $3\frac{1}{4}\%$	**11.** $1\frac{3}{4}\%$	**12.** $12\frac{1}{2}\%$
13. $18\frac{3}{4}\%$	**14.** 6%	**15.** 1%	**16.** $16\frac{2}{3}\%$
17. $17\frac{1}{2}\%$	**18.** $8\frac{1}{3}\%$	**19.** $6\frac{1}{4}\%$	**20.** $83\frac{1}{3}\%$
21. $62\frac{1}{2}\%$	**22.** $37\frac{1}{2}\%$	**23.** $\frac{1}{2}\%$	**24.** $\frac{3}{4}\%$
25. 140%	**26.** 250%	**27.** 180%	**28.** 235%
29. $6\frac{2}{3}\%$	**30.** 4.5%	**31.** 3%	**32.** 1.5%
33. $28\frac{4}{7}\%$	**34.** $1\frac{1}{2}\%$	**35.** $\frac{3}{8}\%$	**36.** 1.85%
37. .8%	**38.** .01%	**39.** $166\frac{2}{3}\%$	**40.** 325%
41. 125%	**42.** 36%	**43.** 4.5%	**44.** .9%
45. $\frac{1}{4}\%$	**46.** $3\frac{1}{3}\%$	**47.** $1\frac{1}{4}\%$	**48.** 160%

PRACTICE EXERCISE 12d

Changing per cents to common fractions. To change a per cent to a common fraction, first change the per cent to a decimal fraction; then change the decimal to a common fraction and reduce it to the lowest terms.

$$35\% = .35 = \tfrac{35}{100} = \tfrac{7}{20}$$
$$3\% = .03 = \tfrac{3}{100}$$
$$7\tfrac{1}{2}\% = .07\tfrac{1}{2} = .075 = \tfrac{75}{1000} = \tfrac{3}{40}$$
$$.7\% = .007 = \tfrac{7}{1000}$$
$$\frac{1}{4}\% = .00\frac{1}{4} = .0025 = \frac{25}{10{,}000} = \frac{1}{400}$$
$$\frac{1}{3}\% = .00\frac{1}{3} = \frac{\frac{1}{3}}{100} = \frac{1}{3} \times \frac{1}{100} = \frac{1}{300}$$
$$1.5\% = .015 = \frac{15}{1{,}000} = \frac{3}{200}$$
$$240\% = 2.40 = 2\tfrac{40}{100} = 2\tfrac{2}{5}$$

Change these per cents to common fractions and reduce to lowest terms:

1. 20%	**2.** 15%	**3.** 75%	**4.** 55%
5. 5%	**6.** 2%	**7.** 1%	**8.** 7%
9. $3\frac{1}{2}\%$	**10.** $4\frac{1}{4}\%$	**11.** $5\frac{3}{4}\%$	**12.** $1\frac{1}{8}\%$
13. $\frac{1}{2}\%$	**14.** $\frac{1}{5}\%$	**15.** $\frac{3}{8}\%$	**16.** $\frac{4}{5}\%$

17. $.6\%$	18. 8%	19. $.1\%$	20. $.5\%$
21. $\frac{1}{7}\%$	22. $\frac{4}{9}\%$	23. $\frac{2}{3}\%$	24. $\frac{5}{6}\%$
25. $\frac{1}{4}\%$	26. 2.8%	27. 2.5%	28. 1.25%
29. 5.3%	30. $3.2\frac{1}{2}\%$	31. $15.6\frac{1}{4}\%$	32. 24.5%
33. 250%	34. 180%	35. 160%	36. 200%
37. 375%	38. 225%	39. 175%	40. $212\frac{1}{2}\%$
41. 45%	42. 18%	43. 150%	44. $2\frac{1}{2}\%$
45. 240%	46. $\frac{1}{8}\%$	47. $.3\%$	48. $2\frac{1}{4}\%$

PRACTICE EXERCISE 12e

Common fractions and their equivalent per cents to be memorized. Certain common fractions are used so frequently that it is important to memorize their equivalent per cents. See Appendix D.

Express each of the following as a common fraction, as a decimal fraction, and as a per cent:

	Common Fraction	*Decimal Fraction*	*Per Cent*
1.	$\frac{9}{10}$	--------	--------
2.	$\frac{3}{5}$	--------	--------
3.	--------	.16	--------
4.	--------	.09	--------
5.	--------	--------	4%
6.	--------	--------	12%
7.	--------	$.87\frac{1}{2}$	--------
8.	$1\frac{1}{2}$	--------	--------
9.	--------	--------	130%
10.	--------	.045	--------
11.	--------	$.14\frac{2}{7}$	--------
12.	--------	--------	$83\frac{1}{3}\%$
13.	--------	--------	$12\frac{1}{2}\%$
14.	$\frac{7}{9}$	--------	--------
15.	--------	1.25	--------
16.	--------	.175	--------
17.	$\frac{8}{25}$	--------	--------
18.	--------	--------	175%
19.	$\frac{1}{100}$	--------	--------
20.	--------	--------	$\frac{2}{5}\%$

PRACTICE EXERCISE 12f

Write the common fraction or the per cent, whichever is missing:

1. $\frac{2}{5} = ?$	7. $3\frac{1}{2} = ?$	13. $\frac{1}{3} = ?$
2. $? = 30\%$	8. $\frac{1}{16} = ?$	14. $\frac{4}{5} = ?$
3. $? = 87\frac{1}{2}\%$	9. $\frac{7}{9} = ?$	15. $? = 70\%$
4. $\frac{1}{12} = ?$	10. $? = 66\frac{2}{3}\%$	16. $? = 83\frac{1}{3}\%$
5. $? = 16\frac{2}{3}\%$	11. $? = 14\frac{2}{7}\%$	17. $\frac{2}{7} = ?$
6. $? = 12\frac{1}{2}\%$	12. $\frac{5}{8} = ?$	18. $? = 2\frac{1}{2}\%$

You should now be able to pass *Inventory Test 12.* Try it.

PRACTICE EXERCISE 13a

Finding a Per Cent of a Number

Factor × factor = product.

Examples

6% of $24 = .06 \times 24 = 1.44$
140% of $60 = 1.4 \times 60 = 84$

Solve these exercises:

1. 5% of 36
2. 20% of 85
3. 17% of 60
4. 51% of 79
5. 73% of 12
6. 11% of 150
7. 9% of 40
8. 8% of 24
9. 40% of 15
10. 45% of 20
11. 20% of 72
12. 25% of 400
13. 1% of 50
14. $\frac{1}{2}\%$ of 400
15. 2% of 80
16. .1% of 60
17. 150% of 400
18. 175% of 200
19. 2.5% of 300
20. 200% of 82
21. 48% of 35
22. 18% of 65
23. $\frac{1}{4}\%$ of 200
24. .6% of 50
25. 17% of 120
26. 180% of 90
27. 79% of 500
28. $2\frac{1}{2}\%$ of 40
29. $7\frac{1}{4}\%$ of 80
30. $1\frac{3}{4}\%$ of 60

PRACTICE EXERCISE 13b

Finding a Per Cent of a Number

Factor × factor = product.

Examples

$$16\frac{2}{3}\% \text{ of } 72 = \frac{1}{\cancel{6}} \times \overset{12}{\cancel{72}} = 12$$

$$6\frac{1}{4}\% \text{ of } 320 = \frac{1}{\cancel{16}} \times \overset{20}{\cancel{320}} = 20$$

Using the equivalent fraction, solve each of these exercises:

1. $33\frac{1}{3}\%$ of 36
2. $12\frac{1}{2}\%$ of 48
3. 20% of 60
4. $87\frac{1}{2}\%$ of 16
5. $66\frac{2}{3}\%$ of 75
6. $83\frac{1}{3}\%$ of 42
7. $37\frac{1}{2}\%$ of 56
8. 60% of 25
9. $14\frac{2}{7}\%$ of 21
10. $62\frac{1}{2}\%$ of 160
11. 10% of 800
12. $8\frac{1}{3}\%$ of 24
13. 25% of 420
14. $16\frac{2}{3}\%$ of 18
15. 40% of 35
16. 75% of 12
17. $28\frac{4}{7}\%$ of 28
18. $11\frac{1}{9}\%$ of 18
19. $41\frac{2}{3}\%$ of 24
20. 90% of 270
21. 30% of 60
22. $6\frac{1}{4}\%$ of 480
23. $22\frac{2}{9}\%$ of 270
24. $42\frac{6}{7}\%$ of 14

25. $18\frac{3}{4}\%$ of 64
26. 110% of 70
27. 125% of 16
28. $137\frac{1}{2}\%$ of 80
29. $133\frac{1}{3}\%$ of 36
30. 120% of 45

PRACTICE EXERCISE 13c

Find What Per Cent One Number Is of Another Number

Example A

14 = ?% of 28

The product (14) and one factor (28) are known. The missing factor can be found by this rule:

Product ÷ known factor = missing factor

14 (product) ÷ 28 (known factor) = .50 or 50% (missing factor)

The division may be in the fraction form, as $\frac{14 \text{ (product)}}{28 \text{ (factor)}} = \frac{1}{2} = .50 = 50\%$ (missing factor).

Example B

25% of ? = 84

84 (product) ÷ .25 (known factor) = 336 (missing factor)

Solve these exercises:

1. 8 = ?% of 8
2. 8 = ?% of 12
3. ?% of 100 = 5
4. ?% of 50 = 25
5. 75% of ? = 150
6. 20% of ? = 80
7. 9 = ?% of 3
8. 12 = ?% of 36
9. 10 = ?% of 40
10. ?% of 80 = 50
11. ?% of 50 = 7
12. 15% of ? = 30
13. 8% of ? = 2
14. 3% of ? = 126
15. 2% of ? = 88
16. ?% of 8 = 16
17. 15 = ?% of 45
18. 40% of ? = 32
19. 150% of ? = 90
20. 200% of ? = 72
21. $\frac{1}{2}\%$ of ? = 45
22. ?% of 16 = 3
23. 8 = ?% of 200
24. 20 = ?% of 8
25. ?% of 25 = 75
26. ?% of 20 = 6
27. 1 = ?% of 8
28. $\frac{1}{2}$ = ?% of 2
29. 16 = ?% of 24
30. ?% of 5 = 3
31. 9 = ?% of 15
32. 10 = ?% of 14
33. 105% of ? = 420
34. 3 = ?% of 3
35. 21% of ? = 42
36. ?% of 20 = 45
37. 125% of ? = 36
38. 2 = ?% of 14
39. 100% of ? = 22
40. ?% of 16 = 1
41. ?% of 12 = 1
42. $\frac{1}{3}$ = ?% of 1
43. 100% of ? = 70
44. 7 = ?% of 50
45. $\frac{1}{4}\%$ of ? = 8

PRACTICE EXERCISE 13d

Per Cent Exercises

Fill in the missing numbers:

1. 30% of $25 = ?$
2. $?\%$ of $30 = 5$
3. 10% of $? = 6$
4. $33\frac{1}{3}\%$ of $186 = ?$
5. $?\%$ of $50 = 15$
6. $18 = ?\%$ of 45
7. $15 = ?\%$ of 60
8. 7% of $50 = ?$
9. 28% of $100 = ?$
10. 125% of $60 = ?$
11. $2 = ?\%$ of 10
12. $\frac{1}{2}\%$ of $800 = ?$
13. 60% of $? = 12$
14. $33\frac{1}{3}\%$ of $? = 9$
15. 2.5% of $8 = ?$
16. $3\frac{1}{2}\%$ of $6 = ?$
17. 12% of $? = 18$
18. 140% of $? = 28$
19. $?\%$ of $20 = 16$
20. $?\%$ of $32 = 8$
21. $5 = ?\%$ of 2
22. $6 = ?\%$ of 300
23. $62\frac{1}{2}\%$ of $64 = ?$
24. $?\%$ of $18 = 12$
25. 250% of $40 = ?$
26. $37\frac{1}{2}\%$ of $24 = ?$
27. $?\%$ of $12 = 2$
28. $.1\%$ of $? = 10$
29. $16\frac{2}{3}\%$ of $72 = ?$
30. $112\frac{1}{2}\%$ of $32 = ?$

If you have done well on these *Practice Exercises,* you should be able to pass *Inventory Test 13.* Try it again.

The present business world demands a thorough mastery of percentage. Are you prepared to meet such a demand? If not, consult your teacher about any part of percentage that is not clearly understood by you and then continue your practice.

PRACTICE EXERCISE 14a

Reduction to Lower Units

Change these measures to the specified units:

> **Example A**
>
> 2 yd. 2 ft. = 6 ft. + 2 ft. = 8 ft.

1. 2 ft. 8 in. = ____ in.
2. 3 yd. = ____ ft.
3. 3 qt. = ____ pt.
4. 5 gal. = ____ qt.
5. 3 lb. 6 oz. = ____ oz.
6. 2 bu. = ____ pk.
7. 2 yd. = ____ in.
8. 2 T. = ____ lb.
9. 15 articles = ____ doz.
10. 2 hr. 15 min. = ____ min.
11. 8 min. = ____ sec.
12. 3 sq. ft. = ____ sq. in.
13. 2 sq. yd. = ____ sq. ft.
14. 2 cu. yd. = ____ cu. ft.
15. 5 cu. yd. = ____ cu. ft.
16. 2 sq. mi. = ____ acres
17. 3 gal. = ____ cu. in.
18. 3 pk. 2 qt. = ____ qt.

> **Example B**
>
> $\frac{1}{4}$ hr. = $\frac{1}{4}$ of 60 min. = 15 min.

19. $\frac{3}{4}$ ft. = ____ in.
20. $\frac{3}{4}$ hr. = ____ min.
21. $\frac{3}{8}$ lb. = ____ oz.
22. $\frac{1}{2}$ mi. = ____ ft.
23. $\frac{1}{4}$ ton = ____ lb.
24. $\frac{1}{8}$ gal. = ____ pt.

25. $\frac{2}{3}$ yd. = ____ in.
26. $\frac{2}{3}$ ft. = ____ in.
27. $\frac{1}{2}$ bu. = ____ pk.
28. $\frac{1}{5}$ hr. = ____ min.
29. $\frac{1}{4}$ sq. ft. = ____ sq. in.
30. $\frac{1}{2}$ gal. = ____ qt.
31. $\frac{2}{3}$ yd. = ____ ft.
32. $\frac{2}{3}$ hr. = ____ min.
33. $\frac{1}{4}$ yd. = ____ in.
34. $\frac{1}{4}$ bu. = ____ pk.
35. $\frac{1}{2}$ qt. = ____ pt.
36. $\frac{7}{8}$ lb. = ____ oz.

PRACTICE EXERCISE 14b

Change to Higher Units

Example A

40 in. = ____ ft. ____ in.
$40 \div 12 = 3$ and 4 remainder
Hence, 40 in. = 3 ft. 4 in.

Example B

4 articles = ____ doz.
$\frac{4}{12} = \frac{1}{3}$
Hence, 4 articles = $\frac{1}{3}$ doz.

Express these measures in the specified units:

1. 22 in. = ____ ft. ____ in.
2. 7 ft. = ____ yd. ____ ft.
3. 90 min. = ____ hr. ____ min.
4. 10 qt. = ____ gal.
5. 15 pt. = ____ qt.
6. 24 oz. = ____ lb.
7. 100 min. = ____ hr.
8. 216 sq. in. = ____ sq. ft.
9. 75 sec. = ____ min.
10. 75 in. = ____ ft.
11. 6 qt. = ____ gal.
12. 6,000 lb. = ____ tons
13. 36 sq. ft. = ____ sq. yd.
14. 54 cu. ft. = ____ cu. yd.
15. 9 articles = ____ doz.
16. 18 oz. = ____ lb.
17. 9 in. = ____ ft.
18. 3 qt. = ____ gal.
19. 8 articles = ____ doz.
20. 27 in. = ____ yd.
21. 15 min. = ____ hr.
22. 54 in. = ____ yd.
23. 6 oz. = ____ lb.
24. 10 articles = ____ doz.

PRACTICE EXERCISE 14c

Mixed Reduction

Change these measures to the specified units:

1. 4 ft. 6 in. = ____ in.
2. 5 lb. 8 oz. = ____ oz.
3. $3\frac{1}{2}$ qt. = ____ pt.
4. 20 oz. = ____ lb.
5. 3 ft. 9 in. = ____ in.
6. 100 in. = ____ ft. ____ in.
7. $2\frac{1}{4}$ hr. = ____ min.
8. 150 sec. = ____ min.
9. 1 bu. 3 pk. = ____ pk.
10. $1\frac{3}{4}$ doz. = ____ articles
11. $\frac{3}{4}$ yr. = ____ mo.
12. 10 yd. = ____ ft.
13. 3 lb. = ____ oz.
14. 11 pk. = ____ bu. ____ pk.
15. 20 articles = ____ doz.
16. $2\frac{1}{3}$ doz. = ____ articles

17. $\frac{5}{8}$ lb. = ____ oz.

18. 7,500 lb. = ____ tons ____ lb.

19. $\frac{3}{4}$ sq. mi. = ____ acres

20. $\frac{1}{8}$ bu. = ____ pk.

Now try *Inventory Test 14* again.

PRACTICE EXERCISE 15a

Denominate Numbers—Addition

Add these units of measure. Reduce each sum to its simplest form:

Example

8 ft. 9 in.
+ 3 ft. 7 in.
11 ft. 16 in. =
12 ft. 4 in.

1. 2 ft. 3 in.
5 ft. 6 in.

2. 4 yd. 2 ft.
5 yd. 1 ft.

3. 7 yr. 10 mo.
4 yr. 2 mo.

4. 2 gal. 3 qt.
3 gal. 1 qt.

5. 30 min. 10 sec.
15 min. 20 sec.
10 min. 40 sec.

6. 18 ft. 9 in.
26 ft. 2 in.

7. 19 hr. 18 min.
16 hr. 17 min.

8. 28 min. 31 sec.
14 min. 28 sec.

9. 42 lb. 13 oz.
76 lb. 9 oz.

10. 8 gal. 2 qt.
13 gal. 3 qt.
17 gal. 2 qt.

11. 8 hr. 10 min.
5 hr. 15 min.

12. 80 mi. 185 rd.
75 mi. 172 rd.

13. 8 lb. 10 oz.
5 lb. 8 oz.

14. 8 qt. 1 pt.
4 qt. 1 pt.

15. 7 yd. 2 ft.
6 yd.
8 yd. 2 ft.

Now try *Inventory Test 15* again.

PRACTICE EXERCISE 16a

Denominate Numbers—Multiplication

Multiply these units of measure. Reduce each product to its simplest form.

1. 2 pk. 2 qt.
3

2. 8 hr. 10 min.
4

3. 3 T. 100 lb.
6

4. 10 yd. 2 ft.
5

5. 4 qt. 1 pt.
4

6. 3 ft. 8 in.
3

7. 10 min. 8 sec.
10

8. 2 mi. 40 rd.
8

9. 18 yd. 2 ft.
7

10. 9 qt. 1 pt.
9

11. 8 ft. 11 in.
7

12. 16 bu. 2 pk.
9

13. 18 mi. 96 rd.
5

14. 52 min. 45 sec.
10

15. 5 yd. 10 ft.
7

Try *Inventory Test 16* again.

PRACTICE EXERCISE 17a

Denominate Numbers—Subtraction

Subtract these units of measure:

1. 12 ft. 6 in.
8 ft. 8 in.

2. 4 pk. 1 qt.
1 pk. 3 qt.

3. 10 T. 1,000 lb.
7 T. 1,800 lb.

4. 40 min. 10 sec.
10 min. 20 sec.

5. 3 pk.
1 pk. 3 qt.

6. 8 ft.
2 ft. 7 in.

7. 45 pk. 3 qt.
16 pk. 7 qt.

8. 21 T. 800 lb.
16 T. 1,000 lb.

9. 70 min. 18 sec.
32 min. 35 sec.

10. 32 yd.
17 yd. 1 ft.

11. 31 hr. 30 min.
18 hr. 42 min.

Example

9 ft. 3 in. = 8 ft. 15 in.
− 4 ft. 8 in. = 4 ft. 8 in.
4 ft. 7 in.

12. 1,900 yr. 9 mo. 5 da.
1,879 yr. 6 mo. 18 da.

13. 18 hr. 10 min. 20 sec.
17 hr. 42 sec.

14. 13 gal. 1 qt.
5 gal. 2 qt. 1 pt.

15. 1,943 yr. 6 mo. 20 da.
1,908 yr. 8 mo. 25 da.

Try *Inventory Test 17* again.

PRACTICE EXERCISE 18a

Division of denominate numbers. In the division of a denominate number, there is often a remainder. If this remainder is not already expressed in the lowest possible unit of measure, it should be (a) changed to the next lower unit, (b) added to the like unit, if any, in the dividend, and (c) divided by the given divisor.

Example A

4 ft. $9\frac{2}{3}$ in.
3)14 ft. 5 in.

(a) 14 ft. ÷ 3 = 4 ft. and 2 ft. remainder.
(b) 2 ft. = 24 in. (changed to lower unit)
(c) 24 in. + 5 in. = 29 in. (added to like unit)
(d) 29 in. ÷ 3 = $9\frac{2}{3}$ in. (divided)
Quotient is 4 ft. $9\frac{2}{3}$ in.

Example B

$$\begin{array}{r} 0 \text{ yd. } 2 \text{ ft. } 6 \text{ in.} \\ 4\overline{)3 \text{ yd. } 1 \text{ ft.}\phantom{ 6 \text{ in.}}} \end{array}$$

(a) 3 yd. ÷ 4 = 0 yd. and 3 yd. remainder.
(b) 3 yd. = 9 ft. (changed to next lower unit)
(c) 9 ft. + 1 ft. = 10 ft. (added to like unit)
(d) 10 ft. ÷ 4 = 2 ft. and 2 ft. remainder
(e) 2 ft. = 24 in. (remainder changed to lower unit)
(f) 24 in. ÷ 4 = 6 in.
Quotient is 2 ft. 6 in.

Divide these units of measure:

1. $5\overline{)10 \text{ hr. } 30 \text{ min.}}$

2. $4\overline{)8 \text{ pk. } 5 \text{ qt.}}$

3. $6\overline{)24 \text{ min. } 10 \text{ sec.}}$

4. $3\overline{)18 \text{ ft. } 8 \text{ in.}}$

5. $2\overline{)5 \text{ qt. } 1 \text{ pt.}}$

6. $3\overline{)4 \text{ yd. } 2 \text{ ft.}}$

7. $5\overline{)1 \text{ yd. } 2 \text{ ft.}}$

8. $4\overline{)18 \text{ yd. } 2 \text{ ft.}}$

9. $5\overline{)21 \text{ ft. } 5 \text{ in.}}$

10. $3\overline{)17 \text{ mi. } 100 \text{ rd.}}$

11. $4\overline{)42 \text{ min. } 4 \text{ sec.}}$

12. $3\overline{)8 \text{ bu. } 1 \text{ pk.}}$

13. $3\overline{)8 \text{ ft. } 7 \text{ in.}}$

14. $5\overline{)1 \text{ qt. } 1 \text{ pt.}}$

15. $4\overline{)6 \text{ mi. } 180 \text{ rd.}}$

Now try *Inventory Test 18* again if you have done well with this *Practice Exercise.* If you *have not* done well, practice more before attempting the test.

APPENDIX B

GLOSSARY OF MATHEMATICAL TERMS*

A

Absolute value: the value of a number without regard to the plus or minus sign preceding it.

Acute angle: an angle that contains less than 90 degrees.

Acute triangle: a triangle all of whose angles are acute angles.

Ad valorem duty: an import tax levied on goods based on their price at the port of shipment.

Altitude: vertical height.

Angle: opening between two intersecting lines at the point where they meet.

Annual: occurring every year.

Annuity: an assured income paid by an insurance company.

Approximate: nearly accurate.

Arc: any section of a curve.

Area: the number of square units in the surface of a plane figure.

Assessed value: the worth assigned to property for the purpose of taxation.

B

Balanced equation: one in which the left member and the right member are equal.

Bank discount: the interest on a loan that is taken from the face of the loan at the time the loan is made.

Base: the line or surface upon which a plane or a solid figure rests.

Beneficiary: the person who is to receive the proceeds from an insurance policy.

Bisect: to cut into two equal parts.

Brokerage: the commission paid for the buying and selling of stocks and bonds.

Budget: a statement of probable expenses based on an expected income.

C

Canceled check: a check that has been paid by the bank and is so indicated on the check.

Capital stock: the amount of money represented by the total face value of the shares of a corporation.

Check: a depositor's written order on a bank for the payment of a certain sum of money on demand.

Circle: a plane bounded by a curved line all points of which are equidistant from the center.

Circumference: the line that bounds a circle; the length of the line that bounds the circle.

Coefficient: a number placed as a multiplier of another quantity.

Collateral: personal property given to be held as security for the payment of a debt.

Common stock: stock that does not pay the holder a stated amount of dividend.

Compasses: an instrument used to draw circles and arcs of circles.

Compound interest: interest on both the principal and the accrued interest.

Concentric circles: two or more circles with the same center.

Cone: a solid figure with a curved surface reaching a point at one end and a circular base at the other.

Consumer: one who consumes or uses a product or service.

Corporation: a group of persons who have been granted a charter to manufacture or sell certain merchandise.

Credit: sold on trust; charged.

Cube: the product obtained when a number is multiplied twice by itself; a solid figure with 6 equal square faces.

*The definitions apply to the use of the words in this book.

Currency: paper money.

Cylinder: a solid figure with a curved surface on the side and two equal circles for bases.

D

Day of maturity: the day on which an obligation becomes due.

Decagon: a plane figure having ten straight sides and ten angles.

Decimal fraction: a part of a unit that is expressed by the use of a decimal point.

Deduction: the amount subtracted.

Deposit: money placed in a bank and subject to withdrawal by the depositor.

Depositor: one who puts money in a bank subject to withdrawal.

Depreciation: decrease in value.

Diagonal: a straight line drawn across a plane figure from one vertex to another vertex, the vertices being not consecutive.

Diameter: a straight line through the center of a circle dividing the circle into halves.

Digit: any one of the ten numerals, as, 1, 2, 3, and so on.

Dimension: extension in a single line or direction, as length, width, and thickness or depth.

Discount: a reduction from the list price of an article or an invoice; a deduction made as an interest charge for the loan of money.

Dissimilar terms: terms whose combinations of letters and exponents are not alike.

Dividend: the amount of the profits paid by a corporation to its stockholders; the amount of the earnings paid by a life insurance company to the holders of its participating policies.

Drawee: the person or organization on which an order is written.

Drawer: the person who writes a check or other demand for payment.

Duty: a government tax levied on the importation, exportation, or use and consumption of goods.

E

Employee: a person who works for another.

Equation: the statement that two quantities are equal in value.

Equilateral triangle: a triangle whose sides are equal in length.

Equity: amount paid on the principal when buying a home.

Equivalent: equal in value.

Estimate: to form an opinion as to certain values.

Evaluate: to find the numerical value of a quantity.

Excise tax: a duty levied on the manufacture, sale, or consumption of articles within a country.

Exemption: a release from some obligation or payment of money.

Extremes: the two outside quantities of a proportion.

F

Face of note: amount stated on a note.

Face of policy: the amount of insurance stated in the policy.

Factor: one of two or more numbers which, when multiplied together, give a certain product.

Formula: the expression of a rule by means of letters and numbers.

Frustum: the remainder of a pyramid or cone after the top has been cut off by a plane parallel to the base.

G

Graph: a representation by means of lines showing relationships.

Gross: whole; twelve dozen.

Gross income: entire receipts without any deductions.

H

Height: the vertical distance from the base to the top of an object.

Hemisphere: one half of a sphere.

Hexagon: a plane figure having six straight sides and six angles.

Horizontal: a position parallel to the horizon.

Hypotenuse: that side of a right-angle triangle that is opposite the right angle.

I

Income: earnings, profit, or interest, coming in regularly.

Income tax: a tax levied upon the yearly receipts or profits.

Indirect measurement: a method of finding the size of an object without applying a measuring instrument directly.

Indorsement: the writing on the back of a check that transfers the ownership of the paper.

Inheritance tax: a charge or duty levied upon the amount of money or property received by an heir.

Installment buying: paid for in a number of stated payments.

Insurance: a system by means of which one may protect himself or his family against a specific loss or damage.

Insured: the person whose life or property is protected from loss or damage.

Interest: money paid for the use of money.

Internal revenue: the Bureau of Internal Revenue is the department of the government which collects the tax on certain articles manufactured in the United States; a tax on articles manufactured (within the United States).

Inventory: an itemized list of merchandise on hand.

Investment: placing of money so as to receive income or profit.

Isosceles triangle: a triangle that has two equal sides.

L

Lateral area: the sum of the areas of the sides, not including the base or bases, of a solid figure.

Lateral face: one of the sides of a solid figure.

Latitude: the number of degrees, on the surface of the earth, that lie between a given place and the equator.

Left member (of an equation): the terms to the left of the equals sign.

Length: the measure of an object from end to end.

Levy: an amount to be raised; to assess a certain amount.

License: a legal permit or right, usually issued upon the payment of a fee.

Literal factor: any letter in an algebraic term.

Longitude: the number of degrees on the earth's surface that lie between a given place and the prime meridian.

Luxury tax: a duty levied by the government on the purchase price of nonessential articles.

M

Market value: the price that a property will bring when offered for sale.

Maximum: greatest quantity or degree attainable.

Means: the two inside quantities of a proportion.

Measurement: dimension.

Meridian: an imaginary circle extending around the earth through both poles.

Minimum: least quantity or degree possible.

Mortgage: assignment of property as security for debt.

N

Negative number: a number preceded by a minus sign, expressing a quantity less than zero.

Negotiable: capable of being transferred from one owner to another.

Net income: the amount of money derived from labor, business, property, or capital remaining after all deductions have been subtracted.

Non-participating policy: a life insurance policy stating that the insured receives no dividends, that is, does not share in the profits of the insurance company.

Note or **promissory note:** a written promise to pay back the amount of a loan.

Numerical factor: in an algebraic term, the number that is a multiplier of the letters following it.

O

Obtuse angle: an angle that contains more than 90 degrees and less than 180 degrees.

Obtuse triangle: a triangle one of whose angles is obtuse.

Octagon: a plane figure having 8 straight sides and 8 angles.

Opposite side: in a triangle, the side that connects the two sides of the angle being considered.

Original: not copied.

Overhead: the operating expenses of a firm.

P

Parallel: lines an equal distance apart no matter how far extended.

Parallelogram: a 4-sided plane figure whose opposite sides are straight, equal, and parallel.

Participating policy: a life insurance policy stating that the insured receives dividends and so shares in the profits of the insurance company.

Par value: the original price stated on a bond or share of stock.

Payee: the person who receives or is to receive a sum of money.

Pentagon: a plane figure having 5 straight sides and 5 angles.

Per capita: for each person in a group.

Per cent: a value expressed in hundredths by means of the words *per cent* or a per cent (%) sign.

Perfect square: a product formed when a number is multiplied by itself.

Perimeter: the sum of the lengths of the sides of a plane figure.

Perpendicular: at right angles to a line or surface.

Pi ($\pi = 3.1416$): the ratio of the circumference of a circle to its diameter.

Plane: a flat surface—if any two points on it are joined by a straight line, the line will lie wholly in the surface.

Plane figure: a shape, drawn on a flat surface, that has length and width but no thickness.

Policy: a written agreement of insurance issued by an insurance company to the insured.

Polygon: a closed figure having any number of straight sides and angles.

Position: the arrangement of an object when in place.

Positive number: a number, sometimes preceded by a plus sign, expressing a quantity greater than zero.

Preferred stock: stock that pays the holder a stated amount of dividends.

Premium: the price of insurance for a stated period.

Proceeds: the amount of money received from a sale.

Promissory note: a written promise to pay a certain sum of money on a stated day to a certain person.

Property tax: a charge or duty based on the assessed value of a certain piece of property.

Protractor: instrument for measuring angles.

Pyramid: a solid figure having triangles for faces.

Q

Quadrilateral: any 4-sided plane figure having straight sides.

R

Radical sign: a mark ($\sqrt{}$) placed before a number to indicate that its square root is to be found.

Radii: (plural of *radius*).

Radius: a straight line from the center of a circle to its circumference.

Reconciliation: the act of making checkbook stubs agree with the bank statement.

Rectangle: a plane figure having 4 straight sides and 4 right angles.

Rectangular prism: a solid figure which has 6 sides, all of which are rectangles.

Registered bond: a bond on which the owner's name is recorded by the company.

Regular figure: a plane figure in which all sides are equal in length.

Retail: the sale of merchandise by a shopkeeper to a consumer.

Revenue: income.

Right angle: an angle that contains exactly 90 degrees.

Right member (of an equation): the terms to the right of the equals sign.

Right triangle: a triangle containing one right angle.

Ruler fractions: fractions usually marked off on a ruler, as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and so forth.

S

Salability: ease with which the item can be sold.

Sales tax: a tax collected by a state or city government from the seller and paid by the customer at the time of the sale.

Scale drawing: a drawing reduced in size with lines in the same proportion as in the original object.

Signature: the name of a person written by his own hand.

Signed number: a number preceded by a plus or minus sign.

Similar terms: two or more terms having the same combination of letters.

Slanting: in a position neither horizontal nor vertical.

Social Security tax: a per cent of a worker's salary added to a like amount paid by the employer and sent to the government, to be paid to retired workers as a pension.

Solid figure: a figure that has length, width, and thickness.

Specific duty: a tax on imported goods at a fixed amount per unit.

Sphere: a circular solid all points of whose surface are equally distant from the center.

Square: a rectangle all of whose sides are equal; the product obtained by multiplying a number by itself.

Square root: a number which, when multiplied by itself, gives a certain product; one of the two equal factors of a product.

Square unit: a square of a certain size that is used to determine the area of a surface.

Stockholder: owner of stock in a corporation.

Straight angle: an angle of 180 degrees.

Symbol: representation by a sign or letter.

T

Tariff: a list of duties levied by the government on goods coming into or going out of a country.

Tax: a compulsory charge or duty on income or property levied for the support of the government.

Tax rate: a fixed standard used to compute the tax to be paid by any individual.

Terms: quantities connected by plus or minus signs.

Thickness: the distance between two parallel sides of a solid figure.

Total area: the sum of the areas of the sides and bases of a solid figure.

Trapezoid: a plane figure with 4 straight sides only 2 of which are parallel.

Triangle: a plane figure having 3 straight sides and 3 angles.

Triangular prism: a solid figure with 3 rectangular faces and 2 parallel triangles for bases.

U

Unit of area: the measurement used to determine the amount of space in a plane figure.

Unknown number: the quantity to be found.

V

Valuation: estimated value.

Vertex: the point where the two sides of an angle meet.

Vertical: a position at right angles to the horizon.

Volume: the number of cubic units in a solid figure.

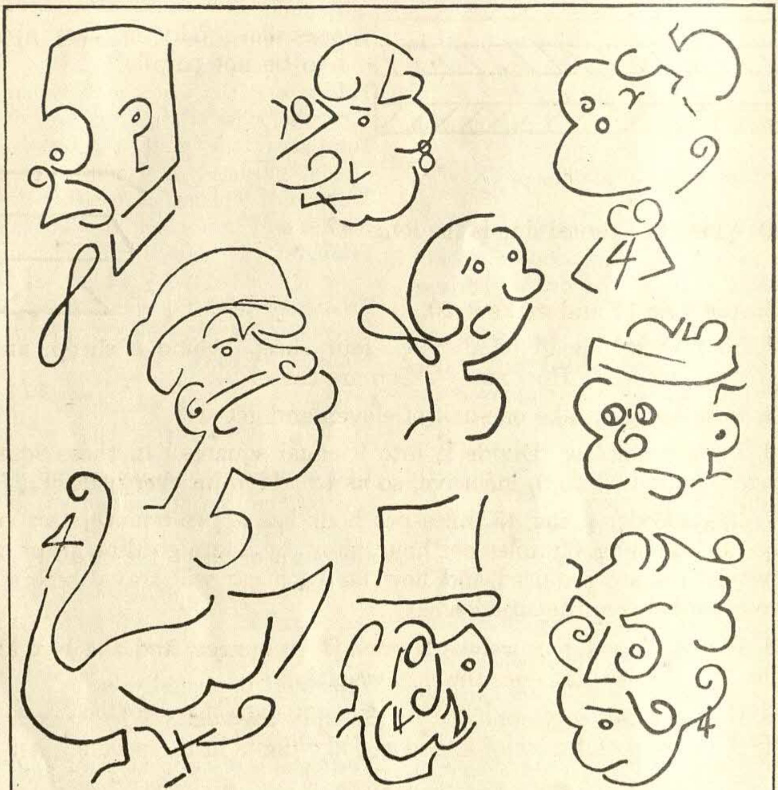
W

Width: the measure of an object from side to side.

Withdrawal: money taken from a depositor's bank account.

APPENDIX C

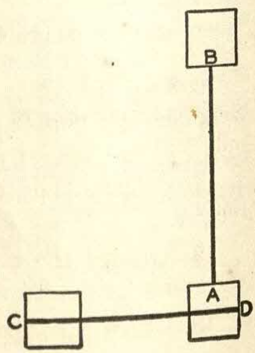
BRAIN TEASERS



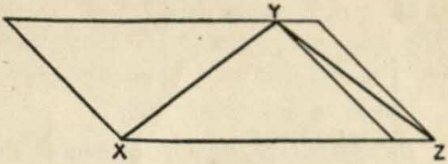
1. These "figureheads" are made from the digits, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Design several different "figureheads."

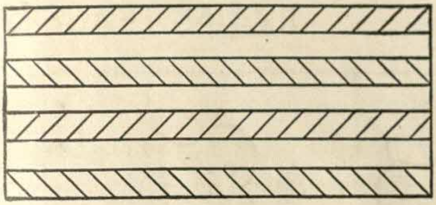
2. Do you agree that "seeing is believing"? Do not be too sure. Take a look at these drawings:

(a) How do *AB* and *CD* seem to compare in length? Now measure the lines.



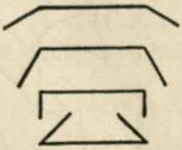
(b) In your judgment, which is the longer: XY or YZ? Measure to be sure.





(c) The horizontal lines are parallel. Why do they appear to be not parallel?

(d) Which horizontal line is the longest?



3. Add 1 to 11 and make it 20.

4. Four sheep ahead of a sheep, four sheep behind a sheep, and a sheep in the middle. How many sheep are there?

5. How can you take one-half of eleven and get six?

6. Draw a square. Divide it into 9 equal squares. In these squares arrange the digits 1 to 9, inclusive, so as to add 15 in every straight line.

7. If a speeder going 45 miles per hour has a two-minute start on a police car traveling 60 miles per hour, show by a straight-line graph how many minutes are required and how far each car will travel before the police car overtakes the speeder.

8. If a pup in a box weighs 2 pounds 10 ounces, and the box alone weighs 10 ounces, what does the pup weigh?

9. How far can you continue to multiply the digits 1 to 9, exclusive of the 8, by 9 or multiples of 9, and get like digits in the product, as:

$$\begin{array}{r}12345679\\ 9\\ \hline 111111111\end{array}\qquad\begin{array}{r}12345679\\ 18\\ \hline 222222222\end{array}\qquad\begin{array}{r}12345679\\ 27\\ \hline 333333333\end{array}$$

10. Two fishermen, each weighing 200 pounds, and their two sons, weighing 100 pounds each, cross a river in a small boat that will carry only 200 pounds. How do they all manage to get across?

11. What is the difference between a two-foot square and two square feet?

12. Arrange the digits 1 to 9, inclusive, in any order. Try dividing the result by 9.

13. Which is correct to say: "12 and 9 *is* 20" or "12 and 9 *are* 20"?

14. An old Indian offered to divide his 23 ponies among his three sons. The oldest was to receive one-half the ponies, the second son one-third, and the youngest one-eighth. How was the division made?

15. Prove that $960 = 1000$.

16. Can you determine the sum of the numbers 1 to 40 inclusive without adding them?

17. Take one from nine and get ten.

18. How many half-inch blocks will it take to fill an inch hole?

19. A $\frac{1}{2}$-mile square is what part of $\frac{1}{2}$ square mile?

20. How can you place three 9's so that they will exactly equal ten?

21. A pan and a lid cost $1.20. The pan cost $1 more than the lid. What is the cost of each?

22. How far can you continue to multiply 37 by multiples of 3 and have like digits in the products, as:

$$\begin{array}{r} 37 \\ \underline{3} \\ 111 \end{array} \qquad \begin{array}{r} 37 \\ \underline{6} \\ 222 \end{array} \qquad \begin{array}{r} 37 \\ \underline{9} \\ 333 \end{array}$$

23. Select any number. Reverse the order of its digits and subtract the two numbers. Try dividing the remainder by 9.

24. Write 20 with two equal figures.

25. John has 36 ears of corn in a box. How many trips will it take a squirrel to carry out all the corn, if it carries out 3 ears each trip?

26. Tom wishes to take his cat, his bird, and his dog across the river to his home. He can take but one at a time. The cat will eat the bird and the dog will kill the cat if they are left together. How can he get them all safely across?

27. A girl went to the pantry with a 5-cup and a 3-cup measure to get exactly 4 cups of sugar. How did she measure it?

28. If it takes 6 minutes to saw a log into 3 pieces, how long will it take to saw the same log into 4 pieces?

29. How many minutes will it take to cut 10 yards of cloth into one-yard towels, if one yard is cut off per minute?

30. Jack bought a bicycle for $30, sold it for $35, and then bought it back for $25. How much is his profit?

31. How much dirt is there in a hole 3 ft. by 2 ft. by 1 ft.?

ANSWERS TO BRAIN TEASERS

3. In XI cross the I with another I and the result is XX.

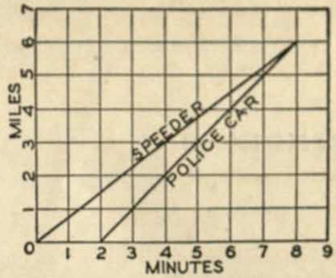
4. 5 sheep.

5. The upper half of XI is VI.

6.

4	3	8
9	5	1
2	7	6

7.



There are other solutions to **6**; however, the numbers will always be in the same relative positions.

8. 2 pounds 10 ounces.

9. Up through 81.

10. Two sons cross, and one son rows back. One man crosses, and other son rows back. Both sons cross, and one son rows back. Other man rows over, and other son rows back. Both sons cross.

11. A two-foot square is a square 2 feet on an edge. Two square feet is an area of any shape that contains 2 square feet.

12. The result will always be divisible by 9.

13. Neither is correct, for 12 and 9 are *21*.

14. The father borrowed a pony from a friend and placed it in the pen, making 24 ponies. The oldest son took one-half the ponies, or 12; the second son took 8, and the third son took 3. The father returned the borrowed pony, and all were satisfied.

15. In counting time, $9{:}60 = 10{:}00$.

16. The sum of each pair of end numbers, as 1 and 40, 2 and 39, and so forth, is 41 in each case. The 20 pairs of these numbers would add the same as $20 \times 41 = 820$.

17. Take I away from IX and X remains.

18. 8.

19. $\frac{1}{2}$.

20. $9\frac{9}{9}$.

21. The pan cost $1.10 and the lid $.10.

22. Up through 27.

23. The remainder will always be divisible by 9. This is used as a test by bookkeepers to locate mistakes in copying in correct order the numbers on the books.

24. XX.

25. 36 trips. He carries his own 2 ears each trip.

26. Tom takes the cat across; returns and takes the dog across; brings the cat back and takes the bird across. He leaves the bird with the dog and returns for the cat.

27. She fills the 3-cup measure and pours the sugar into the 5-cup measure. The 3-cup measure is filled again and enough of it is poured into the 5-cup measure to fill it, leaving 1 cup of sugar in the 3-cup measure. The 5-cup measure is emptied and the one cup of sugar is poured into it. The 3-cup measure is filled and poured into the 5-cup measure, making 4 cups of sugar.

28. 9 minutes.

29. 9 minutes.

30. $10.00.

31. No dirt.

APPENDIX D

USEFUL TABLES

BUSINESS COUNTING UNITS

12 units	=	1 dozen (doz.)
12 dozen	=	1 gross (gr.)
144 units	=	1 gross

BUSINESS WEIGHT UNITS

16 ounces (oz.)	=	1 pound (lb.)
100 pounds (lb.)	=	1 hundredweight (cwt.)
2,000 pounds	=	1 ton (T.)

LINEAR MEASURE

12 inches (in. *or* ″)	=	1 foot (ft. *or* ′)
3 feet	=	1 yard (yd.)
16½ feet	=	1 rod (rd.)
5½ yards	=	1 rod
5,280 feet	=	1 mile (mi.)
320 rods	=	1 mile

SQUARE MEASURE

144 square inches (sq. in.)	=	1 square foot (sq. ft.)
9 square feet	=	1 square yard (sq. yd.)
272¼ square feet	=	1 square rod (sq. rd.)
30¼ square yards	=	1 square rod
160 square rods	=	1 acre (A.)
640 acres	=	1 square mile (sq. mi.)

CUBIC MEASURE

1,728 cubic inches (cu. in.)	=	1 cubic foot (cu. ft.)
27 cubic feet	=	1 cubic yard (cu. yd.)

DRY MEASURE

2 pints (pt.)	=	1 quart (qt.)
8 quarts	=	1 peck (pk.)
4 pecks	=	1 bushel (bu.)

LIQUID MEASURE

4 gills (gi.)	=	1 pint (pt.)
2 pints	=	1 quart (qt.)
4 quarts	=	1 gallon (gal.)
31½ gallons	=	1 barrel (bbl.)
2 barrels	=	1 hogshead (hhd.)

ANGLES AND ARCS

60 seconds (″)	=	1 minute (′)
60 minutes	=	1 degree (°)
90 degrees	=	1 right angle
360 degrees	=	1 circle

EQUIVALENTS

1 bushel	$= 2{,}150$ cu. in. or $1\frac{1}{4}$ cu. ft. (approximately)
1 gallon	$= 231$ cu. in.
1 cubic foot	$= 7\frac{1}{2}$ gallons
1 cubic foot water	$= 62\frac{1}{2}$ pounds
1 cubic foot ice	$= 57\frac{1}{2}$ pounds
π	$= 3.1416$ or $3\frac{1}{7}$ (approximately)
1 meter	$= 39.37$ inches or 1.1 yd.
1 centimeter	$= .4$ inch (approximately)
1 decimeter	$= 4$ inches
1 kilometer	$= .6$ mile or $\frac{5}{8}$ mile (approximately)
1 yard	$= .914$ meter
1 mile	$= 1.609$ kilometers

FORMULAS

$A = bh$, area of a rectangle.
$A = s^2$, area of a square.
$A = bh$, area of a parallelogram.
$A = \frac{1}{2}\,bh$, area of a triangle.
$A = \frac{1}{2}\,h(b + b')$, area of a trapezoid.
$A = \pi r^2$, area of a circle.
$C = \pi d$ or $2\,\pi r$, circumference of a circle.
$A = 4\,\pi r^2$, area of the surface of a sphere.
$S = Ph$, lateral area of a rectangle.
$S = Ch$, or $2\,\pi rh$, lateral area of a cylinder.
$V = Bh$ or lwh, volume of a rectangular prism.
$V = Bh$ or πr^2h, volume of a cylinder.
$V = \frac{1}{3}\,Bh$ or $\frac{1}{3}\,\pi r^2h$, volume of a cone.
$V = \frac{1}{3}\,Bh$, volume of a pyramid.
$V = \frac{4}{3}\,\pi r^3$, volume of a sphere.
$A + B + C = 180°$, sum of the angles of a triangle.
$P = 2\,b + 2\,h$, perimeter of a rectangle.
$P = 4\,s$, perimeter of a square.
$P = a + b + c$, perimeter of a triangle.
$D = 16\,t^2$, distance covered by a falling object.
$a = \frac{S}{n}$, average of a sum.
$S = a \times n$, sum of numbers.
$p = .0005\,V^3$, pounds of pressure of air exerted upon wings of a moving airplane.
$C = \frac{F - 32°}{9} \times 5$, Fahrenheit to Centigrade.
$F = \frac{C}{5} \times 9 + 32°$, Centigrade to Fahrenheit.
$p = \frac{2\,wh}{s + 1}$, an engineer's formula.
$d = \frac{v^2}{2\,r}$, a mechanic's formula.
$L = d + \frac{2\,r^2}{5\,d}$, a pendulum formula.
$h = \frac{V}{b^2}$, height of a square prism.

COMMON PER CENTS AND THEIR EQUIVALENT FRACTIONS

1/64	.015625
1/32	.03125
3/64	.046875
1/16	.0625
5/64	.078125
3/32	.09375
7/64	.109375
1/8	.125
9/64	.140625
5/32	.15625
11/64	.171875
3/16	.1875
13/64	.203125
7/32	.21875
15/64	.234375
1/4	.25
17/64	.265625
9/32	.28125
19/64	.296875
5/16	.3125
21/64	.328125
11/32	.34375
23/64	.359375
3/8	.375
25/64	.390625
13/32	.40625
27/64	.421875
7/16	.4375
29/64	.453125
15/32	.46875
31/64	.484375
1/2	.5
33/64	.515625
17/32	.53125
35/64	.546875
9/16	.5625
37/64	.578125
19/32	.59375
39/64	.609375
5/8	.625
41/64	.640625
21/32	.65625
43/64	.671875
11/16	.6875
45/64	.703125
23/32	.71875
47/64	.734375
3/4	.75
49/64	.765625
25/32	.78125
51/64	.796875
13/16	.8125
53/64	.828125
27/32	.84375
55/64	.859375
7/8	.875
57/64	.890625
29/32	.90625
59/64	.921875
15/16	.9375
61/64	.953125
31/32	.96875
63/64	.984375
1	1.

345
9a-1-10
9b-1-8

APPENDIX E

ANSWERS TO PRACTICE EXERCISES

Practice Exercise 1a (page 331)

12	9	13	7	12	4	10	10	9	17	(10)
11	7	8	15	14	10	16	6	9	15	
14	7	17	9	13	9	11	9	10	8	(30)
6	6	12	8	8	10	11	14	8	5	
12	13	12	13	11	8	11	11	15	7	(50)
8	13	10	14	7	11	9	7	11	0	
10	15	16	12	14	10	18	12	16	13	(70)

Practice Exercise 1b (page 331)

24	23	27	25	15	
31	35	31	31	31	(10)
48	42	46	43	41	
54	52	50	53	51	(20)
64	63	63	66	65	
77	69	75	69	73	(30)
82	83	81	78	79	
93	91	88	94	91	(40)
104	102	102	101	105	
71	91	55	81	100	(50)

Practice Exercise 1c (page 332)

35	38	32	37	34	34	33	39	(8)
28	35	28	35	29	32	33	38	(16)
39	35	42	45	40	41	40	42	(24)

Practice Exercise 1d (page 332)

3,084	2,537	1,722	1,349	1,776	2,330	(6)
2,811	2,132	2,124	1,575	2,577	2,225	(12)
2,588	3,529	4,112	3,081	4,515	3,834	(18)
31,350	18,013	16,907	15,122	17,092	18,967	(24)

Practice Exercise 2a (page 333)

35	45	48	10	72	14	16	21	(8)
56	18	0	63	25	49	54	15	
0	30	32	0	15	24	14	28	(24)
0	42	18	72	12	20	0	16	
48	18	21	12	7	35	8	9	(40)
12	54	24	30	0	28	40	27	
40	27	0	18	6	20	24	10	(56)
42	9	36	12	16	56	63	36	
36	64	32	81	24	45	6	0	(72)

Practice Exercise 2b (page 333)

9,000	22,800	16,200	34,400	41,300	
48,000	35,000	1,000	36,000	54,000	(10)
1,060	12,320	56,400	45,810	42,560	
52,200	32,560	50,400	68,600	63,540	(20)
639,000	3,293,600	554,200	1,468,800		
863,583	916,458	2,907,630	1,522,048		(28)

Practice Exercise 2c (page 334)

31,110	103,824	278,718	42,840	217,536	
74,340	217,756	66,272	64,163	366,025	(10)
105,456	429,654	166,816	326,432	641,956	
428,542	824,464	145,728	359,664	10,201	(20)

Practice Exercise 2d (page 334)

24,928	32,956	37,196	59,677	(4)
23,199	32,214	49,898	10,960	
42,108	25,198	5,076	16,376	(12)
427,000	56,980	419,219	299,874	
21,654	40,948	584,576	180,072	(20)

Practice Exercise 3a (page 335)

2	1	4	4	6	5	1	1	7	3	(10)
0	0	6	3	1	9	3	2	2	0	(20)
2	6	4	7	2	0	5	6	3	5	(30)
1	4	5	3	3	1	2	0	4	0	(40)
8	8	0	0	1	5	1	4	3	7	(50)

Practice Exercise 3b (page 335)

20	32	42	51	18	11	
20	16	24	18	24	36	(12)
44	25	10	26	13	17	
13	24	31	10	49	30	(24)
4	4	7	8	8	7	
18	29	2	39	17	6	(36)
47	55	8	15	13	8	
7	9	12	5	16	6	(48)
24	48	65	24	9	6	
65	19	29	9	9	34	(60)

Practice Exercise 3c (page 335)

278	73	134	456	17	67	(6)
379	540	77	182	55	198	
297	175	13	328	196	399	(18)
432	263	136	476	397	21	
103	74	287	485	113	225	(30)

Practice Exercise 3d (page 336)

58,392	8,129	16,768	36,650	
10,378	30,381	16,667	61,274	(8)
6,449	7,402	2,316	14,013	
39,290	5,037	27,596	32,297	(16)
12,169	79,062	18,121	17,459	
16,018	4,667	25,295	48,056	(24)
14,465	5,806	25,255	11,015	
167,615	649,676	292,838	67,080	(32)

Practice Exercise 4a (page 336)

7	4	4	0	
2	8	8	3	(8)
4	4	2	5	

3	8	9	7	(16)
6	5	0	6	
5	2	7	0	(24)

9	2	6	5		6	9	7	6	(40)
6	3	0	7	(32)	8	9	4	8	
5	9	8	3		0	9	0	9	(48)

Practice Exercise 4b (page 337)

210	141	102	$50\frac{1}{2}$	$111\frac{1}{3}$	
141	120	114	104	$1178\frac{4}{5}$	(10)
182	$105\frac{2}{3}$	76	$41\frac{5}{6}$	1010	
$80\frac{5}{8}$	$50\frac{3}{5}$	$92\frac{1}{6}$	$90\frac{1}{2}$	$1022\frac{1}{3}$	(20)
97	116	$252\frac{1}{3}$	$89\frac{5}{6}$	1311	
$241\frac{2}{3}$	$100\frac{2}{3}$	$81\frac{1}{4}$	51	$1045\frac{4}{5}$	(30)
$181\frac{1}{2}$	$113\frac{5}{6}$	$89\frac{3}{7}$	$90\frac{1}{3}$	$627\frac{5}{6}$	
$50\frac{1}{2}$	$75\frac{1}{4}$	$75\frac{1}{2}$	$27\frac{5}{6}$	$769\frac{4}{5}$	(40)

Practice Exercise 4c (page 337)

25	27	76	82	
24	28	48	94	(8)
22	18	81	34	
19	15	72	52	(16)
16	12	36	50	
14	42	64	34	(24)

Practice Exercise 4d (page 338)

358	2001	$1173\frac{41}{72}$	
$715\frac{44}{63}$	$1559\frac{5}{12}$	$958\frac{7}{8}$	(6)
$1257\frac{28}{51}$	$819\frac{31}{35}$	$895\frac{11}{30}$	
$1108\frac{5}{19}$	2003	$1073\frac{1}{27}$	(12)
720	396	2630	
$127\frac{7}{17}$	$395\frac{53}{91}$	$296\frac{31}{62}$	(18)

Practice Exercise 5a (page 338)

$1\frac{1}{10}$		$1\frac{5}{8}$		$\frac{7}{12}$		$\frac{7}{24}$	
$\frac{3}{4}$		$1\frac{1}{2}$		$1\frac{1}{6}$		$\frac{8}{15}$	(8)
$1\frac{1}{12}$	$\frac{5}{8}$	$\frac{1}{2}$	$1\frac{5}{12}$	$\frac{11}{15}$	$\frac{13}{18}$	$1\frac{1}{4}$	(15)
$1\frac{17}{24}$	$\frac{13}{16}$	$1\frac{1}{6}$	$1\frac{23}{42}$	$1\frac{5}{24}$	$1\frac{13}{16}$	$1\frac{1}{36}$	(22)
$2\frac{1}{8}$			$2\frac{5}{18}$			$1\frac{21}{40}$	(25)

Practice Exercise 5b (page 339)

$\frac{5}{12}$	$\frac{7}{8}$	$1\frac{1}{40}$	$1\frac{5}{24}$	$\frac{23}{24}$	$1\frac{1}{2}$	$1\frac{1}{2}$	(7)
$1\frac{1}{16}$	$1\frac{1}{6}$	$1\frac{1}{12}$	$1\frac{1}{5}$	$1\frac{1}{6}$	$\frac{13}{24}$	$1\frac{1}{4}$	(14)
$1\frac{3}{5}$	$\frac{7}{8}$	1	$2\frac{11}{84}$	$1\frac{9}{20}$	$1\frac{3}{4}$	$1\frac{11}{12}$	(21)
$2\frac{1}{6}$	$1\frac{7}{15}$	$1\frac{7}{20}$	$1\frac{13}{28}$	$1\frac{3}{4}$	$1\frac{17}{24}$	$2\frac{1}{18}$	(28)

Practice Exercise 5c (page 339)

$1\frac{1}{2}$	$1\frac{1}{4}$		$1\frac{13}{24}$		$1\frac{13}{40}$	(4)
$1\frac{5}{14}$	$1\frac{5}{18}$		$\frac{7}{12}$		$1\frac{1}{4}$	
$\frac{1}{2}$	$\frac{17}{24}$		$1\frac{11}{15}$		$1\frac{1}{2}$	(12)
$1\frac{13}{30}$	$\frac{4}{9}$		$1\frac{3}{8}$		$1\frac{1}{30}$	
$1\frac{11}{12}$		$2\frac{1}{8}$		$1\frac{41}{60}$		(19)
$1\frac{9}{20}$		$1\frac{1}{8}$		$2\frac{5}{16}$		
$1\frac{2}{9}$		$1\frac{1}{16}$		$1\frac{5}{12}$		(25)

Practice Exercise 5d (page 339)

$11\frac{1}{6}$	$13\frac{4}{15}$	$14\frac{2}{9}$	$18\frac{3}{10}$	$16\frac{19}{24}$	$16\frac{1}{12}$	(6)
$17\frac{5}{6}$	$14\frac{3}{8}$	$18\frac{13}{14}$	$16\frac{1}{6}$	$14\frac{5}{9}$	$11\frac{4}{5}$	(12)
$5\frac{7}{12}$	$12\frac{11}{12}$	$10\frac{15}{16}$	$17\frac{4}{9}$	13	$14\frac{15}{16}$	(18)
$7\frac{11}{24}$	$14\frac{1}{6}$	$10\frac{5}{12}$	$13\frac{3}{8}$			(22)
$13\frac{1}{3}$	9	$8\frac{8}{15}$	3			(26)
$16\frac{11}{15}$	$10\frac{5}{12}$	$9\frac{7}{8}$	$10\frac{11}{12}$			(30)

Practice Exercise 5e (page 340)

$41\frac{1}{3}$	$72\frac{1}{2}$	$156\frac{5}{6}$	$142\frac{13}{20}$	$110\frac{4}{9}$	(5)
121	$73\frac{1}{2}$	$104\frac{3}{4}$	$170\frac{11}{15}$	$75\frac{5}{6}$	(10)
$33\frac{11}{16}$	$110\frac{2}{3}$	$106\frac{2}{5}$	$122\frac{13}{24}$	$74\frac{1}{3}$	(15)
$34\frac{7}{12}$	$40\frac{3}{4}$	$55\frac{13}{40}$			
45	$55\frac{11}{12}$	$65\frac{5}{6}$			(21)
$297\frac{17}{20}$	$405\frac{5}{6}$	$540\frac{1}{10}$	$466\frac{1}{5}$	$360\frac{8}{9}$	(26)
$1547\frac{5}{6}$	$802\frac{3}{14}$	$896\frac{8}{15}$	$1157\frac{1}{6}$	$765\frac{7}{12}$	(31)

Practice Exercise 6a (page 340)

$\frac{1}{4}$	$\frac{2}{15}$	$\frac{1}{4}$	$\frac{3}{20}$	$\frac{1}{6}$	$\frac{3}{8}$	
$\frac{5}{8}$	$\frac{1}{2}$	$\frac{7}{15}$	$\frac{1}{9}$	$\frac{5}{14}$	$\frac{19}{48}$	(12)
$\frac{3}{4}$	$\frac{5}{8}$	$\frac{1}{6}$	$\frac{5}{12}$	$\frac{1}{3}$	$\frac{7}{24}$	
$\frac{13}{30}$	$\frac{5}{16}$	$\frac{3}{10}$	$\frac{1}{24}$	$\frac{1}{4}$	$\frac{11}{16}$	(24)

Practice Exercise 6b (page 341)

$\frac{1}{2}$	$\frac{3}{8}$	$\frac{3}{5}$	$\frac{1}{2}$	
$\frac{5}{24}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{9}{16}$	(8)
$\frac{1}{6}$	$\frac{2}{5}$	$\frac{5}{12}$	$\frac{5}{12}$	
$\frac{1}{9}$	$\frac{7}{20}$	$\frac{7}{24}$	$\frac{2}{5}$	(16)
$\frac{5}{24}$	$\frac{13}{18}$	$\frac{5}{12}$	$\frac{11}{20}$	
$\frac{1}{6}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{13}{24}$	(24)

Practice Exercise 6c (page 341)

$6\frac{1}{3}$	$6\frac{1}{24}$	$5\frac{7}{10}$	$2\frac{11}{12}$	$\frac{4}{5}$	$4\frac{8}{15}$	
$6\frac{7}{8}$	$4\frac{5}{8}$	$5\frac{1}{2}$	$4\frac{13}{16}$	$5\frac{2}{7}$	$3\frac{1}{2}$	(12)
$3\frac{2}{5}$	$4\frac{23}{24}$	$7\frac{9}{10}$	$4\frac{4}{7}$	$3\frac{2}{3}$	3	
$5\frac{43}{60}$	$4\frac{11}{12}$	$\frac{3}{8}$	$4\frac{1}{10}$	$5\frac{13}{16}$	$5\frac{17}{20}$	(24)

Practice Exercise 6d (page 341)

$11\frac{1}{5}$	17	$20\frac{5}{8}$	$16\frac{7}{8}$	$28\frac{11}{12}$	$57\frac{1}{5}$	
$27\frac{9}{16}$	$94\frac{11}{24}$	$57\frac{8}{9}$	$37\frac{5}{6}$	$3\frac{1}{20}$	$12\frac{7}{8}$	(12)
$27\frac{2}{3}$	$10\frac{7}{10}$	$23\frac{7}{8}$	$38\frac{3}{7}$	$13\frac{2}{3}$	$390\frac{5}{12}$	
$75\frac{7}{9}$	$102\frac{13}{24}$	$598\frac{2}{15}$	$289\frac{3}{10}$	$63\frac{3}{8}$	$11\frac{5}{8}$	(24)

Practice Exercise 7a (page 342)

$\frac{4}{9}$	$\frac{12}{25}$	$\frac{1}{2}$	$\frac{2}{3}$	
$\frac{3}{11}$	$\frac{1}{4}$	$\frac{8}{33}$	$\frac{1}{16}$	(8)
$\frac{10}{21}$	$\frac{1}{3}$	$\frac{1}{6}$	$1\frac{1}{20}$	
$\frac{2}{5}$	$\frac{7}{12}$	$\frac{9}{64}$	$\frac{3}{7}$	(16)
$\frac{3}{25}$	$\frac{1}{5}$	$\frac{2}{63}$		
$\frac{8}{27}$	$1\frac{3}{4}$	$\frac{4}{5}$		(22)

Practice Exercise 7b (page 342)

$4\frac{1}{2}$	$1\frac{1}{2}$	$6\frac{2}{3}$	6	(4)
$3\frac{1}{2}$	$7\frac{1}{2}$	16	$6\frac{2}{3}$	
$11\frac{2}{3}$	$3\frac{1}{5}$	$2\frac{1}{4}$	$36\frac{2}{3}$	(12)
33	40	39	$172\frac{1}{2}$	
$37\frac{1}{2}$	$52\frac{1}{2}$	18	35	(20)

Practice Exercise 7c (page 343)

$7\frac{1}{3}$	15	$1\frac{7}{8}$	$2\frac{1}{4}$	
$15\frac{1}{3}$	12	25	$5\frac{1}{16}$	(8)
$10\frac{1}{2}$	$15\frac{5}{8}$	$18\frac{3}{4}$	$11\frac{1}{9}$	
5	$16\frac{1}{2}$	$1\frac{1}{2}$	5	(16)
$4\frac{1}{2}$	$2\frac{3}{4}$	$156\frac{1}{4}$	$56\frac{1}{4}$	
$7\frac{7}{24}$	$126\frac{2}{3}$	45	$11\frac{1}{4}$	(24)

Practice Exercise 8a (page 343)

$1\frac{1}{6}$	$2\frac{2}{3}$	$1\frac{1}{4}$	$5\frac{1}{3}$	
$\frac{7}{9}$	$1\frac{7}{9}$	1	6	(8)
$\frac{3}{4}$	$1\frac{1}{6}$	2	$\frac{8}{9}$	
$1\frac{1}{2}$	$1\frac{1}{4}$	1	$\frac{4}{5}$	(16)
$1\frac{1}{3}$	2	$\frac{14}{15}$	$\frac{3}{14}$	
$\frac{2}{3}$	$1\frac{1}{3}$	$\frac{7}{10}$	$1\frac{1}{6}$	(24)

Practice Exercise 8b (page 344)

36	7	$6\frac{3}{4}$	$\frac{11}{36}$	(4)
72	27	25	48	
$\frac{1}{3}$	$\frac{5}{48}$	$\frac{1}{18}$	$\frac{1}{14}$	(12)
$\frac{1}{24}$	$\frac{2}{27}$	$\frac{1}{24}$	$\frac{5}{192}$	
$\frac{1}{15}$	$\frac{1}{16}$	18	30	(20)

Practice Exercise 8c (page 344)

$\frac{14}{27}$	$\frac{3}{4}$	$3\frac{3}{4}$	$\frac{9}{10}$	(4)
$3\frac{3}{4}$	$1\frac{2}{3}$	$1\frac{1}{4}$	2	
$6\frac{2}{5}$	$3\frac{1}{2}$	8	$4\frac{3}{5}$	(12)
$3\frac{1}{5}$	$7\frac{1}{9}$	$7\frac{3}{7}$	2	
$1\frac{1}{2}$	$11\frac{1}{5}$	4	$6\frac{2}{85}$	(20)

Practice Exercise 9a (page 345)

1. 1.8	2. 20.7	3. 21.56	4. 25.097	5. 7.675
6. 63.06	7. 41.695	8. 9.612	9. 127.199	10. 19.375

Practice Exercise 9b (page 345)

1. 5.64	2. 24.30	3. 16.887	4. 2.928	5. 3.0
6. 10.46	7. 6.0	8. 102.97	9. 250.675	10. 872.87
11. 15.667	12. 38.806	13. 26.45 ($26\frac{9}{20}$)	14. $126.3\frac{7}{12}$ ($126.358\frac{1}{3}$)	15. $22.5\frac{2}{3}$
16. $12.68\frac{1}{2}$	17. 34.2	18. 103.008		

Practice Exercise 9c (page 345)

1. .42	2. 3.8	3. 2.3	4. 8.06	5. 6.079	6. .28	7. 2.131
8. 7.138	9. 14.83	10. 56.77	11. 23.824	12. 6.9	13. 17.982	14. 2.17
15. .61	16. 9.9752	17. .195	18. 757.04	19. .652	20. .3255	

Practice Exercise 9d (page 346)

1. 8.2	2. .31	3. 1.53	4. 16.9	5. 1.37	6. 6.6
7. 4.8	8. 1.545	9. 23.2	10. $58.8\frac{1}{2}$	11. 34.461	12. 574.62
13. 80.1	14. $4.2\frac{2}{3}$	15. 99.63	16. 3.268	17. 195.5	18. 149.632

Practice Exercise 10a (page 346)

1. .536	2. 1.53	3. .0268	4. .0084	5. 780
6. 6.01	7. 39,200	8. .14920	9. .268	10. .6813
11. 2.4192	12. .33258	13. 7.7688	14. 329.184	15. 14.6224
16. 70	17. .5535	18. .000008	19. .483	20. 78.6
21. 748	22. 2.8268	23. 21.528	24. 355.25	25. 25.086

Practice Exercise 10b (page 346)

1. 7.4	2. .042	3. 3150	4. .656	5. .00125	6. 6.24
7. 39	8. .19	9. .074	10. 650	11. 10	12. 1616
13. 9	14. 34.32	15. 6.25	16. 3.6	17. 13.9	18. .216
19. 10.404	20. 64.00	21. .06	22. 7200	23. 32.4	24. 234.56
25. .00072	26. .11025	27. .0121	28. .00762	29. 8.67	30. 4.2112

Practice Exercise 11a (page 347)

1. 1.8	2. 28	3. 160	4. .4	5. 20	6. 20	7. .02
8. .002	9. 200	10. 2	11. .02	12. .02	13. .02	14. .002
15. .0002	16. 200	17. .02	18. .02	19. 2	20. 20	

Practice Exercise 11b (page 348)

1. 4210	2. 42300	3. 10.7	4. .9	5. .469	6. $26.68\frac{4}{9}$	7. 28.1
8. 4	9. 78.7	10. .00004	11. .154	12. .03	13. 20.2	14. .004
15. 15.4	16. 4.03	17. 11.1	18. 4	19. 303	20. 720	

Practice Exercise 11c (page 348)

1. 274	2. 1.27	3. 3.43	4. 250	5. .034	6. 32.4
7. 50.9	8. 128	9. 286	10. 128.3	11. 31.07	12. .91
13. 7.8	14. 101	15. 67.4	16. .798	17. 4.69	18. .006
19. .0346	20. .005	21. .5678	22. .0005	23. .074	24. .5

Practice Exercise 12a (page 348)

1. 18%	2. 75%	3. 42%	4. 64%	5. 3%	6. 5%
7. 1%	8. 8%	9. $17\frac{1}{2}\%$	10. 25.6%	11. $62\frac{1}{2}\%$	12. $56\frac{1}{4}\%$
13. 80%	14. 10%	15. $72\frac{3}{4}\%$	16. 135%	17. 118%	18. $206\frac{1}{4}\%$
19. $312\frac{1}{2}\%$	20. 400%	21. 130%	22. 250%	23. 95%	24. $62\frac{1}{2}\%$
25. $133\frac{1}{3}\%$	26. $14\frac{1}{2}\%$	27. $31\frac{1}{4}\%$	28. 220%	29. .2%	30. $100\frac{1}{2}\%$
31. 16.6%	32. 108%	33. 115%	34. 20%	35. $12\frac{1}{2}\%$	36. $37\frac{1}{2}\%$
37. $162\frac{1}{2}\%$	38. 150%	39. $2\frac{1}{2}\%$	40. 225%		

Practice Exercise 12b (page 349)

1. 50%	2. 75%	3. 20%	4. 30%	5. 35%	6. $12\frac{1}{2}\%$
7. 24%	8. $33\frac{1}{3}\%$	9. $44\frac{4}{9}\%$	10. $16\frac{2}{3}\%$	11. $18\frac{3}{4}\%$	12. $87\frac{1}{2}\%$
13. $26\frac{2}{3}\%$	14. 2%	15. $14\frac{2}{7}\%$	16. $66\frac{2}{3}\%$	17. $57\frac{1}{7}\%$	18. $62\frac{1}{2}\%$
19. $83\frac{1}{3}\%$	20. 70%	21. $37\frac{1}{2}\%$	22. 60%	23. $42\frac{6}{7}\%$	24. $11\frac{1}{9}\%$
25. 150%	26. 240%	27. 125%	28. $131\frac{1}{4}\%$	29. $233\frac{1}{3}\%$	30. $166\frac{2}{3}\%$
31. 175%	32. 140%	33. $177\frac{7}{9}\%$	34. $41\frac{2}{3}\%$	35. $43\frac{3}{4}\%$	36. $71\frac{3}{7}\%$
37. $266\frac{2}{3}\%$	38. $3\frac{1}{3}\%$	39. 16%	40. $112\frac{1}{2}\%$		

Practice Exercise 12c (page 349)

1. .50	2. .75	3. .18	4. .20	5. .10	6. .02
7. .09	8. .07	9. .025	10. .0325	11. .0175	12. .125
13. .1875	14. .06	15. .01	16. $.16\frac{2}{3}$	17. .175	18. $.08\frac{1}{3}$
19. .0625	20. $.83\frac{1}{3}$	21. .625	22. .375	23. .005	24. .0075
25. 1.4	26. 2.5	27. 1.8	28. 2.35	29. $.06\frac{2}{3}$	30. .045
31. .03	32. .015	33. $.28\frac{4}{7}$	34. .015	35. .00375	36. .0185
37. .008	38. .0001	39. $1.66\frac{2}{3}$	40. 3.25	41. 1.25	42. .36
43. .045	44. .009	45. .0025	46. $.03\frac{1}{3}$	47. .0125	48. 1.60

Practice Exercise 12d (page 350)

1. $\frac{1}{5}$	2. $\frac{3}{20}$	3. $\frac{3}{4}$	4. $\frac{11}{20}$	5. $\frac{1}{20}$	6. $\frac{1}{50}$
7. $\frac{1}{100}$	8. $\frac{7}{100}$	9. $\frac{7}{200}$	10. $\frac{17}{400}$	11. $\frac{23}{400}$	12. $\frac{3}{800}$
13. $\frac{1}{200}$	14. $\frac{1}{500}$	15. $\frac{3}{800}$	16. $\frac{1}{125}$	17. $\frac{3}{500}$	18. $\frac{2}{25}$
19. $\frac{1}{1000}$	20. $\frac{1}{200}$	21. $\frac{1}{700}$	22. $\frac{1}{225}$	23. $\frac{1}{150}$	24. $\frac{1}{120}$
25. $\frac{1}{400}$	26. $\frac{7}{250}$	27. $\frac{1}{40}$	28. $\frac{1}{80}$	29. $\frac{53}{1000}$	30. $\frac{13}{400}$
31. $\frac{5}{32}$	32. $\frac{49}{200}$	33. $2\frac{1}{2}$	34. $1\frac{4}{5}$	35. $1\frac{3}{5}$	36. 2
37. $3\frac{3}{4}$	38. $2\frac{1}{4}$	39. $1\frac{3}{4}$	40. $2\frac{1}{8}$	41. $\frac{9}{20}$	42. $\frac{9}{50}$
43. $1\frac{1}{2}$	44. $\frac{1}{40}$	45. $2\frac{2}{5}$	46. $\frac{1}{800}$	47. $\frac{3}{1000}$	48. $\frac{9}{400}$

Practice Exercise 12e (page 351)

Common Fraction	Decimal Fraction	Per Cent	Common Fraction	Decimal Fraction	Per Cent
1. $\frac{9}{10}$	.9	90%	11. $\frac{1}{7}$	$.14\frac{2}{7}$	$14\frac{2}{7}\%$
2. $\frac{3}{5}$	.6	60%	12. $\frac{5}{6}$	$.83\frac{1}{3}$	$83\frac{1}{3}\%$
3. $\frac{4}{25}$	.16	16%	13. $\frac{1}{8}$	.125	$12\frac{1}{2}\%$
4. $\frac{9}{100}$	.09	9%	14. $\frac{7}{9}$	$.77\frac{7}{9}$	$77\frac{7}{9}\%$
5. $\frac{1}{25}$	.04	4%	15. $1\frac{1}{4}$	1.25	125%
6. $\frac{3}{25}$	.12	12%	16. $\frac{7}{40}$	.175	$17\frac{1}{2}\%$
7. $\frac{7}{8}$	$.87\frac{1}{2}$	$87\frac{1}{2}\%$	17. $\frac{8}{25}$	.32	32%
8. $1\frac{1}{2}$	1.5	150%	18. $1\frac{3}{4}$	1.75	175%
9. $1\frac{3}{10}$	1.3	130%	19. $\frac{1}{100}$	.01	1%
10. $\frac{9}{200}$	.045	$4\frac{1}{2}\%$	20. $\frac{1}{250}$	.004	$\frac{2}{5}\%$

Practice Exercise 12f (page 351)

1. 40%	2. $\frac{3}{10}$	3. $\frac{7}{8}$	4. $8\frac{1}{3}\%$	5. $\frac{1}{6}$	6. $\frac{1}{8}$
7. 350%	8. $6\frac{1}{4}\%$	9. $77\frac{7}{9}\%$	10. $\frac{2}{3}$	11. $\frac{1}{7}$	12. $62\frac{1}{2}\%$
13. $33\frac{1}{3}\%$	14. 80%	15. $\frac{7}{10}$	16. $\frac{5}{6}$	17. $28\frac{4}{7}\%$	18. $\frac{1}{40}$

Practice Exercise 13a (page 352)

1. 1.8	2. 17	3. 10.2	4. 40.29	5. 8.76	6. 16.5	7. 3.6	8. 1.92
9. 6	10. 9	11. 14.4	12. 100	13. .5	14. 2	15. 1.6	16. .06

17. 600	**18.** 350	**19.** 7.5	**20.** 164	**21.** 16.8	**22.** 11.7	**23.** .5	**24.** .3
25. 20.4	**26.** 162	**27.** 395	**28.** 1	**29.** 5.8	**30.** 1.05		

Practice Exercise 13b (page 352)

1. 12	**2.** 6	**3.** 12	**4.** 14	**5.** 50	**6.** 35	**7.** 21	**8.** 15
9. 3	**10.** 100	**11.** 80	**12.** 2	**13.** 105	**14.** 3	**15.** 14	**16.** 9
17. 8	**18.** 2	**19.** 10	**20.** 243	**21.** 18	**22.** 30	**23.** 60	**24.** 6
25. 12	**26.** 77	**27.** 20	**28.** 110	**29.** 48	**30.** 54		

Practice Exercise 13c (page 353)

1. 100%	**2.** $66\frac{2}{3}\%$	**3.** 5%	**4.** 50%	**5.** 200	**6.** 400	**7.** 300%
8. $33\frac{1}{3}\%$	**9.** 25%	**10.** $62\frac{1}{2}\%$	**11.** 14%	**12.** 200	**13.** 25	**14.** 4200
15. 4400	**16.** 200%	**17.** $33\frac{1}{3}\%$	**18.** 80	**19.** 60	**20.** 36	**21.** 9000
22. $18\frac{3}{4}\%$	**23.** 4%	**24.** 250%	**25.** 300%	**26.** 30%	**27.** $12\frac{1}{2}\%$	**28.** 25%
29. $66\frac{2}{3}\%$	**30.** 60%	**31.** 60%	**32.** $71\frac{3}{7}\%$	**33.** 400	**34.** 100%	**35.** 200
36. 225%	**37.** 28.8	**38.** $14\frac{2}{7}\%$	**39.** 22	**40.** $6\frac{1}{4}\%$	**41.** $8\frac{1}{3}\%$	**42.** $33\frac{1}{3}\%$
43. 70	**44.** 14%	**45.** 3200				

Practice Exercise 13d (page 353)

1. 7.5	**2.** $16\frac{2}{3}\%$	**3.** 60	**4.** 62	**5.** 30%	**6.** 40%	**7.** 25%
8. 3.5	**9.** 28	**10.** 75	**11.** 20%	**12.** 4	**13.** 20	**14.** 27
15. .2	**16.** .21	**17.** 150	**18.** 20	**19.** 80%	**20.** 25%	**21.** 250%
22. 2%	**23.** 40	**24.** $66\frac{2}{3}\%$	**25.** 100	**26.** 9	**27.** $16\frac{2}{3}\%$	**28.** 10,000
29. 12	**30.** 36					

Practice Exercise 14a (page 354)

1. 32 in.	**2.** 9 ft.	**3.** 6 pt.	**4.** 20 qt.	**5.** 54 oz.
6. 8 pk.	**7.** 72 in.	**8.** 4000 lb.	**9.** $1\frac{1}{4}$ doz.	**10.** 135 min.
11. 480 sec.	**12.** 432 sq. in.	**13.** 18 sq. ft.	**14.** 54 cu. ft.	**15.** 135 cu. ft.
16. 1280 acres	**17.** 693 cu. in.	**18.** 26 qt.	**19.** 9 in.	**20.** 45 min.
21. 6 oz.	**22.** 2640 ft.	**23.** 500 lb.	**24.** 1 pt.	**25.** 24 in.
26. 8 in.	**27.** 2 pk.	**28.** 12 min.	**29.** 36 sq. in.	**30.** 2 qt.
31. 2 ft.	**32.** 40 min.	**33.** 9 in.	**34.** 1 pk.	**35.** 1 pt.
36. 14 oz.				

Practice Exercise 14b (page 355)

1. 1 ft. 10 in.	**2.** 2 yd. 1 ft.	**3.** 1 hr. 30 min.	**4.** $2\frac{1}{2}$ gal.	**5.** $7\frac{1}{2}$ qt.
6. $1\frac{1}{2}$ lb.	**7.** $1\frac{2}{3}$ hr.	**8.** $1\frac{1}{2}$ sq. ft.	**9.** $1\frac{1}{4}$ min.	**10.** $6\frac{1}{4}$ ft.
11. $1\frac{1}{2}$ gal.	**12.** 3 tons	**13.** 4 sq. yd.	**14.** 2 cu. yd.	**15.** $\frac{3}{4}$ doz.
16. $1\frac{1}{8}$ lb.	**17.** $\frac{3}{4}$ ft.	**18.** $\frac{3}{4}$ gal.	**19.** $\frac{2}{3}$ doz.	**20.** $\frac{3}{4}$ yd.
21. $\frac{1}{4}$ hr.	**22.** $1\frac{1}{2}$ yd.	**23.** $\frac{3}{8}$ lb.	**24.** $\frac{5}{6}$ doz.	

Practice Exercise 14c (page 355)

1. 54 in.	2. 88 oz.	3. 7 pt.	4. $1\frac{1}{4}$ lb.	5. 45 in.
6. 8 ft. 4 in.	7. 135 min.	8. $2\frac{1}{2}$ min.	9. 7 pk.	10. 21 articles
11. 9 mo.	12. 30 ft.	13. 48 oz.	14. 2 bu. 3 pk.	15. $1\frac{2}{3}$ doz.
16. 28 articles	17. 10 oz.	18. 3 tons 1500 lb.	19. 480 acres	20. $\frac{1}{2}$ pk.

Practice Exercise 15a (page 356)

1. 7 ft. 9 in.	2. 10 yd.	3. 12 yr.
4. 6 gal.	5. 56 min. 10 sec.	6. 14 yd. 2 ft. 11 in.
7. 1 da. 11 hr. 35 min.	8. 42 min. 59 sec.	9. 119 lb. 6 oz.
10. 39 gal. 3 qt.	11. 13 hr. 25 min.	12. 156 mi. 37 rd.
13. 14 lb. 2 oz.	14. 13 qt.	15. 22 yd. 1 ft.

Practice Exercise 16a (page 356)

1. 1 bu. 2 pk. 6 qt.	2. 1 da. 8 hr. 40 min.	3. 18 T. 600 lb.
4. 53 yd. 1 ft.	5. 4 gal. 2 qt.	6. 3 yd. 2 ft.
7. 1 hr. 41 min. 20 sec.	8. 17 mi.	9. 130 yd. 2 ft.
10. 85 qt. 1 pt.	11. 20 yd. 2 ft. 5 in.	12. 148 bu. 2 pk.
13. 91 mi. 160 rd.	14. 8 hr. 47 min. 30 sec.	15. 58 yd. 1 ft.

Practice Exercise 17a (page 357)

1. 3′ 10″	2. 2 pk. 6 qt.	3. 2 T. 1200 lb.
4. 29 min. 50 sec.	5. 1 pk. 5 qt.	6. 5 ft. 5 in.
7. 28 pk. 4 qt.	8. 4 T. 1800 lb.	9. 37 min. 43 sec.
10. 14 yd. 2 ft.	11. 12 hr. 48 min.	12. 21 yr. 2 mo. 17 da.
13. 1 hr. 9 min. 38 sec.	14. 7 gal. 2 qt. 1 pt.	15. 34 yr. 9 mo. 25 da.

Practice Exercise 18a (page 357)

1. 2 hr. 6 min.	2. 2 pk. 1 qt. $\frac{1}{2}$ pt.	3. 4 min. $1\frac{2}{3}$ sec.	4. 6 ft. $2\frac{2}{3}$ in.
5. 2 qt. $1\frac{1}{2}$ pt.	6. 1 yd. 1 ft. 8 in.	7. 1 ft.	8. 4 yd. 2 ft.
9. 4 ft. $3\frac{2}{5}$ in.	10. 5 mi. $246\frac{2}{3}$ rd.	11. 10 min. 31 sec.	12. 2 bu. 3 pk.
13. 2 ft. $10\frac{1}{3}$ in.	14. $\frac{3}{5}$ pt.	15. 1 mi. 205 rd.	

INDEX

(References are to pages.)

A

Absolute value, 318
Achievement Tests, procedure for, 76
(*see* Testing and improvement program)
Acute angle, 100
Acute triangle, 268
Addition:
  common fractions, 23, 77, 78, 123, 125, 201, 273, 338, 339, 340
  decimal fractions, 35, 77, 78, 123, 125, 201, 273, 344, 345
  denominate numbers, 63, 77, 78, 123, 125, 201, 273, 356
  integers, 5, 77, 78, 123, 125, 201, 273, 331, 332
  of non-ruler fractions, 25
  of ruler fractions, 23
  proving, 6
  signed numbers, 321, 323, 325
  terms of, 5
Ad valorem duty, 213
Algebra:
  addition of signed numbers, 321, 323, 325
  equations, 302–318
  formulas, making, 289, 292
  graphing, 297, 327
  subtraction of signed numbers, 326
Algebraic expressions:
  evaluating, 291
  symbols, 283, 314
  terms, 285
Aliquot parts, 40
Altitude, 239, 259
Angles:
  acute, 100
  base, 268
  bisector of, 271
  drawing of, 102
  kinds of, 100
  lettering, 99
  measuring, 99, 101
  obtuse, 100
  reading, 99
  reflex, 100
  right, 100
  straight, 100
  sum of, 269
  symbol of, 99
  vertex of, 99
  vertical, 101
Annuities, 133
Arc, 230
Area, 236
  lateral, 243
  meaning of, 236
  of circle, 254
  of parallelogram, 244
  of rectangle, 12, 239
  of square, 242
  of trapezoid, 247
  of triangle, 245
  units of, 236
Arithmetic, beginnings of, 1
Arithmetic, plan of improvement in fundamentals of, 2
Arranging numbers in a series according to size, 34
Assessed valuation, 205
Automobile insurance, 142
Averages, 17

B

Balancing the cash record, 180
Banks, 145
  checkbook, 149
  checking account, 145
  commercial, 145
  deposit ticket, 146
  discount, 190
  loans by, 190
  personal check, 150
  savings account, 158
  signature card, 146
  statement, 152
Bar graph, 111–113
Base, 239, 259
Base angles, 268
Beginnings of arithmetic, 1
Beginnings of measurement, 81
Beneficiary, 128
Bisection:
  of angles, 271
  of lines, 270
Bisector, perpendicular, 270–271
Bonds, 165
  corporation, 165
  U. S. Government, 166
Borrowing money, 190
Brain teasers, 364–366
  answers to, 366–367
Broken-line graph, 107

Broker, 168
Brokerage, 168
Budget, 173–177, 206
family, 176 (*table*), 177
government, 206
making of a, 173, 176
pupil, 173
Building and loan association, 163

C

Calculating grades, 55
Canceled check, 152
Capital stock, 168
Cash record, 177
balancing, 180
making, 178
Change, making, 197
Check:
book, 149
canceled, 152
indorsement, 151
personal, 150
stub, 149
writing, 150
Checkbook stub, 149
Circles:
arc of, 118, 230
area of, 254
circumference of, 226, 250
concentric, 229
definition, 226
designs, 230
diameter of, 226, 250
drawing of, 227
graphs, 107, 117
radius, 226
sector, 118
Circumference, 226, 250
Clothing, thrift in buying, 185
Coefficient, 285
Coinsurance clause, 139
Collision insurance, 142
Common fractions, changing form of, 20
Common stock, 168
Compasses, use of:
to bisect a line, 270
to bisect an angle, 271
to draw circles, 227
to erect perpendicular, 270, 272
Compound interest, 159 (*table*), 161
Concentric circles, 229
Cone, 232
frustum of, 232
volume of, 265, 266
Construction, 268–273
bisect a line, 270
bisect an angle, 271
of perpendicular lines, 272, 273
Consumer:
borrowing, 190
buying, 181–190
Corporation, 165
Credit and installment buying, 187
Cube, 231
Cubic measure, 260
Cubit, 83
Currency, 146
Cylinder, 232
lateral area of, 258
volume of, 264

D

Day of maturity, 191
Decagon, 225
Decimal fractions:
in shop measurements, 36
reading and writing, 32
to common fractions, 33
to per cents, 49, 348, 349
Denominate numbers, 62–67, 354–358
Deposit ticket:
for commercial bank, 146
for savings bank, 159
Depreciation, 196
Designs:
border, 221, 230
circle, 230
plane figure, 227
straight line, 221
Diameter, 250
Digit, 1
Direct measurement, 82–103
Direct tax, 210
Discount, 190
Dissimilar terms, 285
Dividend:
insurance, 130
stock, 168
Division:
estimates, 43
of common fractions, 30, 77, 79, 123, 126, 202, 273, 343, 344
of decimal fractions, 42, 77, 79, 123, 126, 202, 273, 347, 348
of denominate numbers, 67, 77, 79, 123, 126, 202, 273, 358
of integers, 16, 77, 79, 123, 126, 202, 273, 337, 338
proving, 17
terms of, 16
Drawee, 150
Drawer, 150
Duty, customs, 213
ad valorem, 213
specific, 213

E

Endowment life policy, 132–133
Equations:
  forming, 303–315
  meaning of, 302
  members of, 302
  solving, 308–316
  terms used in, 302
  writing, 303
Equilateral triangle, 268
Equity, 194
Estimating results, 43, 50
Evaluating algebraic expressions, 291
Excise tax, 215
Expenditures, plan for, 181–185

F

Face of note, 191
Face of policy, 128
Factor:
  literal, 285
  numerical, 285
Federal income tax, 209
Figure:
  plane, 223–226
  regular, 226
  solid, 231
Fire insurance, 139, 142
  coinsurance clause, 139
Food, thrift in buying, 182
Formula, 239
  area of:
    circle, 255
    parallelogram, 244
    rectangle, 240
    square, 242
    trapezoid, 248
    triangle, 246
  finding the value of a, 294
  graphing a, 297
  lateral area:
    of cylinder, 258
    of a rectangular prism, 243
  making a, 292
  volume of:
    cone, 265
    cylinder, 264
    pyramid, 265
    rectangular prism, 260
  writing a, 283
Fractions:
  addition of, 23, 35, 338–340, 344, 345
  changing common fractions to per cent, 349, 351
  changing the form of, 20
  decimal:
    reading and writing, 32
Fractions (*cont.*):
  decimal (*cont.*):
    to common fractions, 33, 351
    to per cents, 49, 348, 349, 351
  definition of, 20
  division of, 30, 42, 343, 344, 347, 348
  multiplication of, 28, 38, 39, 40, 41, 342, 343, 346
  non-ruler, 25
  ruler, 23
  subtraction of, 25, 35, 340, 341, 345, 346
Frustum:
  of cone, 232
  of pyramid, 233
Fundamentals of arithmetic, 1–43
  (*see* Testing and improvement program)

G

Glossary, 359–363
Grades, calculating, 55
Graphs:
  bar, 106, 111, 112
  broken line, 107
  circle, 107, 117
  formulas, 297
  interest, 157
  of signed numbers, 327
  pictograph, 104, 109
  straight line, 115
  uses of, 108
Group life insurance, 133

H

Health insurance, 136
Hemisphere, 233
Hexagon, 225
Horizontal line, 218
Hospital insurance, 136
Hypotenuse, 280

I

Import duty, 134
Income taxes, 209
Indirect measurement, 275
  by right triangles, 280
Indirect tax, 215
Indorsement, 151
Installment buying, 187
Insurance:
  accident, 131
  annuities, 133
  automobile, 142
  beneficiary, 128
  coinsurance clause, 139
  collision, 142
  dividend, 130
  endowment, 132

Insurance (*cont.*):
fire, 139, 142
group, 133
health, 136
hospital, 136
kinds of, 128
liability, 142
life, 129
limited payment, 131
need for, 128
non-participating, 130
ordinary life, 131
participating, 130
policy, 128
premium, 129
property damage, 142
Social Security, 136
terms of, 128
theft, 142
workmen's compensation, 136
Interest:
compound, 159
graph, 157
loans, 190
simple, 155
six per cent method, 155
tables, 161
Inventory tests (*see* Testing and improvement program)
Investments, 164
Isosceles triangle, 268

K

Known number, 302

L

Lateral area, 243
of cylinder, 258
of rectangular prism, 243
Lateral faces, 243
Latitude, 222
Left member of equation, 302
Life insurance, 129–136
annuities, 133
benefits of, 129
dividends, 130
endowment, 132, 133
group, 133
investing in, 170
kinds of, 131
limited payment, 131
ordinary life, 131
premium, 129
rates (*table*), 135
Limited payment life insurance, 131
Lines:
bisector of, 270
designs, 221
Lines (*cont.*):
graph, 107, 114
horizontal, 218
parallel, 219
perpendicular, 220
slanting, 218
vertical, 218
Literal factor, 285
Loans, bank, 190
Longitude, 222

M

Making change, 197
Map reading, 221
Mathematics, importance of, 2
Maturity, day of, 191
Measurement:
angles, 101
cubit, 81, 83
digit, 82, 83
direct, 82–103, 275
foot, 85
history of, 81
inch, 82, 85
indirect, 275, 280
metric system, 89
of angles, 101
of diameters, 253
of lines, 85
origin of, 81
palm, 85
rod, 86
ruler, 83
scale, 83
with protractor, 101
yard, 82, 86
Meridian, 221
Metric system, 89–93
Mortgage, 194
Multiplication:
of common fractions, 28, 29, 77, 79, 123, 126, 199, 202, 273, 342, 343
of decimal fractions, 38, 39, 40, 41, 77, 79, 123, 126, 199, 202, 273, 346
of denominate numbers, 64, 77, 79, 126, 199, 202, 273, 356
of integers, 10, 77, 79, 123, 126, 199, 202, 273, 333, 334
proving, 11–12
special types of, 29, 30, 40, 41
terms of, 10

N

Negative numbers:
addition of, 321, 323, 325
changes indicated by, 319
graphing of, 327
meaning of, 317

Negative numbers (*cont.*):
  opposites indicated by, 318
  subtraction of, 326
Non-participating policy, 130
Note, 191
Numbers, arranging series according to size, 34
Numerical factor, 285

O

Obtuse angle, 100
Obtuse triangle, 268
Octagon, 225
Ordinary life policy, 131

P

Palm, 85
Parallel lines, 219
Parallelogram, 224, 244
Parallels, map, 222
Participating policy, 130
Par value, 165
Pass book for savings account, 160
Payee, 150
Pentagon, 225
Per cent:
  exercises, 351–354
  meaning of, 48
  to common fractions, 48, 350, 351
  to decimal fractions, 48, 349, 350, 351
  use of, 49
Percentage and problem solving, 53–58
Perimeter, 7, 237
Perpendicular lines, 220
  to erect, 270–272
Personal check, 150
Personal records:
  balancing the cash record, 180
  budget, 173–177, 206
  cash record, 177
Pi, 251
Pictograph, 104, 109
Plane figures, 223–229
  in designs, 227
  size of, 236–259
Policy, insurance, 128
  non-participating, 130
  participating, 130
Position:
  lines, 218–220
  places on a map, 221
Positive numbers:
  addition of, 321, 323, 325
  changes indicated by, 319
  graphing of, 327
  meaning of, 317
  opposites indicated by, 318
  subtraction of, 326
Practice exercises in fundamentals of arithmetic, 331–358 (*see also* Testing and improvement program)
  answers to, 372–380
Preferred stock, 168
Premium, 129
Prime meridian, 221
Prisms, 231, 232
Problem solving, 51–58
Proceeds, 191
Promissory note, 191
Proofs in arithmetic:
  addition, 6
  division, 17
  multiplication, 11–12
  subtraction, 14
Property damage insurance, 142
Property taxes, 206
Proportion, 71, 72
Protractor, 100
  drawing an angle with, 102
  measuring an angle with, 101
  studying the, 100
Pyramid, 232–233
  frustum of, 233
  volume of, 265, 266

Q

Quadrilateral, 223

R

Radical sign, 276
Radius, 226
Ratio, 68, 251
  and proportion, 68–76
  meaning of, 68
  use of, 69
Reading:
  decimal fractions, 32
  maps, 221
Recipes, changing amounts of, 28
Reconciliation, 153
Rectangle, 224
  area of, 239
  perimeter of, 237
Rectangular prism, 231
  lateral area of, 243
  total area of, 243
  volume of, 260, 261
Remedial program, fundamentals of arithmetic, 331–358
  (*see* Testing and improvement program)
Renting or owning a home, 193–195
Review of fundamentals of arithmetic (*see* Testing and improvement program)
Right angle, 100
Right member of equation, 302
Right triangle, 268

Rod, 86
Rounding off numbers, 44
Ruler:
to draw a line with, 83, 88
to measure with, 83, 87

S

Sales tax, 208
Saving:
for a purpose, 158
in a building and loan association, 163
in life insurance, 170
in savings bank, 158
Savings and loan association, 163
Savings bank, 158
Scale drawing, 94–98
Sector, 118
Shareholder, 168
Shares, 168
Short cuts, 39–42
Signature card, 146
Signed numbers:
addition of, 321, 323, 325
changes indicated by, 319
graphing of, 327
meaning of, 317
opposites indicated by, 318
subtraction of, 326
Similar terms, 285, 286
Six per cent method, 155
Slanting lines, 218
Social Security, 136
Solid figure, 231
in buildings and nature, 231–233
Span, 83
Special types of multiplication, 29, 30, 40, 41
Specific duty, 213
Sphere, 233
Square (of a quantity), 275
table of, 277
Square (plane figure), 224, 242
Square root, 275, 276
table of, 277
uses of, 279
Statement, bank, 152
State sales tax, 208
Steel tape, 89
Stock, shares of, 168
Stockbroker, 168
Stockholder, 168
Straight angle, 100
Straight line:
designs, 221
graph, 114, 115
Stub, check, 149
Subtraction:
of common fractions, 25, 77, 78, 123, 125, 199, 201, 273, 340, 341
Subtraction (*cont.*):
of decimal fractions, 35, 77, 78, 123, 125, 199, 201, 273, 345, 346
of denominate numbers, 66, 77, 78, 123, 125, 199, 201, 273, 357
of integers, 13, 77, 78, 123, 125, 199, 201, 273, 335, 336
of signed numbers, 326
proving, 14
terms of, 13
Symbol:
algebraic, 283
for angles, 99
for triangles, 268

T

Tables:
aliquot parts, 371
decimal equivalents, 371
formulas, 370
interest, 161
premium rates, life insurance, 135
square roots, 275–282
squares, 277
useful, 369
weights and measures, 369
Tapeline, 89
Tariff, 213
Tax, 204–217
ad valorem duty, 213
deductions, 211
direct, 210
excise, 215
exemptions, 211
import duties, 213
income, 209
indirect, 215
kinds, 204
personal property, 205
property, 205–206
rate of, 206
real estate, 205
special, 215
state sales, 208
why necessary, 204
Terms:
algebraic, 285
dissimilar, 285
of credit, 187
similar, 285
Testing and improvement program:
Achievement Tests:
all processes, 77, 123, 199, 273
procedure for, 76
Inventory Tests:
common fractions:
addition of, 23
division of, 30

Testing & improvement program (*cont.*):
  Inventory Tests (*cont.*):
    common fractions (*cont.*):
      multiplication of, 28
      subtraction of, 25
    decimal fractions:
      addition and subtraction of, 35
      division of, 42
      multiplication of, 38
    integers:
      addition, 5
      division, 16
      multiplication, 10
      subtraction, 13
    denominate numbers:
      addition of, 63
      division of, 67
      multiplication of, 64
      relationships, 62
      subtraction of, 66
    per cent, 49, 50
  plan of, 2
  Practice Exercises, 331–358
  Review of Fundamentals:
    addition and subtraction, 78, 125, 201
    multiplication and division, 79, 126, 202
    percentage, 80, 126, 202
Theft insurance, 142
Thrift in buying:
  clothing, 185
  food, 182
Total area, 243
Trapezoid, 225
  area of, 247
Triangles, 225
  acute, 268
  altitude of, 239
  area of, 245
Triangles (*cont.*):
  base of, 239
  equilateral, 268
  isosceles, 268
  kinds of, 268
  obtuse, 268
  right, 268
  rigidity of, 270
  sum of angles of, 269
  symbol for, 268
  use of, 270
Triangular prism, 232

U

Unit:
  of area, 236
  of length, 237
  of volume, 260
Unknown number, 302

V

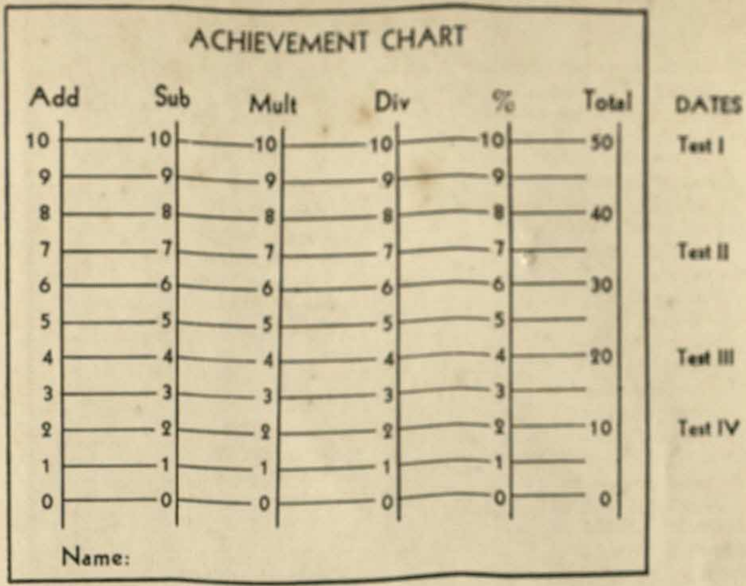
Vertex of angle, 99
Vertical angles, 101
Vertical lines, 218
Volume:
  of cone, 265, 266
  of cylinder, 264
  of pyramid, 265, 266
  of rectangular prism, 260, 261
  of solid figures, 259, 260
  unit of, 260

W

Workmen's compensation, 136
Writing a check, 150

Y

Yardstick, 89


ACHIEVEMENT CHART
Add
Sub
Mult
Div
%
Total
DATES
Test I
Test II
Test III
Test IV
10 9 8 7 6 5 4 3 2 1 0
50 40 30 20 10 0
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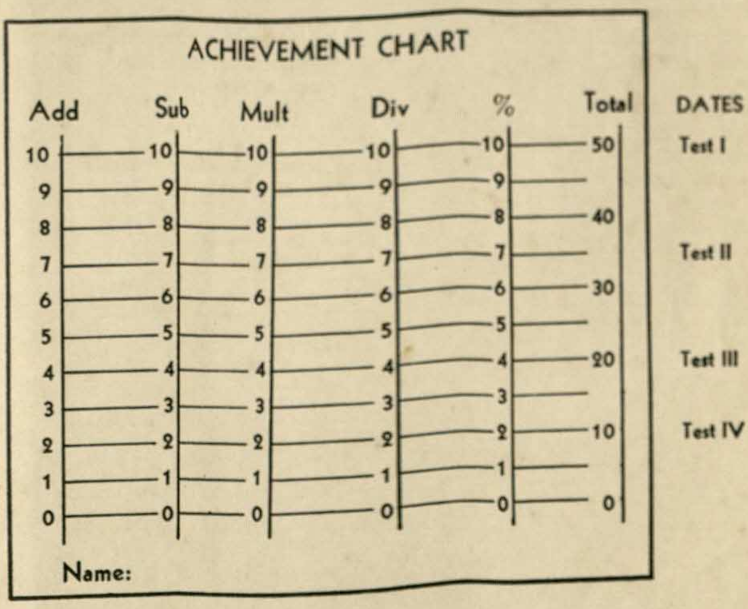
ACHIEVEMENT CHART
Add
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Div
%
Total
DATES
Test I
Test II
Test III
Test IV
10 9 8 7 6 5 4 3 2 1 0
50 40 30 20 10 0
Name: